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Journal of the Royal Statistical Society

SERIES A (GENERAL)

PART I, 1949

Some Statistical Aspects of Road Safety Research

By R. J. Smeed, Ph.D.

(Department of Scientific and Industrial Research, Road Research Laboratory)

[Read before the Royal Statistical Society, November 11th, 1948, the President, Dr. David Heron, in the Chair]

(1) *Introduction*

The Road Research Laboratory of the Department of Scientific and Industrial Research has recently had its terms of reference expanded to include research into road safety. Statistical investigations are obviously fundamental in this work, in indicating the paths in which the research efforts should be directed and in assessing the effects of the resulting actions taken. The aim of this paper is to survey some aspects of road accident statistics, and to draw attention to some of the matters into which investigation is required. It will be realized that the statistical side constitutes only a small proportion of the work of the Laboratory on road safety, its main value being its basic importance to the other sections of the Laboratory.

I think it will be helpful to start by putting the subject into its broader perspective, and will, therefore, begin by comparing the accident rates in different countries. I then propose to consider the trend in the accident rate in this country, the economic cost of road accidents, and some of the available evidence of the success of various accident prevention methods used.

(2) *Comparison of National Accident Rates*

The statistical year-books of various countries give the number of fatalities due to road accidents, the number of mechanically propelled vehicles licensed and the population. Unfortunately, these figures are not always strictly comparable owing to slight differences in the terminology used. For example, in some countries accidents between trains and road vehicles are not classed as road accidents; in others, the natives are excluded, e.g. Maoris in New Zealand. From data given in those published year-books for 1930 onwards, contained in the Library of this Society, and, for Great Britain, from data taken from other official sources, the number of fatalities per 100,000 population and per 10,000 registered motor vehicles have been calculated. The results, together with the number of registered vehicles in each country, are given in Tables 1, 2 and 3. It will be seen that both accident rates vary erratically, but that certain features are consistent:

(i) The number of registered vehicles in all countries increased fairly steadily or changed only slightly until the outbreak of war.

(ii) There were violent fluctuations in the accident rate in many of the countries during the war period.

(iii) If the accident rate in any one of the countries was higher than that of another in any one year of the period 1930–38, it tended to be higher in all the years of the period.

(iv) The U.S.A. has the highest fatality rate calculated on the basis of the population, but almost the lowest when calculated on the basis of the number of vehicles.

(v) Australia and Great Britain have the next highest accident rates on the basis of the population. Australia's rate is fairly low when based on the number of vehicles, Great Britain's rate is fairly high.

TABLE 1.—*Number of Registered Motor Vehicles*

Country	1930	1931	1932	1933	1934	1935	1936
Great Britain	2,274	2,201	2,227	2,285	2,405	2,507	2,758
Northern Ireland	36	36	37	38	40	42	48
Eire	55	57	57	57	54	58	61
United States	26,719	25,986	24,295	24,104	25,223	26,515	28,523
Australia	604	588	617	645	688	743	791
Canada	1,232	1,201	1,114	1,083	1,130	1,176	1,240
South Africa	185	188	193	205	242	273	312
New Zealand	195	199	196	193	197	210	228
Denmark	134	145	141	144	151	157	163
Finland	42	42	40	38	39	41	43
Norway	54	58	59	64	68	74	82
Sweden	202	205	199	192	197	205	218
Belgium	211	232	246	255	263	267	278
France	1,962	2,178	2,214	2,397	2,564	2,694	2,791
Netherlands	175	191	204	214	221	227	234
Italy	..	390	418	456	510	546	570
Germany	..	..	..	..	..	..	..
Portugal	..	..	37	38	..	..	49
Spain	..	..	..	..	..	..	..
Switzerland	124	130	..	118	125	124	119

TABLE 2.—*Road Deaths per* 100,000

Country	1930	1931	1932	1933	1934	1935	1936
Great Britain	16·4	14·9	14·8	16·0	16·2	14·3	14·3
Northern Ireland	9·2	9·1	9·4	11·1	10·4	9·7	10·5
Eire	6·0	6·2	6·7	6·1	6·4	7·6	6·9
United States	26·7	27·2	23·6	25·0	28·6	28·6	29·7
Australia	..	14·1	12·5	13·8	14·3	16·4	20·0
Canada	12·6	12·7	10·7	9·0	10·4	11·3	12·0
South Africa (Union)	..	..	..	..	..	10·4	10·6
New Zealand	15·4	11·0	10·9	8·2	10·3	11·2	13·3
Denmark	6·6	6·1	5·2	5·3	6·8	8·0	8·5
Finland	..	3·2	3·0	4·3	4·2	3·9	5·4
Norway	4·3	4·5	3·3	3·6	3·5	4·0	3·9
Sweden	4·5	5·7	6·2	5·3	5·4	5·6	7·0
Belgium	5·8	4·8	4·5	4·1	4·4	7·4	7·5
France	9·7	9·8	10·0	10·8	11·3	10·5	11·0
Netherlands	8·2	8·8	8·8	9·7	9·9	10·1	..
Italy	4·7	5·0	5·7	6·6	7·5	7·9	5·4
Germany	..	..	..	..	..	..	12·5
Portugal	..	..	..	..	..	..	..
Spain	8·1	8·5	7·0	6·6	5·8	5·3	6·2
Switzerland	10·2	9·8	11·2	12·3	15·1	14·5	15·1

in Various Countries, 1930–46 (*Thousands*)

1937	1938	1939	1940	1941	1942	1943	1944	1945	1946
2,929	3,085	3,149	2,325	2,478	1,840	1,537	1,593	2,553	3,041
52	55	60	49	56	48	41	43	62	67
65	70	74	91	67	52	26	27	34	64
30,041	29,853	..	..	..	..	..	..	..	..
857	900	895	872	..	..	..	..	..	..
1,320	1,395	1,439	1,501	1,573	1,524	1,512	1,503	..	..
352	376	..	..	..	..	..	..	..	..
255	274	299	306	300	292	287	277	284	295
174	180	192	32	34	33	31	30	81	177
49	56	61	..	..	..	..	..	..	..
95	111	123	106	101	101	99	..	..	..
236	263	298	121	81	85	85	92	135	317
296	..	..	..	56	46	43	54	151	158
2,821	2,863	..	..	..	..	..	..	..	..
247	265	295	..	..	..	..	..	..	..
615	666	..	..	..	..	..	..	..	..
..	3,403	..	..	..	..	..	..	..	..
52	54	56	67	67	57	52	..	..	..
182	..	..	..	..	..	..	..	..	..
121	124	127	102	39	41	42	43	46	121

Population for Various Countries, 1930–46

1937	1938	1939	1940	1941	1942	1943	1944	1945	1946
14·4	14·4	18·2	19·3	21·0	16·1	13·6	15·1	12·4	11·0
9·8	9·1	9·7	13·9	21·0	17·7	11·7	11·6	9·3	8·6
7·3	7·7	6·5	6·9	5·2	5·9	3·7	4·4	3·9	5·6
30·8	25·1	24·7	26·1	30·0	21·1	17·8	18·3	21·3	24·1
20·4	21·6	20·5	22·0	18·5	18·1	10·6	..	..	..
14·9	13·9	14·1	15·0	16·1	12·1	12·2	11·5	12·7	..
10·7	10·8	9·5	8·8	8·5	7·1	5·9	5·5	..	..
12·9	15·0	13·9	12·9	11·4	10·8	9·4	..	..	..
7·1	8·8	8·4	5·3	4·5	4·7	4·6	4·0	..	7·5
7·0	7·0	5·3	5·8	5·9	6·3	6·9	6·5	..	..
3·9	6·1	4·6	4·7	6·4	..	..	..	..	..
7·5	9·3	9·1	4·1	4·0	4·3	3·8	3·8	4·3	..
7·4	7·1	7·6	..	..	10·2	..	13·1	16·4	..
..	10·4	..	..	..	..	..	..	..	..
8·6	9·0	9·4	8·7	6·1	..	..	..	..	..
6·2	5·7	5·0	4·0	2·3	2·3	..	..	..	..
11·3	..	..	..	..	..	..	..	..	..
4·9	5·4	5·5	5·7	5·3	2·9	3·3	..	..	..
8·1	9·3	6·6	5·7	5·3	4·3	5·0	4·2	3·8	..
14·3	15·1	12·5	10·9	8·3	5·0	7·1	5·6	4·8	10·3

TABLE 3.—*Road Deaths per* 10,000 *Motor*

Country	1930	1931	1932	1933	1934	1935	1936
Great Britain	32·1	30·4	30·0	31·5	30·5	25·3	23·8
Northern Ireland	32·1	31·9	32·1	36·9	33·1	29·6	28·4
Eire	31·8	31·5	34·8	31·3	35·1	39·2	33·8
United States	12·4	13·0	12·2	13·2	14·4	13·9	13·5
Australia	..	15·2	13·9	14·8	14·8	15·9	18·2
Canada	10·5	11·0	10·1	8·8	9·9	10·4	10·6
South Africa (Union)	..	..	..	..	..	32·5	32·6
New Zealand	11·3	8·0	8·1	6·2	7·7	7·9	8·9
Denmark	17·6	15·0	13·2	13·5	16·6	18·8	19·4
Finland	..	28·7	27·4	42·9	41·0	36·2	48·1
Norway	22·5	21·7	15·9	16·3	14·6	15·6	13·9
Sweden	13·5	17·0	19·2	17·0	17·2	17·1	20·3
Belgium	22·1	17·0	15·1	13·8	13·8	23·0	22·5
France	20·7	18·9	18·9	18·8	18·5	16·4	16·5
Netherlands	37·1	37·0	35·1	37·6	37·4	37·7	..
Italy	..	52·9	57·0	60·8	62·0	61·6	40·7
Germany (excluding motor-cycles)	..	..	..	..	..	..	65·0
Portugal	..	..	..	..	..	..	..
Spain (excluding motor-cycles)	..	..	..	..	..	..	..
Switzerland	33·4	30·7	35·6	43·8	49·9	48·6	53·1

The gaps in these tables are

(vi) Norway's accident rate is low in respect to either basis. Sweden and Denmark also have low rates.

(vii) New Zealand, Canada, Belgium and France all have low rates based on the number of vehicles. Portugal's rate is very high.

(viii) Omitting the war years, the population fatality rate varied from 3·0 to 30·8 per 100,000 population, a range of 10 to 1. The vehicle fatality rate varied from 6·2 to 112·2 per 10,000 vehicles, a range of 18 to 1. The proportion of vehicles to population, for 1938 only, is given in Fig. 1(*a*) and 1(*b*) and varies from 7·2 to 231 per 1,000 population, a range of 32 to 1. It will be realized that the inclusion of bicycles would modify some of the comparisons between accident rates per vehicle in different countries.

Denoting by D the number of deaths in any country in a given year, by N the number of registered vehicles and by P the population, scatter diagrams showing the relationships between D/N, D/P and N/P have been prepared for 1938 and are given in Fig. 1(*a*) and 1(*b*). Britain, Australia and Switzerland all have rather higher rates than would be expected on the basis of the limited factors here considered; Norway, Canada and New Zealand have rather lower rates.

The diagrams show considerable evidence of statistical relationships between the three quantities D/N, D/P, N/P—much greater than would be expected if D, N and P were uncorrelated, and constants A, α and β have been chosen by Least Squares so as to make a formula of the form $D = AN^{\alpha}P^{\beta}$ fit the data as closely as possible. The resultant formula is

$$D = 0{\cdot}0000993N^{0{\cdot}3377}P^{0{\cdot}7323}.$$

The actual values of D, N and P together with the values of D, calculated by the formula, are given in Table 4 below. The differences between the calculated and actual values are also given.

Vehicles for Various Countries, 1930–46

1937	1938	1939	1940	1941	1942	1943	1944	1945	1946
22·6	21·6	26·3	37·0	37·0	37·6	37·7	40·3	20·6	16·6
24·4	21·4	22·6	37·2	46·0	56·7	35·9	35·6	20·1	17·1
33·0	32·5	26·0	22·3	23·2	33·9	42·6	47·8	34·2	..
13·3	11·1	10·6	10·8	11·6	8·7	7·8	8·1	9·2	9·9
17·5	17·3	15·9	17·4	14·9	..	..	..	..	..
12·4	11·1	11·0	11·4	11·8	9·2	9·5	9·1	..	..
29·6	28·5	24·5	23·2	23·6	17·6	17·7	13·4	..	..
7·8	8·4	7·2	6·6	5·8	5·7	5·1	5·1	4·5	6·4
15·4	18·5	16·5	63·3	51·3	55·3	58·3	..	..	17·6
54·5	47·9	33·7	..	..	..	..	..	..	..
11·8	15·9	10·9	13·0	18·7	..	..	..	..	..
19·8	22·2	19·3	21·8	31·8	32·6	29·1	27·4	..	..
21·0	..	..	..	..	..	..	..	..	..
..	14·9	..	..	..	..	..	..	..	..
29·8	29·4	28·0	..	..	..	..	..	..	..
44·0	37·4	..	..	..	..	..	..	..	..
50·2	..	..	..	..	..	..	..	..	..
70·1	74·8	75·5	66·1	61·0	39·6	50·2	..	..	..
112·2	..	93·7	..	..	..	..	..	..	..
49·6	51·4	41·3	45·0	90·8	82·5	72·5	57·3	45·7	38·3

due to data not being available.

TABLE 4.—*Comparison of Estimated and Actual Fatalities in Various Countries*

Country	*D*	*N* (*thousands*)	*P* (*thousands*)	*Estimate of D from formula* $D = 0{\cdot}0000993\, N^{0{\cdot}3377} P^{0{\cdot}7323}$	*Error absolute*	*Error* %
Great Britain	6,648	3,085	46,208	6,327	− 321	4·8
N. Ireland	117	55	1,286	118	+ 1	0·9
Eire	226	70	2,937	234	+ 8	3·5
U.S.A.	32,582	29,853	129,825	29,018	− 3,564	10·9
Australia	1,483	857	6,864	1,016	− 467	31·5
Canada	1,545	1,395	11,152	1,709	+ 164	10·6
South Africa	1,074	376	9,980	1,012	− 62	5·8
New Zealand	230	273	1,530	230	0	0
Denmark	334	180	3,777	387	+ 53	15·9
Finland	268	56	3,855	265	− 3	1·1
Norway	177	111	2,921	273	+ 96	54·2
Sweden	584	263	6,297	640	+ 56	9·6
Belgium (1936)	625	278	8,331	801	+ 176	28·2
France	4,263	2,863	41,100	5,662	+ 1,399	32·8
Netherlands	780	265	8,680	811	+ 31	4·0
Italy	2,490	666	43,771	3,624	+ 1,134	45·5
Germany (1937)	7,636	3,403	68,425	8,720	+ 1,084	14·2
Portugal	406	54	7,506	427	+ 21	5·2
Spain	2,380	182	25,493	1,574	− 806	33·9
Switzerland	634	124	4,192	369	− 265	41·8

It will be seen that of the 20 calculated figures, 9 are within 10 per cent. of the actual figures, 19 are within 50 per cent. and there is one difference of 54 per cent. It is possible to get an easily remembered, but quite good approximation to the above formula. It is $D = 0{\cdot}00030\ (NP^2)^{\frac{1}{3}}$. Ten of the values calculated by this formula are within 15 per cent. of the actual values, 19 are within 40 per cent. One is in error by 67 per cent. of its actual value. This equation can be put into the alternative forms $D/P = 0{\cdot}00030\ (N/P)^{\frac{1}{3}}$ and $D/N = 0{\cdot}00030\ (N/P)^{-\frac{2}{3}}$. The curves corresponding to these forms of the equation are shown in figures 1(*a*) and 1(*b*).

It follows from the above formula that, on the whole, the greater N/P, the proportion of vehicles to the population of a country, the greater D/P, the death rate per unit of population, this death rate increasing roughly as the cube root of the proportion of vehicles to population. On the other hand, D/N, the death rate per registered vehicle, decreases as the proportion of vehicles in the population increases, this death rate varying approximately inversely as the two-thirds power of the proportion of vehicles to the population.

It should be pointed out that if conditions in any region were sensibly uniform over a period, except for the number of mechanically propelled vehicles and the population, we should expect the number of vehicle-fixed-object collision accidents in any year to be approximately proportional to the number of vehicles using the roads in that year. We should expect the number of vehicle-pedestrian accidents to be proportional to the product of the number of vehicles and the number of pedestrians, and since the chance of any given vehicle being involved in a collision with another vehicle would be expected to be proportional to the number of other vehicles, we should expect the number of vehicle-vehicle accidents to be proportional to the square of the number of vehicles. If the number of cycles was varying we should expect the number of vehicle-cycle accidents to be proportional to the product of the number of motor vehicles and the number of cycles, but, omitting any term for pedal cycles and for other types of vehicle, we should expect the total number of accidents in the region to obey a law of the form—

$$D = aN + bN^2 + cNP.$$

If the population is sensibly constant, the formula becomes—

$$D = aN + bN^2.$$

There is evidence that, in some circumstances, a law of this form actually holds. A laboratory investigation, not yet published, into accidents on the classified roads of Buckinghamshire shows that, for non-built-up areas, the number of accidents per mile of road does increase more than linearly with the number of vehicles using the road. Other evidence is also available from the United States. Table 5 below, taken from a paper by D. M. Baldwin (1, 2), shows that, for certain selected two-lane highways in the United States, the accident rate per vehicle mile increases with the increase of traffic until congestion becomes important. Unfortunately, in Baldwin's paper only the "adjusted" totals of accidents are given. "Adjusted" total is defined as the sum of accidents reported in the various States, with State totals increased by varying amounts to compensate for incomplete reporting of accidents. The increase was such as to bring the ratio of non-fatal to fatal accidents in each State up to the level of the highest ratio reported for States in the study.

TABLE 5.—*Accidents on Straight Two-Lane Highways*

Average daily volume (vehicles per day)	*Accidents ("adjusted" total)*	*Accident rate (per 1,000,000 vehicle-miles)*
Less than 1,000	329	3·2
1,000 to 2,000	1,779	3·2
2,000 to 3,000	1,990	3·4
3,000 to 4,000	1,469	3·9
4,000 to 5,000	1,235	3·9
5,000 to 6,000	616	4·1
6,000 to 7,000	291	4·3
7,000 to 8,000	195	4·6
8,000 to 9,000	249	6·2
9,000 and over	117	2·6

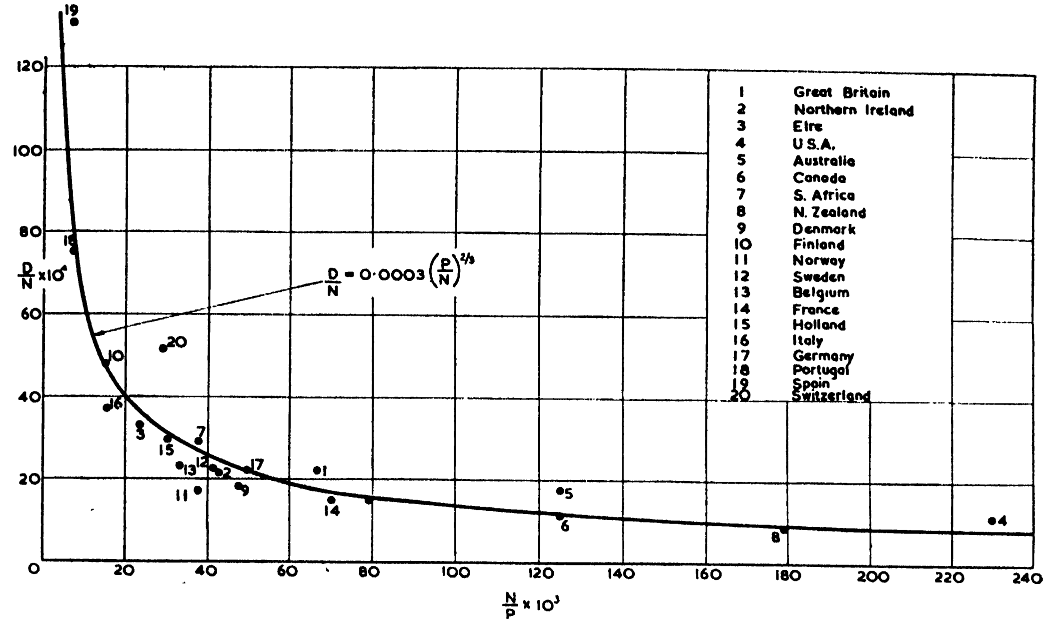

FIG. 1A

Relation between Number of Deaths per 10,000 Registered Motor Vehicles and Number of Vehicles per 1,000 Population for 1938

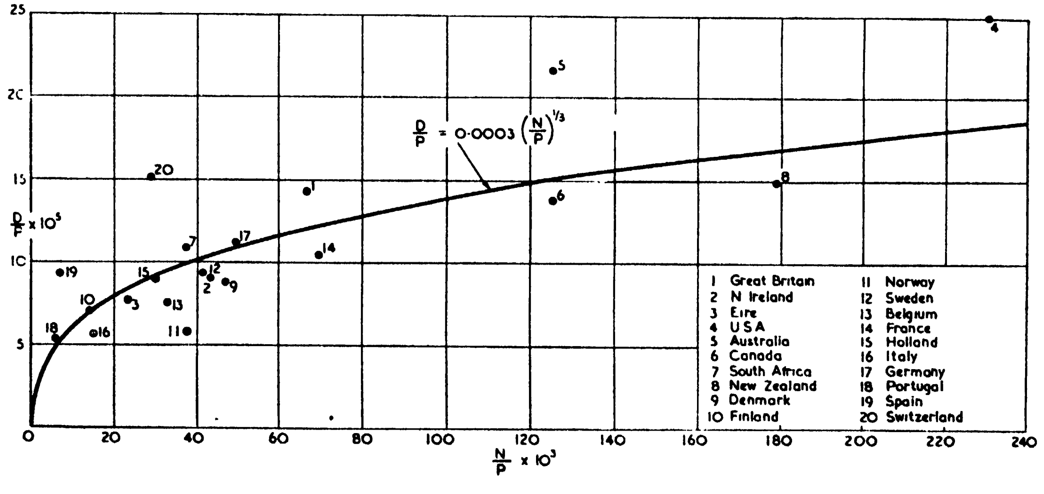

FIG. 1B

Relation between Number of Fatalities per 100,000 Population and Number of Registered Vehicles per 1,000 Population for 1938

The comparative goodness of fit of a formula of the form $D = A(NP^2)^{\frac{1}{3}}$ over quite a wide range of P and N, when it might be expected that D would increase more than linearly with the number of registered vehicles, suggests that there are major counter-balancing factors. It is shown later that this formula does give an approximation, although not a very good one, to the number of deaths in Great Britain, year by year since 1909. It is probable that as the population accident rate becomes higher the urge to do something about it becomes greater, and that something is in fact done. In addition, as the number of motor vehicles increases, which is in practice as time goes on, people are growing up and becoming more used to dealing with the situations which motor traffic causes.

(3) *Accident Rate in Great Britain*

It is desirable to analyse in detail the accident rate in Great Britain. Sufficient data for many purposes are, unfortunately, not available owing to the absence of any adequate system of reporting and analysing accidents in this country. It is hoped that a fairly comprehensive system for recording and analysing personal injury accidents will be started in 1949, but even then the records will not be nearly as complete and systematic as those in some of the States of America. However, it is possible to gain useful knowledge from the records at present available, although not as much as one would wish.

Between 1909 and 1938 the Home Office or the Ministry of Transport published annually the numbers of fatal and injury accidents. From 1926 onwards they published the numbers of fatalities and persons injured in these accidents. For the years in which both the fatalities and the number of fatal accidents are given, the ratio of the two figures averages 1·028. The number of fatal accidents in the years 1909–25 have, therefore, been multiplied by this factor to give estimates of the number of fatalities in the earlier years. Similarly the factor 1·170 was used to convert the number of injury accidents to a number of injuries. The total numbers of deaths and injuries found in this way for the earlier years are given in Table 6. It will be seen that, except for the period of the First World War, the number of deaths increased annually between 1909 and 1930, since when they varied erratically but without much overall change until the Second World War.

The dot diagrams showing the relations between D/P, the number of deaths per unit of population, D/N, the number of deaths per registered vehicle, and N/P, the number of registered vehicles per unit of population, are given in Fig. 2(*a*) and 2(*b*) covering all the years since 1909. It will be seen that the curves which were found above to give estimates of the numbers of deaths in any of the countries considered give approximate estimates of road deaths—not the number of road deaths due to motor vehicles—in Great Britain throughout the period covered. These curves are therefore shown on the diagrams, although it is clear that no curves would give really good fits. It will be noticed that the number of road accident deaths per 1,000 registered vehicles has decreased fairly consistently during the years considered, from 9·0 in 1909 to 2·1 in 1938 and 1·4 in 1947.

It is very desirable to learn as much as we can about the way various types of traffic accidents are varying at the moment, and nearly all the factual information for Great Britain as a whole has been summarized and is given in Table 7. A number of conclusions follow immediately from these figures.

The number of accidents to pedestrians increased up to 1930 and, except for the war years, has been decreasing since 1934. These changes are presumably connected with the changes in the number of vehicles on the road and with the results of the Road Traffic Acts of 1930 and 1934. The former introduced compulsory third party insurance, and consequently probably reduced the number of poor quality vehicles on the road. The passing of the Act was certainly followed by a large reduction in the number of registered motor cycles. The 1934 Act introduced the 30 m.p.h. speed limit and driving tests, and was followed by the introduction of pedestrian crossings.

The number of pedal cyclists killed increased continuously until 1934, and has been decreasing except for one of the war years, since 1936. There are no data on the number of cycles in use, year by year, but the aggregate number of pedal cycles enumerated at a number of Ministry of Transport census points was 5,343,000 in August, 1929, and 10,123,000 in 1936 (3). The

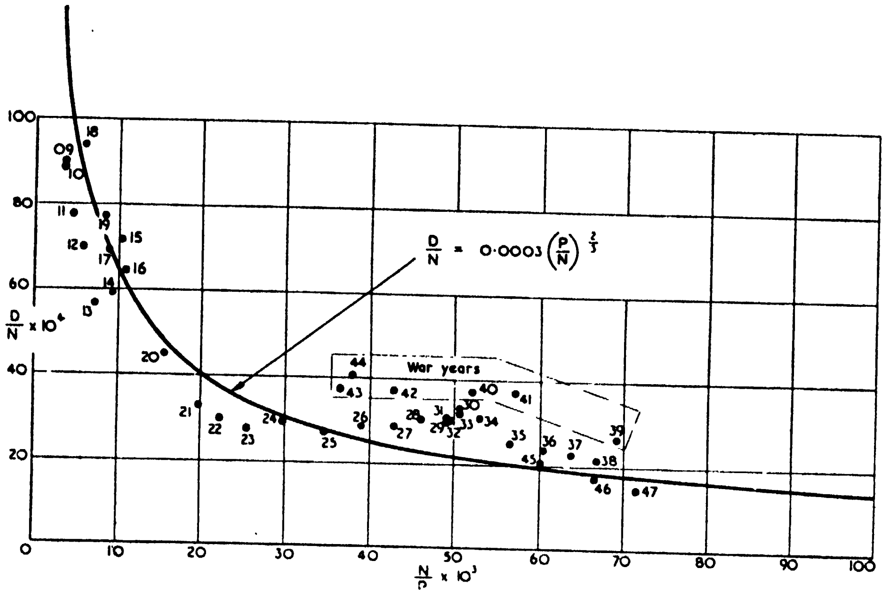

FIG. 2A

Relation between Number of Deaths per 10,000 Registered Vehicles and Number of Vehicles per 1,000 Population for Great Britain, Period 1909-1947. (The curve shown is the same as that shown in Fig. 1a. It is not a curve of best fit for the points of this diagram)

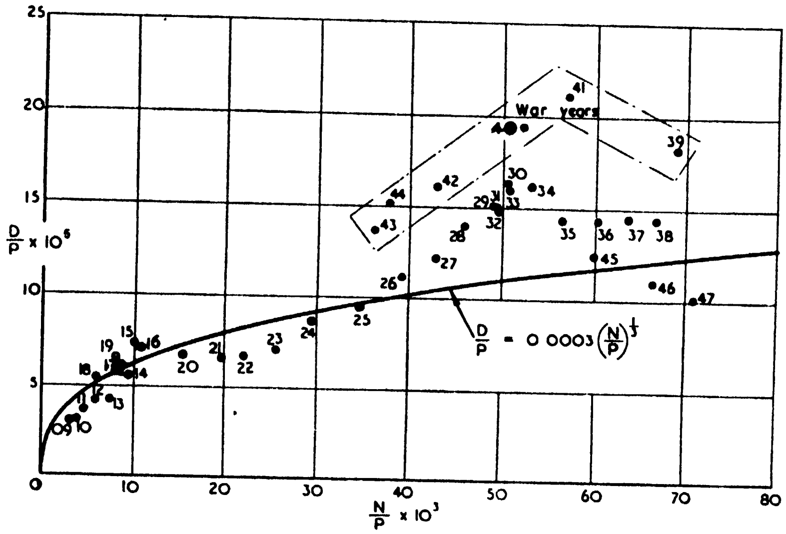

FIG. 2B

Relation between Number of Road Accident Deaths per 100,000 Population and Number of Registered Motor Vehicles per 1,000 Population, Great Britain, 1909-1947. (The curve shown is the same as that shown in Fig. 1b. It is not a curve of best fit for the points of this diagram)

TABLE 6.—*Persons Killed and Injured in Road Accidents in Great Britain,* 1909–1947

Year	*Persons killed*	*Persons injured*	*Persons killed by motor vehicles*
1909	1,100	30,526	..
1910	1,276	33,872	665
1911	1,490	38,234	872
1912	1,711	41,817	1,078
1913	2,017	48,309	1,365
1914	2,282	67,422	1,524
1915	2,929	68,395	2,134
1916	2,753	58,477	2,060
1917	2,359	48,858	1,781
1918	2,153	40,471	1,591
1919	2,558	55,297	1,993
1920	2,780	62,869	2,267
1921	2,753	70,133	2,261
1922	2,846	78,892	2,410
1923	3,062	93,743	2,616
1924	3,733	110,663	3,235
1925	4,082	130,457	3,641
1926	4,886	133,888	4,346
1927	5,329	148,575	4,719
1928	6,138	164,838	5,489
1929	6,696	170,917	6,017
1930	7,305	177,895	6,551
1931	6,691	202,119	5,936
1932	6,667	206,450	5,745
1933	7,202	216,328	6,066
1934	7,343	231,603	6,040
1935	6,502	221,726	5,137
1936	6,561	227,813	5,188
1937	6,633	226,402	5,266
1938	6,648	226,711	5,401
1939	8,272	..	..
1940	8,609	..	..
1941	9,169	..	..
1942	6,926	140,618	..
1943	5,796	116,740	..
1944	6,416	124,458	..
1945	5,256	133,042	..
1946	5,062	157,484	..
1947	4,881	161,318	..

Notes.—(1) 1909–1913. Accidents attributed to pedal cyclists excluded.

(2) 1909–1925. Persons killed and persons injured estimated from ratio of fatal accidents to persons killed 1926–38, and from ratio of non-fatal accidents to persons injured 1926–38. Persons killed by motor vehicles estimated from ratio of persons killed by motor vehicles to fatal accidents attributed to motor vehicles 1926–38.

(3) During the war the reporting of slightly injured cases probably not complete.

TABLE 7.—*Road Users Killed and Injured in Great Britain, 1928–1947*

	Pedestrians		*Pedal cyclists*		*Motor cyclists and pillion riders*		*Drivers and passengers of vehicles other than motor cycles and pedal cycles*		*Children killed*	
	Killed	*Injured*	*Killed*	*Injured*	*Killed*	*Injured*	*Killed*	*Injured*	*in all road accidents*	*Pedestrians and cyclists*
1928	3,255	69,142	691	27,680	1,395	36,266	797	31,750	..	..
29	3,523	70,056	795	28,193	1,582	38,376	796	34,292	..	..
30	3,722	71,155	887	29,666	1,832	40,375	864	36,699	1,685	..
31	3,467	81,462	926	38,502	1,499	43,268	799	38,887	1,525	..
32	3,385	80,636	1,046	45,005	1,558	43,946	678	36,863	1,489	..
33	3,504	80,238	1,354	55,347	1,569	40,668	775	40,075	1,474	..
34	3,529	80,800	1,536	67,705	1,430	39,642	848	43,456	1,438	..
35	3,073	74,709	1,400	69,052	1,277	34,880	752	43,085	1,243	..
36	3,068	74,576	1,498	71,193	1,187	35,004	808	47,040	1,206	..
37	3,002	72,647	1,416	67,748	1,151	32,413	1,064	53,594	1,115	..
38	3,046	74,193	1,401	65,727	1,145	31,626	1,056	55,165	..	1,069
39	4,497	..	1,374	..	1,231	..	1,170	..	..	1,034
40	4,724	..	1,363	..	1,270	..	1,252	..	..	1,177
41	4,781	..	1,355	..	1,412	..	1,621	..	..	1,462
42	3,650	49,336	1,134	32,253	895	16,328	1,247	42,701	..	1,315
43	3,058	41,265	1,069	29,921	568	9,059	1,101	36,495	..	1,124
44	3,314	42,484	1,185	29,064	574	9,356	1,343	43,554	..	1,335
45	2,602	45,840	918	29,221	553	12,039	1,183	45,942	..	1,219
46	2,489	53,495	833	31,158	772	22,001	968	50,830	1,060	986
47	2,380	52,850	812	34,477	783	22,991	906	51,000	958	891

increase in the death rate seems, therefore, to have been of the same order as the increase in the number of cycles in use.

The number of motor cyclists killed has shown a downward tendency since 1930.

There are, however, no signs of any tendency for the number of deaths and injuries to drivers and passengers of motor vehicles to fall.

The number of children killed in road accidents fell steadily between 1930 and 1937, the overall reduction in this period being 34 per cent. The reduction in the number of children living in Great Britain between the census of 1931 and 1939 was 8·3 per cent.

(4) *The Cost of Road Accidents*

It is still not generally realized that in addition to the cost of road accidents in human suffering, the economic cost to the community is very great. It has been computed by Professor Jones (4) that road accidents cost the country about £60,000,000 in 1938, and that the same number of accidents would cost about £100,000,000 at present prices. From data given in his report it seems that on the average, at present prices, each serious, including fatal accident, costs about £1,230, each slight injury accident £54, and each non-injury accident £18. The economic cost of each fatality is £1,900, of each serious injury £1,020, and of each slight injury £30. These and other figures should assist in making possible a comparison between the cost of any road improvement scheme and the economic saving that the improvement is likely to achieve. If a reliable method can be worked out for translating the economic gains or losses due to a speeding up or slowing down of traffic into monetary terms, as has already been attempted both here and in America, we shall be well on the way to putting the whole subject on a rational basis. It is very important that this should be done. Measures to promote either road safety or smooth traffic flow are frequently expensive, sometimes very expensive, and the expense is clear to all. It is very desirable that the expense of not making the improvements should be made equally clear.

The significance of these figures for the cost of road accidents is, I think, emphasized if an attempt is made to relate the figures for the cost of road accidents to the vehicle mileage involved. There are many difficulties in doing this; the accident rate per vehicle mile varies very greatly with the driver, the vehicle, the road, the time of day, etc., but a rough overall average figure is, I think, of some value. Anyway, I hope that the result of the preliminary investigation given below will encourage others to do some further work on the subject.

It was mentioned above that the Laboratory is at present carrying out an investigation into accidents in Buckinghamshire. During the course of this investigation it was necessary to find the vehicle mileage on the classified roads of the county, and a summary of the results is given in the following table:

TABLE 8.—*Vehicle Mileage on Classified Roads in Buckinghamshire During the Year* 1946–47

Type of vehicle	*Millions of vehicle miles*
Pedal cycles	76
Lorries	77
Private cars	167
Motor cycles	21
Buses	9
Total	350

No figures are available for vehicle mileage on unclassified roads, but 31 per cent. of accidents in the county took place on these roads. Other comparable figures for Buckinghamshire and Great Britain are given in Table 9.

TABLE 9

	Mileage of classified road	*Persons killed and injured in* 1946	*Persons killed and injured in* 1947
Buckinghamshire	430	1,769	1,884
Great Britain	45,400	162,546	166,199

Buckinghamshire has, therefore, just less than 1 per cent. of the country's classified road mileage, and just more than 1 per cent. of the casualt'es.

Assuming—

(i) that the cost of accidents in Buckinghamshire was 1 per cent. of the total cost of accidents in the country,

(ii) that 69 per cent. of this cost was due to accidents on classified roads,

(iii) and that the total cost of accidents during the year was £100,000,000 multiplied by the ratio of the road casualties in 1947 to that in 1938, i.e. 166,199 to 233,359,

it is found that road accidents cost the country about 0·33d. per vehicle mile.

(5) *The Cure*

We have seen that—*ab initio*—we might expect the number of accidents to vary as some power between one and two of the number of registered vehicles, or more correctly of the number of vehicle miles. We have found that, in practice, although accident rates along given stretches of road may increase as fast as this in their short term fluctuations of traffic volumes, the long term accident rate over the country as a whole does not increase nearly as fast as this, and the opinion was expressed that this is due to a considerable extent to the active steps that are taken to reduce this rate.

It is frequently argued that it is a waste of energy to take many of these steps to reduce accidents. There is a body of opinion that holds that the provision of better roads, for example, or the increase in sight lines merely enables the motorist to drive faster, and the result is the same number of accidents as previously. I think there will nearly always be a tendency of this sort, but I see no reason why this regressive tendency should always result in exactly the same number of accidents as would have occurred in the absence of active measures for accident reduction. Some measures are likely to cause more accidents and others less, and we should always choose the measures that cause less.

The number of different views that are put forward to increase road safety is very high. A view commonly held by road engineers is that accidents can largely be avoided by improvements in road surfaces and in the design of the roads. The police frequently take the view that what is required is the enforcement of the law and the presence of more policemen on the highway. Another widely held police view is that more education in driving is required. A view which has considerable official support is that more propaganda is required. Many motorists believe that the trouble is that certain people cannot or will not drive properly and courteously and that certain people are accident prone. Another body of opinion is that almost the entire trouble is that speeds are too high, and that the speed limit in built-up areas should be reduced and speed limits put on other roads. Others believe that a high proportion of the trouble is due to defective vehicles, whilst others believe that more control over pedestrians and cyclists is required.

The evidence in support of these views is, generally, not nearly as great as the emphasis with which they are expressed, but it should be realized that many of the suggestions are not mutually contradictory, and that it is conceivable that accidents could be reduced by varied and different methods and reduced even more by a combination of them.

In fact, if the number of accidents can be reduced by fractions $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n$ by n different and mutually exclusive methods, the resultant reduction will, of course, be to $(1 - \varepsilon_1)(1 - \varepsilon_2) \ldots (1 - \varepsilon_n)$ of its former value. It is by a combination of different methods that I think that a really large reduction in the number of accidents is most likely to be achieved.

Some of the evidence for various methods of accident prevention will now be reviewed. This

TABLE 10.—*Number of Fatal Accidents Distributed by Time of Day*

								Daylight Hours																
Hour	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24	*Total*
Jan.	4	1	—	—	—	4	15	17	13	8	14	20	18	22	26	12	38	67	54	35	26	14	20	7	435
Feb.	2	1	—	—	1	3	6	14	13	7	12	12	15	16	17	23	29	40	38	35	15	18	19	6	342
Mar.	1	—	—	—	1	1	4	8	7	9	13	9	19	21	12	19	19	18	21	21	12	15	12	4	246
April	—	1	1	—	1	—	1	5	6	11	11	15	21	17	19	19	22	25	18	25	14	14	13	5	264
May	6	2	2	—	2	1	3	13	12	6	13	13	17	16	21	14	39	33	15	16	17	8	19	8	296
June	3	1	—	—	—	2	6	10	11	9	10	14	22	22	21	14	17	31	15	14	5	12	12	9	260
July	6	7	—	—	—	2	7	20	14	8	10	10	21	11	22	22	29	39	28	24	18	11	23	14	346
Aug.	2	2	3	—	—	1	5	15	13	19	11	20	21	12	26	23	23	28	17	18	20	23	23	14	339
Sept.	1	2	2	—	1	5	7	27	14	7	19	12	31	13	15	27	28	36	23	24	32	28	32	12	398
Oct.	2	3	3	—	—	4	5	22	15	9	18	16	18	19	19	30	31	41	37	39	17	19	27	6	400
Nov.	3	2	—	—	1	3	9	19	22	4	7	16	22	18	11	28	39	77	53	39	15	22	23	6	439
Dec.	3	2	1	1	—	3	13	22	9	8	12	13	20	14	14	19	38	54	33	32	17	15	12	9	364
Total	33	24	12	1	7	29	81	192	149	105	150	170	245	201	223	250	352	489	352	322	208	199	235	100	4,129
Probability of discrepancies as large as those observed, arising by random chance							·004	·003	·18	·10	·52	·60	·68	·60	·20	·095	·015	·000	·000	·001	·002	·03	·015	·05	

evidence is not always as strong as one would wish, and it is important that more should be collected. Perhaps, if statisticians take more interest in the subject, something will be done.

The data do seem to show that active steps to reduce accidents can be effective, but more data must be collected before it will be possible to forecast even approximately the quantitative effects of any given action.

(6) *Accidents at Night*

The Ministry of Transport has published three reports on accident causation covering the years 1933, 1935 and 1936/37. These all indicate that a very appreciable proportion of accidents occur during the hours of darkness. In the 1937 report, for example, it is stated that 57 per cent. of accidents occur in daylight, 5 per cent. at dusk, and 38 per cent. during the hours of darkness. Few data are available as to the relative vehicle mileages in these three periods, so that it is not possible to get a direct comparison of the accident rates, but it is certain that less than 38 per cent. of the total vehicle mileage takes place at night.

However, the 1935 report gives the number of fatal accidents in separate hourly periods for each month of the year, and from these data an estimate can be made of the relative road accident hazards during daylight and darkness. Table 10 gives these accident data for the week-days Monday to Friday, excluding public holidays. The times of civil twilight, i.e. when the sun is 6° below the horizon, have been inserted into the table, and for the purposes of this paper will be regarded as the dividing lines between the dark and light periods of the day. The total number of accidents for the year occurring during each hourly period are given in the table, and also the probability—when this can be found from the χ^2 test—of discrepancies from expectation, as large as those observed, arising by random chance, the expectation for each month being taken as the total number of accidents occurring during the year divided by 12. It will be seen that the distributions of accidents by month of the year for particular hours of the day are not very different from the distributions that would be expected to occur by random chance, except for those hourly periods which were dark for some months of the year and light for others. In these cases the differences are most marked.

TABLE 11.—*Comparison of Accidents in Light and Dark Hours*

Period	5–6 a.m.	6–7 a.m.	5–6 p.m.	6–7 p.m.	7–8 p.m.	8–9 p.m.	9–10 p.m.
Accidents per hour in light (*a*)	1·2	4·75	30	19·3	19·2	15	11·5
Accidents per hour in dark (*b*)	3·67	14	66	43·0	33·5	19·1	18·7
$\frac{b}{a}$	3·1	3·0	2·1	2·2	1·7	1·3	1·6

Table 11, calculated from Table 10, gives the number of accidents per month for the light and dark months for those parts of the day which are dark in some months and light in others. Hourly periods in which there was a change of light conditions have been omitted from the table unless at least four-fifths of the time was either light or dark. It will be seen that the evidence is strong that the accident rate is much higher in the dark hours than in the light hours, the most probable value of the relative rates being about 2 to 1. Ratios of 2 and 3 to 1 are sometimes quoted in the U.S.A. Accepting the figure of 2 to 1 and noting that about 40 per cent. of accidents take place at night, it would appear conceivable that 20 per cent. of total accidents might be eliminated by sufficient improvements of street and vehicle lighting.

We might, therefore, expect adequate street lighting to make a difference to the accident rate, and this is, in fact, what we sometimes find. An example, taken from the U.S.A., is given below.

The lighting of 10 miles of street in Hartford, Connecticut, was changed from 0·18 to 0·43 lumens per sq. ft. in 1937. The accident records on the streets concerned, together with the accidents on the unlighted and unchanged portions of the city streets, are taken from a report on the subject by R. E. Simpson (5), and are given in Table 12.

TABLE 12.—*Accidents in Hartford, Connecticut, Before and After the Lighting in Certain Streets was Improved*

	Relighted sections		*Unchanged sections*		*Rest of city*	
	Day	*Night*	*Day*	*Night*	*Day*	*Night*
One year before	232	248	222	195	686	480
One year after	217	133	246	203	680	477

The night accidents in the streets concerned were almost halved in number, but hardly changed in the other streets. In fact, the constancy of these other accidents is almost too good to be true.

(7) *The Effect of Physical Separation of Opposing Streams of Traffic*

The before and after accident experience on the dividing of a highway in New Jersey is described in reference 6. The portion of State Highway Route No. 26 in the Townships of North Brunswick, South Brunswick and Plainsboro, extending for a distance of 10·64 miles, was reconstructed from a 4-lane highway into a dual carriageway in 1935 and 1936. In the table below the number of accidents in 1933 and 1934 representing the "before" period and 1937 and 1938 representing the "after" period are given.

TABLE 13.—*Accident Experience State Highway Route No.* 26 *Before* (1933 *and* 1934) *and After* (1937 *and* 1938) *the Conversion to a Dual Highway*

Accident severity	*N. Brunswick*		*S. Brunswick*		*Plainsboro'*		*Total accidents*	
	Before	*After*	*Before*	*After*	*Before*	*After*	*Before*	*After*
Fatal	7	—	18	4	5	1	30	5
Non-fatal injury	30	13	81	45	31	15	142	73
Property damage	26	17	63	49	19	23	108	89
Total accidents	63	30	162	98	55	39	280	167

Unfortunately, the data of accidents on other stretches of this road for the corresponding period are not given in the above-mentioned report, but in New Jersey as a whole for the period covered there was an increase of 17·3 per cent. in motor vehicle registrations, an increase of 44 per cent. in petrol consumption and an increase of 6·7 per cent. in accidents.

(8) *The Effect of Winding Roads on the Accident Rate*

An attempt at finding the effect of winding roads on the accident rate has been made by D. M. Baldwin (1). Results are given below:

TABLE 14.—*Accident Rates on Two-Lane Highways of Different Curvatures* (8 *States*)

Degrees of curvature per 100 ft. of road	*Accidents ("Adjusted" total)*		*Accident rate (per 1,000,000 vehicle-miles)*	
	Traffic flow under 5,000 vehicles per day	*Traffic flow 5,000 to 10,000 vehicles per day*	*Traffic flow under 5,000 vehicles per day*	*Traffic flow 5,000 to 10,000 vehicles per day*
Less than 2	250	37	2·4	1·9
2 to 2·9	221	47	3·3	2·5
3 to 3·9	152	65	3·5	3·5
4 to 4·9	158	64	3·7	3·7
5 to 5·9	139	35	4·3	3·3
6 to 6·9	119	34	3·9	2·8
7 to 9·9	115	25	3·1	2·5
10 to 13·9	132	24	3·7	2·6
14 to 19·9	108	12	6·3	*
20 and over	129	13	7·6	*

* Sample considered too small for reliability.

It will be seen that there is good reason to believe that the accident rate of a road depends materially on its curvature. It is to be presumed that, if the rélationship can be assessed accurately, accidents can be reduced by suitable alterations to the curvature of the roads.

(9) *The Effect of Road Widths*

The effect of the road widths on the accident rate is also given in the paper by Baldwin. A copy of his table is given below. Increasing the width of the road up to 23 ft. certainly seems to have a most desirable effect on the accident rate.

TABLE 15.—*Effect of Width of Road on Accidents in Two-Lane Highways*

Pavement width	*Accidents ("adjusted" total)*	*Accident rate (Per 1,000,000 vehicle-miles)*
Less than 18 ft.	274	5·2
18 ft. to 19·9 ft.	2,774	3·8
20 ft. to 22·9 ft.	4,338	3·5
23 ft. and over	822	3·4

(10) *The Effect of Improving Road Surfaces*

Two small stretches of the Embankment, London, which largely consist of wood blocks, known to be slippery when wet, were surface dressed with non-skid material in 1938 (7). The accidents in a number of adjacent stretches of the road during the period November 8th, 1937–February 7th, 1938, before these portions were surface dressed, and in the period November 8th, 1938–February 7th, 1939, afterwards are given in Table 16.

TABLE 16.—*Accidents on Sections of the Embankment, London, Before and After Certain Portions Were Surface Dressed*

	A	B	C	D	E	F	G	*Total*
Before	11	15	11	51	14	12	6	120
	Portions D *and* F *were Surface Dressed*							
After	14	28	27	3	6	1	11	90

It will be seen that the reduction in the accident rate in the surface-dressed portions was enormous. The increase in the number of accidents in the non-surface-dressed portions was probably connected with the greater rainfall in the second period compared with that of the first. In the first period there were 45 days in which no rain fell, in the second period 29. The total rainfall in the first period was 7·38 in., in the second period it was 11·23 in.

(11) *The Effect of Vehicle Characteristics*

The Ministry of Transport report for 1936–37 (8), attributed 3·3 per cent. of fatal accidents to defects in vehicles, but the Ministry's Interim Report of the Committee of Road Safety (9), published in 1945, states that they believe that vehicle defects have been in part the cause of a much larger number of accidents. An investigation carried out by Major Bayly Pike, of the Army Operational Research Group (10), into accidents in which Service vehicles were involved, states that vehicle defects were contributory causes in 23 per cent. of accidents. It is quite conceivable that improvements in the design of the vehicles and accessories might contribute largely to a reduction in the accident rate.

Another investigation carried out by the Army Operational Research Group (11), gave the following figures for accidents to six different types of motor-bicycles:

TABLE 17.—*Accidents to Six Different Types of Motor Cycles*

Type	*No. in use*	*No. involved in injury accidents*	*No. involved in injury accidents per 1,000 vehicles*
A	5,340	51	9·6
B	1,130	12	10·6
C	3,000	45	15·0
D	9,900	176	17·8
E	9,000	248	27·6
F	1,030	38	36·9

The most probable explanation of the large and significant difference between these accident rates is that some types are more accident prone than others. The matter is being investigated.

(12) *The Effect of Propaganda*

Very little evidence on the effect of propaganda alone on the accident rate exists, but there is good evidence that propaganda, in conjunction with other measures, has a valuable effect. The Home Office Experimental Motor Patrol Scheme was tried out by the Lancashire Constabulary from April 1st, 1938, and immediately gave encouraging results. The scheme consisted of constant advice and warning being given to all road users by both motor patrols and foot officers, coupled with the education of road users in general, including "Safety First" propaganda of various kinds in the press, cinema and shop windows, and the education of children in road safety by school teachers and police officers.

A comparison of the accident rate in Lancashire and the rest of the country, taken from a report by the Chief Constable of Lancashire (12) is given in Table 18.

TABLE 18.—*Road Accident Casualties in Lancashire*

Year	*Casualties per 1,000 population*	
	Whole country	*Lancashire*
1932	4·9	4·6
1933	5·2	4·9
1934	5·5	5·1
1935	5·3	4·8
1936	5·4	4·7
1937	5·4	4·6
1938–39	5·4	2·5
1946	3·4	2·7
1947	3·5	2·9

A comparison of the casualties during the year before the scheme was started and during its first year is given in Table 19.

TABLE 19.—*Comparison of Casualties Before and After Introduction of Police Road Patrol in Lancashire*

Casualties	*April 1st, 1937, to March 31st, 1938*	*April 1st, 1938, to March 31st, 1939*	*Reduction*	*Percentage reduction*
Killed	303	248	55	18
Seriously injured	1,839	1,050	789	43
Slightly injured	5,200	2,807	2,393	46
Total	7,342	4,105	3,237	44

It will be seen that there was an appreciable reduction in all three classes of casualties. Calculation shows each reduction separately to be significant. It is worth while using the figures given above for the cost of road accidents to convert this saving in casualties to an economic saving. The saving is 55 × £1,900 + 789 × £1,020 + 2,393 × £30, i.e., £980,000, the saving in "damage only" accidents being omitted as no figures are available. The report describing the scheme, and from which the above accident figures were taken, states that the Police Force was augmented for the purpose of the experiment by 331 men, 92 motor vehicles and 50 light-weight motor cycles. Even although it is quite possible that a part of the reduction in casualties may not have been due to the patrol scheme, and even although a more precise translation of the reductions in casualties to economic saving is really required, it would appear that further experiment on the lines of the Motor Patrol Scheme is desirable in spite of the present serious economic situation of the country. It is, however, extremely desirable that proper records of the detailed steps taken to reduce accidents should be kept. The experimental motor patrol scheme was tried out in other areas besides Lancashire, but similar success was not always obtained. No really satisfying explanation is available as to the reasons for success in one place and comparative failure in others.

(13) *Accident Proneness*

Perhaps more scientific papers have been written on the subject of accident proneness than on any other aspect of road safety. Most of them have been concerned with the detection of the "accident prone" by means of psychological tests. The literature is too extensive to review in a general paper, and I cannot, therefore, do much more in this paper than express the hope that this Society will find time for a further discussion on the subject. However, few papers give the distribution of accidents sustained by different drivers, and even fewer give the data for drivers driving under similar conditions, and this basic information is necessary to determine the importance of the concept of accident proneness. The available numerical information is given in Table 20. Some details of the various accident distributions in the table are given below. The details of Groups C–K are all taken from the report (13), but some additional references are also given.

Group A—London Bus Drivers.—Farmer and Chambers (14), in the course of an investigation into the correlation between certain tests of hand and eye co-ordination and road accidents, collected the accident records of 166 bus drivers. The accident distribution, covering a period of 5 years, is given in the first column of the table.

Group B—Accidents sustained by a firm delivering goods in the London area.—Parkinson, in the course of an investigation into the accidents sustained by two transport organizations centred in London (15), not yet published, found the distribution for accidents to 67 drivers, each of whom had been driving for a period of 10 years.

Groups C and D.—J. S. Baker (16 and 13) has given two distributions:

(i) personal injury accidents in which a random sample of New York commercial drivers were involved during a 3-year period;

(ii) personal injury accidents in which a random sample of New York general drivers were involved during a 3-year period.

Group E.—J. S. Baker (17) has also given the number of accidents sustained by 123 drivers employed by a motor transport company in California during a period of 3 years, in the course of which each driver drove 100,000 miles.

Group F.—The accidents sustained by 2,300 operators of the Boston Elevated System during the year 1937 have been collected by Slocombe and Brakeman. It will be noticed that the average rate of accidents, 3·13 per driver per annum, is very high. However, a public carrier has to record all accidents no matter how trivial, even though they may involve nothing worse than a passenger being bumped or shaken

Group G.—The accidents sustained during the period 1930–35 by 313 drivers who worked for a Public Utility Co. for a period of 10 years are also given in the report on the accident prone driver referred to above.

Group H.—C. S. Slocombe obtained the records of 5,000 general drivers in Connecticut during the period 1926–30 in the following manner: From the drivers' cards as arranged in alphabetical

order in the commissioner's licence files, approximately 200 consecutive cards were extracted from each section of the alphabet. The records were next searched to determine whether the drivers thus selected had driven continuously throughout the 5-year period. Those drivers whose records were defective were eliminated from consideration. It is important to note that the definition of "reportable accidents" in Connecticut varied during the years in question. From 1926 to 1929 the law required all accidents to be reported which caused death or injury to any person, or which resulted in property damage apparently exceeding $10. In 1929 the property damage minimum was raised from $10 to $25. This change resulted in a reduction in the number of accidents reported per annum.

TABLE 20.—*Frequency Distributions of Numbers of Accidents Sustained by Various Groups of Drivers and Numbers of Drivers Having These Number of Accidents*

Accidents per driver	*Group A*	*Group B*	*Group C*	*Group D*	*Group E*	*Group F*	*Group G*	*Group H*	*Group I*	*Group J*	*Group K*
0 .	1	..	13,157	33,777	14	217	81	3,140	23,881	901	6,812
1 .	2	..	2,956	4,770	15	326	44	1,202	4,503	549	848
2 .	3	..	433	213	34	569	68	423	936	276	110
3 .	14	..	102	17	18	458	41	155	160	164	15
4 .	17	..	34	3	11	258	25	50	33	79	6
5 .	21	2	14	..	14	145	20	15	14	41	2
6 .	17	1	2	..	3	99	13	5	3	28	..
7 .	14	2	2	..	3	86	7	3	1	17	..
8 .	14	3	..	..	1	46	5	2	..	10	..
9 .	12	3	..	..	4	27	4	3	..	7	..
10 .	13	2	..	..	3	24	3	1	..	4	..
11 .	9	1	..	..	..	24	2	..	..	5	..
12 .	6	9	..	..	3	7	..	..	..	8	..
13 .	2	6	..	..	..	5	..	..	..	..	..
14 .	6	1	..	..	..	2	..	..	..	..	..
15 .	1	4	..	..	..	2	..	..	..	..	..
16 .	6	6	..	..	..	2	..	..	..	..	..
17 .	3	6	..	..	..	2	..	1	..	..	..
18 .	..	2	..	..	..	1	..	..	..	..	..
19 .	1	3	..	..	..	..	..	..	..	..	..
20 .	..	3	..	..	..	..	..	..	..	..	..
21 .	3	1	..	..	..	..	..	..	..	..	..
22 .	..	1	..	..	..	..	..	..	..	..	..
23 .	..	4	..	..	..	..	..	..	..	..	..
25 .	..	1	..	..	..	..	..	..	..	..	..
26 .	..	3	..	..	..	..	..	..	..	..	..
27 .	..	1	..	..	..	..	..	..	..	..	..
32 .	1	1	..	..	..	..	..	..	..	..	..
36 .	..	1	..	..	..	..	..	..	..	..	..
Total No. of drivers .	166	67	16,700	38,780	123	2,300	313	5,000	29,531	2,089	7,793
Total No. of accidents.	1,330	1,056	4,360	5,259	400	7,197	758	2,909	7,082	2,735	1,147
Accidents per driver per year .	1·60	1·58	0·087	0·045	1·08	3·13	0·40	0·12	0·040	0·264	0·037

Group I.—The Connecticut Report.—The most extensive data on the subject of accident liability are given in a report (13) describing work carried out under the auspices of the American Bureau of Public Roads. The law of Connecticut requires that every accident involving personal injury or in which damage is sustained and mounting to more than $25 should be reported to the police. It is claimed that the well-kept records revealed, in a comparatively few years, a class of high accident rate drivers, and that the accident repeater group in the first few years tended to repeat again in the next few years.

But the repetition of accidents to the same individuals found in the report may not—as is claimed—have been due to the accident proneness of the individuals concerned. It may have been at least partly due to the different liabilities to accident on the routes on which they were driving, the vehicles which they drove and the variation in mileage. It must also have been partly due to the corruptness of the data. Some drivers would tend to report their accidents better than others. The writers of the report believe that practically all the fatal and personal injury accidents are reported, and a little less than half of the property damage accidents. On the average, drivers sustain a number of damage only accidents for every personal injury accident in which they are involved, and if, as is likely, some drivers in Connecticut report their "damage only" accidents and others do not, large differences in the numbers of accidents reported by different drivers are to be expected, quite apart from accident proneness.

Group J.—C. S. Slocombe has also given the distribution of accidents involving personal injury or damage in excess of $10 sustained by insured general drivers of Connecticut during the period 1926–30. The investigators obtained the names of drivers from the companies in which they were insured, along with their accident histories. The investigators then checked these records with the records in the department of motor vehicles. They found that both sets of records were incomplete, each office having some information which the other office lacked, and lacking some information which the other office had. This population is selected, all the drivers being insured against liability for personal injury.

Group K.—In the course of an accident survey in Massachussetts in 1934, an analysis was made by E. G. Allen on fatal and personal injury accidents sustained by a sample of drivers registered in a State during the period 1930–33. The cards containing the particulars of 10,000 drivers were taken in blocks of 100 in alphabetical order fairly evenly distributed throughout the alphabet. Of the 10,000 drivers selected, 7,793 were found to have been licensed each year from 1930 to 1933, and those were selected for study.

(14) *The Distribution of Accident Proneness Amongst the Population*

Let λ be the expectation of the number of accidents which a person has under given exposure conditions. If the usual bold assumption is made that the probability of his having a certain number of accidents is given by the terms of a Poisson series with mean λ, it is possible to deduce some properties of the λ distribution from the distribution of the number of accidents sustained. In particular, it can be shown (18) that the moments of the λ distribution are equal to the factorial moments of the accident distribution. Using this result, the variances and third moments about the mean for the distributions of λ have been calculated from the above accident data and are given below. Each of the distributions has been scaled so as to make the mean value of λ unity:

TABLE 21.—*Indirect Estimates of Second and Third Moments of* λ *Distributions*

	Group	μ_2	μ_3
A.	London Bus Drivers	0·21	0·19
B.	Parkinson data	0·10	0·02
C.	New York Commercial Drivers	1·03	5·77
D.	New York General Drivers	−0·21	1·42
E.	Californian Motor Transport Co. Drivers	0·40	0·34
F.	Drivers of Boston Elevated System	0·31	0·40
G.	Drivers of an American Public Utility Co.	0·54	0·29
H.	5,000 General Drivers, Connecticut	1·15	6·96
I.	29,531 Connecticut Drivers	1·14	3·33
J.	Insured General Drivers, Connecticut	1·20	3·40
K.	General Drivers, Massachusetts	1·50	8·75

There is little sign of agreement amongst the data although, since the coefficient of variation is appreciable in each case, they nearly all indicate accident proneness to be an important phenomenon. Even with the Parkinson data, the least variable of all except for distribution D, the standard deviation of λ is 0·31 when the mean is unity, so that the more accident prone persons must suffer several times the accident rate of the least accident prone. However, this conclusion is based on the Poisson hypothesis, on which serious doubt is thrown by the Parkinson data and even more by distribution D, personal injury accidents amongst New York General Drivers. In the latter case the calculation of the variance on the assumption of a Poisson hypothesis yields a negative quantity. Although this negative figure is not significantly different from zero, this variance is undoubtedly small, and it does appear likely that the hypothesis is not valid, and that after a driver has had one accident, his chances of having a second are reduced. If there is an effect of this sort, mixed up with the wide variability in accident liabilities shown in the other distributions, the actual variability must be even greater than is indicated by the large coefficient of variation for λ shown above. It will be noticed from the above table that groups of "general" drivers usually show much greater variability in accident liability than do groups of commercial drivers.

However, all that the above shows is that the difference in the liability to accidents of different drivers is very great. The next need is to find the cause, and I think that a hopeful method of tackling the problem would be to compare drivers with very good and very bad accident records. The causes of the large differences in the values of μ_2 and μ_3 amongst the different distributions should also be investigated. It is also desirable to obtain more reliable and complete data and, for commercial drivers who stay with the same firms for a long period of years, these should not be very difficult to obtain.

(15) *Some Directions in which Statistical Research is Necessary*

It is necessary to build up a large collection of facts. Measures of various kinds are continually being taken to reduce road accidents and to improve traffic flow. It is very desirable that a proportion of these should be adequately recorded so that the proper lessons may be learned and imparted to others. Examples can be given of measures having the reverse effect to that intended, and very few of the changes that have been made in recent years have been adequately recorded. One-way streets have been made, refuges, guard rails, pedestrian crossings, etc., have been put down, with no adequate effort to decide whether they have or have not been successful in achieving the object for which they were intended. Success or otherwise has usually been decided by the personal impressions of those responsible for the changes. There is no doubt that these impressions are most valuable, but they are not enough, and important lessons which may be learned by the personnel responsible cannot be imparted to others. It is important to the nation's well-being to reduce the present heavy cost of road accidents and to speed up traffic flow. The measures likely to be successful will be found more quickly if some of the changes at present being made are properly analysed. Relative costs could then also be readily assessed. It is possible for statisticians to help considerably by making proper records and analysing data of all kinds relating to these subjects.

We, at the Road Research Laboratory, are hoping that investigations of this kind will be made by research students at Universities, and for interested persons I should mention that there are a number of mathematical probability problems in connection with these matters to be solved.

We also hope that the more scientifically minded Road Safety Committees will try to assess the effects of their efforts by measuring the behaviour of road users before and after Road Safety Weeks, etc.

The staff of the Road Research Laboratory will always be glad to assist persons doing this type of work.

Acknowledgments

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DISCUSSION ON DR. SMEED'S PAPER

SIR GEORGE MADDEX: Dr. Smeed has performed a service to the Society in bringing together in one paper a variety of statistics on a difficult subject. That alone is a sufficient reason for the vote of thanks which it is my pleasure to propose.

The causation and prevention of road accidents is a subject in which we all, as both statisticians and men in the street, may be expected to take a lively interest. It is not only very complex, but highly controversial, and our decided opinions about what ought to be done vary as we change from being pedestrians to being cyclists, and again to motorists. This is a matter in which we all suffer from a split personality, and we are fortunate if we can retain a balanced view as statisticians. There is hardly less variety of opinion regarding the relative importance of road construction and lay-out, speed limits, propaganda, accident proneness, and so forth—all of them regarded by somebody as the vital factor in the problem. I look forward to an animated discussion from special points of view, and I shall make my own remarks brief and general.

The paper begins with an international review of fatal road accidents. The number of such accidents occurring in a particular country in a particular year is the result of so many different factors that I do not suppose the author imagined for one moment that there was a high degree of direct comparability between the heterogeneous figures for all these various countries, or that any formula involving merely the population and the number of vehicles would give a very good fit. There are one or two things about his formulae which puzzle me. Taking D as the number of deaths in any country in a given year, N as the number of registered vehicles, and P as the population, he obtains the formulae for D in terms of N and P; the indices of N in both formulae are almost identical and the indices of P are not markedly different, but the constant multiplier is three times as great in one case as in the other. I cannot see how in these circumstances one formula can very well be described as an approximation to the other, and it is clear that with the heterogeneous data like those from the United States one cannot arrive in this way at very satisfactory conclusions. It is easy to see that the two formulae which the author gives at the beginning of his paper could not be identical unless there was an inverse relationship between N and P, which is not only contrary to common sense but is not borne out by the figures in Table 4. For a number of countries, it is true, the second formula gives much the same result as the first, but no doubt one could find other formulae which would do this, for example, the simple expression $D = \frac{4}{5}N + \frac{1}{15}P$. It is not surprising that when the author later develops a formula

from certain broad *a priori* assumptions it is of quite a different type, and it would be interesting to be told how this formula fitted the data given in Table 4.

To an actuary the most baffling characteristic of road accident statistics is the extreme difficulty of constructing an "exposed to risk" formula against which to consider them. Unless accident rates can be derived, related to an appropriate and significant denominator, we are in danger of accumulating collections of facts, interesting in themselves, no doubt, but unmanageable and embarrassing. We may therefore be led to false conclusions when studying the statistics for different classes of the population, times of day, seasons of year, etc. There is an instance of this in Table 20, in which the frequency distributions of numbers of accidents are set out for eleven different experiences. These are interesting series of figures, but they cannot be compared one with another if only for the reason that the length of the period covered differs, and hence the "exposed to risk" of incurring a given number of accidents. The author could hardly be expected to provide, even if it were available, the considerable amount of further information which would be required in order to analyse and appraise these statistics, but he might well have given a warning on this point.

Apart from the "exposed to risk" question, as regards the accident itself, i.e. the numerator of the fraction, one would think there would be a practical limit to what can be recorded accurately in the difficult circumstances attendant upon such incidents. One gathers however, from a remark of the author, that the limit in recordable information is very much higher in the United States than it is—or rather than it will be next year—in this country. The Road Safety Committee in their final Report, published in the autumn of 1947, whilst recommending for general adoption a much more comprehensive system of records of road accidents than the best we have now, indicated that they were conscious that it was experimental, and that it could only justify itself if it was found possible to analyse the results satisfactorily. The success of that scheme will depend very largely on the effective co-operation of the police forces throughout the country, and when it has been working for a year or two the officials of the Ministry of Transport and other statisticians will find plenty of material for analysis and research as regards the accidents, but I do not quite know what they will do as regards the denominators.

The Road Safety Committee's Reports, like Professor Jones's Report on the economic cost of road accidents, were produced under the dominating idea that the end of the war would see an alarming rise in road accidents, for a number of obvious reasons. Fortunately this fear has not yet been fulfilled, as the figures in Table 7 show, though some of the reasons for the present position may not be so gratifying. Nevertheless, the numbers involved, whether fatalities or injuries, are still so heavy as to justify not only the introduction of a system of records such as that proposed by the Road Safety Committee—assuming that satisfactory recording is achieved, and that the records receive an appropriate statistical treatment and are not merely tabulated for general information—but also the undertaking of special investigations into individual factors such as change of lighting or road surfaces, to name only two of those quoted by Dr. Smeed, which seem likely to yield good statistical results.

The analysis of the statistical results from the new system will not be easy. The number of ways in which the data to be recorded in respect of each accident can be permuted and combined is very considerable, and if one regards the subsequent processes and analyses as exercises in multivariate regression the emphasis is undoubtedly on the "multi."

May I revert to the problem of relative "exposed to risk" and contribute one more to the collection of facts? If you compare, age-group by age-group, the deaths from road motor accidents of men and women respectively as given in the national statistics, you get some peculiar results. In a table in the Registrar-General's Statistical Review for England and Wales for 1938–39, giving this information for a number of years immediately before the war, certain general features appear. For ages under 5 years the male death rate from motor accidents is nearly twice the female rate. There is an increasing ratio, up to seven times in the age groups for the twenties and thirties, and then there is a fall to a level ratio of about 2 to 1 at ages over 65. One wonders whether the relationship between the two sexes for the old ages is entirely due to the degree of physical exposure to the risk, or whether, in addition to these differences, there may be some psychological or other factor which makes the male more prone to accident than the female, or less tough to survive an accident. After making allowance for the possibility that old men get about more than old women, would one expect the accident rates for men over 70 or 75 to be as much over the female rates as the figures I have quoted indicate? At these extreme ages we must be concerned with pedestrians almost entirely—the proportion of car drivers and cyclists being then relatively small—but at almost any age over 65 (or indeed much younger) the death rate from all causes is about 50 per cent. higher for men than for women. With that little puzzle I will conclude as I began by proposing that we accord our thanks to Dr. Smeed for an interesting and stimulating paper.

Professor MAJOR GREENWOOD said that they were all interested in the question of increasing road safety, but he did not propose to air his own crotchets, so he would make no comment on the first two-thirds of the paper. In the latter part of the paper the author dealt with the question of accident proneness. The speaker himself disclaimed any expert knowledge of this subject. He belonged to that diminishing class of citizens in this country who were not experts or authorities on anything whatever, but were merely inquisitive people. The present author had a characteristic which was not always found in contributors to the proceedings of the Society; he had a desire to save paper and printing, and in that laudable desire, like an earlier author, striving for brevity, he became a little obscure. Dr. Smeed told them that the power moments of the λ distribution could be obtained in terms of the factorial moments of the accident distribution. A beginner—and, after all, everyone began sometime—on reading this paper might suppose that the power moments of the λ distribution could not be obtained in terms of the power moments of the accident distribution.

Twenty-one years ago the late Dr. Ethel Newbold gave the first 4 power moments of the λ distribution in a memoir which, he still thought, was one of the classics of the subject. Again, in the Society's *Supplement* in 1941 there was a paper by Mr. Yule and Mr. Chambers, together with an invaluable comment by Dr. Irwin. That comment, with Dr. Irwin's characteristic lucidity, placed accurately before them the present position of what he might call the statistical side of the accident proneness question.

He thought that both those papers deserved citation. A beginner might infer from Dr. Smeed's paper that the unmasking of accident proneness was a simple matter of fitting a negative binomial to such distribution of accidents as were given in Table 20 of the present paper; in other words, that if a good fit was found for a negative binomial, well, there one was, and if a bad fit, well, then proneness could be excluded. But life was not as simple as all that. A negative binomial could arise in a great many ways, and if one had a negative binomial and it was a good fit, accident proneness might be involved or might not. If one had a desperately bad fit one could be fairly sure, not *quite* sure, that accident proneness had no part in the business. Unfortunately when there was a good fit it did not follow that the distribution was of a continuous variable. For example, just to see how it worked out, he took four Poissons with parameters of 1, 2, 3 and 4 and slumped them together, a rectangular distribution, and the negative binomial was a pretty fit. If one accepted the total as being less than 2,000 it would give excellent values of P.

He drew attention to Group I in Table 20. Fitting a negative binomial to that he got the following results: for the first frequency the observed value was 23,881 and the fitted value 23,891, only 10 away. For one accident per driver the observed value was 4,503 and the fitted value 4,497. For two accidents, 936 observed and 907 calculated; for three accidents, 160 observed and 187 calculated; for four accidents, 33 observed and 39 calculated; for five or more accidents, 18 observed as against 10. The value of P was less than $0 \cdot 007$. One had 30,000 observations, whereas with 3,000 observations, and the same relative values, one would have got a P of over $0 \cdot 5$. He accepted the inference from the large set that the data were not homogeneous. Whether the partial success of the negative binomial is a reflection of proneness or whether it is due to clubbing Poissons with different parameters cannot be determined by mere curve fitting. This, of course, was mere commonplace to those who investigated proneness. They did their utmost to secure homogeneity of exposure. He had avoided his crotchets so far, but would sin in his last sentences. He believed in proneness, and thought Mr. Farmer and his colleagues had done admirable work, but that we had not yet found practical tests which correlated so highly with what our ancestors called temperament that a satisfactory elimination of potentially dangerous drivers which did not inflict hardship on individuals was possible. But when we remembered the enormous increase of efficiency in tests of the cognitive side of human nature since the beginning of this century, it was surely not utopian to expect equally great improvements in our measuring of the conative side of man.

He was sure, as Sir George Maddex had said, that the present author's contribution made an extremely valuable addition to their knowledge, and he seconded the vote of thanks with great pleasure.

Sir CHARLES GOODEVE said that he had followed the work of the author and his team at the Road Research Laboratory for a number of years, and therefore it gave him special pleasure to hear some part of that work described in this paper. The elucidation of the relations between accident rates and fundamental parameters such as the numbers of vehicles and the population was of the greatest importance and it was satisfactory to find that consistent results could be obtained by using the variation in these parameters in the statistics of various countries. However, there was a danger in using data from such a wide range of sources because some non-random extraneous factors might be present. The same relations could be deduced from the fluctuations

in vehicles and population occurring inside a particular country such as our own, and such a method would have the advantage that extraneous factors, if present, would be more likely to be noticed. In addition, there would be the important advantage in that the data would all be related to unit area or unit road length. Such data were not so accessible in national statistics, but it was believed that the author has made studies along this line.

The simple theoretical equation given by the author for the total number of accidents, $D = aN + bN^2 + cNP$, was obviously very attractive, even though it was not meant to indicate more than a part of the picture. It was of sufficient importance to justify an attempt to have all accident statistics divided under these three terms. The chief factor missing in the second and third terms was that of "awareness" of danger. During the war the speaker came across an extreme example of this. One of the highest points of traffic density, especially for pedestrians, was in the Strand between Charing Cross station and Trafalgar Square. For the first two years of the war the erection of certain bomb protection structures diverted practically all the pedestrian traffic into the roadway. The scene in the black-out at rush-hour would have had to be seen to be believed. The quantities N and P (in terms of densities) were extremely high and one would have expected a high accident rate. The Borough Engineer of the Westminster City Council stated that, to his own surprise, this point had an exceptionally low accident rate; apparently the "awareness" of people to the danger more than compensated for the increased basic probability of a collision. The speaker believed that the ratio of urban to rural accidents was much less than would be expected from the ratio of densities of traffic and people, a fact that could be attributed to a big increase in "awareness" in urban areas. The size of the particular factor and of its variations was perhaps not surprising when it was realized that, even under the worst conditions, the number of collisions was a thousand or a million times less than it would be under the hypothetical condition of, say, "invisible pedestrians."

Mr. W. F. ADAMS said that he would try to confine himself to technical points. He thought the fact that statistics were of use and must be employed in investigating road accidents was obvious. He found particularly interesting the author's proposed formula connecting population (P), number of vehicles (N), and deaths (D). He would like to consider the effect of applying the formula to different counties in this country, for example, to see whether, treating this formula as a basis, one could find some indication of which were good and which were bad counties from the point of view of road accidents, and, if possible, to find out why. He was very glad that the formula was not put forward as anything more than a rough approximate rule. If it applied strictly it would follow that the number of deaths was simply dependent on the number of vehicles and the size of population, which would mean that improvements, police activity and propaganda counted for nothing.

He would like to know what was meant in Table 6 by "persons killed by motor vehicles," because if that were taken literally the odd result was arrived at that the number of people who had been killed other than by motor vehicles fell from the year 1911 to a minimum in 1922, and since then had been continually increasing up to a high figure. On this basis, motor vehicles were responsible in 1910 for 52 per cent. of the accidents on the road; by 1929 they were responsible for 90 per cent., but by 1938 the proportion had fallen back to 82 per cent.

In section 4 the author dealt with the cost of road accidents. In attempting to relate the cost of road accidents at a particular site to what might be done in the way of improvements, any measures which might be taken were not likely to be 100 per cent. successful. The cost of the improvement had to be balanced, not against the whole cost of all the accidents, but against some proportion of it, and clearly research was needed on how much reduction of accidents could be expected from each type of improvement. He knew of some preliminary research on that point, and he thought that the author was alive to it in the reference he had made in the paper to the fact that more data must be collected before it would be possible to forecast even approximately the quantitative effects of any given action.

The author's statement in section 5 of the paper beginning, "in fact, if the number of accidents can be reduced by fractions . . . ," etc., might give a wrong impression unless one put a special meaning on the words "mutually exclusive." Each method must be independent of and have no effect on the results of the others. Suppose that at a particular junction an improvement were effected. It might cause drivers at that place to lessen their care, and to that extent reduce the effect of propaganda in the area. Two such effects might quite well react upon each other and even operate against each other.

The author referred to the lighting of 10 miles of street in Hartford, Connecticut, which was changed from 0·18 to 0·43 lumens per sq. ft. in 1937. That information was practically meaningless to the speaker. If it referred only to the gross amount of light which was emitted by all the lanterns, it left out of account the way in which that light might be distributed or focused on the

road surface. If the actual effect on the surface of the road was increased from 0·18 to 0·43 lumens per sq. ft. it represented a ratio of 1 to 2½—a just perceptible improvement.

For comparison with the figure he had mentioned, the gross lighting of Ministry of Transport Group A was 1·17 to 2·7 lumens per sq. ft., and of Group B 0·2 to 0·8 lumens. If only 0·43 lumens were emitted from the lanterns the improvement was only to a relatively low Group B standard.

Investigations which had been made in this country on quite a number of roads which were improved to Group A standard lighting gave no significant improvement in accident records, and the lighting in practically all these cases was previously below Group B standard.

The Hartford figures given in Table 12 were remarkable. It appeared obvious that the worst section was picked out for experiment. This section might have been so bad—so notoriously bad—that the installation was given a considerable amount of publicity and attention was drawn to the need for care, i.e. the accompanying publicity may have been responsible for part of the improvement.

In Table 14 degrees of curvature of the road were given in relation to their bearing on accident rates. The expression "degrees of curvature" had different meanings. In American practice it was usually a measure of the radius of the curve. In the Table it appeared to mean the amount by which the road in question deviated from the straight, i.e. the sum of the deflection angles. This left out the radius of the curve, which was a very important factor.

Table 15 dealt with the effect of the width of the road, and the author suggested that increasing the pavement width to 23 ft. and over was desirable. This seemed to be higher than the Table justified. A difference which must be borne in mind was that under the Pan-American Agreement vehicles could be up to 8 ft. in width and in many States even wider. In this country the restriction was to 7 ft. 6 in.

In section 15 he would take exception to the statement that "success or otherwise has usually been decided by the personal impressions of those responsible for the changes." That imported a suggestion of bias which he did not accept. In fact, in the majority of these cases success or failure had been decided by the collected opinions of all those interested—the police forces, engineers, road users' organizations, special committees, and so on—and in a number of important cases by statistical comparisons of journey times and delays.

Dr. F. Garwood said that in connection with the relationship between road deaths, the number of vehicles and the population referred to by Sir Charles Goodeve, it might be of interest to point out some of the results which could be obtained fairly simply from the existing statistics of the number of casualties in different police districts. It was found that the population casualty rates in the town police districts, that is, those generally formed by the county boroughs and the larger cities, were rather lower than the same rate for the remaining part of the country.

For instance, the casualty rate per 1,000 in the town districts was, in 1947, 3·3, and in the remaining part of the country 3·7. The same difference existed if one calculated death rates. Expressing these as per million, the rate in town districts was 82 per million, compared with 120 for the remaining parts of the country. It was interesting to note that the same difference existed in America, though there only the death rates were available. In towns with a population larger than 10,000 the death rate was about 119 per million, and in the whole of the United States it was 225, indicating that the rural parts of the country had a much higher death rate. The same distinction existed in Scotland, but the rates there were rather lower than those for England and Wales. One would like to associate these differences with differences in traffic conditions. The author had taken the number of registered vehicles, but what one would like to get was the total vehicle mileage on the roads travelled. There were obvious practical difficulties in obtaining that figure, but one could get some idea of its effect from existing statistics of traffic flow.

Before the war a census of traffic on classified roads was taken every three years during a week in August by the Ministry of Transport, and some of the figures for Class 2 roads had been collected together and published for different counties. If one took the population accident rate in the county police districts of England and Wales one found quite a high correlation with the average traffic on the Class 2 roads. The correlation was highest if one merely considered casualties to pedestrians, it was lowest for drivers and occupants of vehicles, and intermediate for pedal cyclists. He thought that was what one might expect. Roughly speaking, the casualty rate was a measure of the chances of being killed or injured on the road. It might be assumed that pedestrians were not likely to wander far out of the district in which they were registered for census purposes. This was less likely in the case of cyclists, and could certainly not be assumed in the case of motorists.

He would elaborate a little the work of the Laboratory in Buckinghamshire. In one investigation they studied that part of the Bath Road (a road with very heavy traffic) going through the

neighbouring borough of Slough. This was divided into several sections, and the traffic and vehicles involved in accidents were classified by the various sections. Then they divided the number of vehicles of different kinds involved in a personal injury accident by the vehicle-miles travelled in each section. They found that the central part, which was very busy from the point of view of pedestrian and shopping activity, had an accident rate of 14·5 vehicles involved per million vehicle-miles travelled, whereas in the part of the road which was not built-up the rate was about 5 vehicles involved per million vehicle-miles.

The same investigation revealed that motor-cycles were involved in accidents about three times as frequently as the rest of the vehicles on the road per vehicle-mile. That did not necessarily suggest more carelessness on the part of motor-cyclists; the reason was presumably that, once involved in an accident, motor-cyclists were much more likely than motorists to suffer injury. But it was interesting to find that the rates of the order of from 5 to 10 vehicles involved per 1,000,000 vehicle-miles were very similar to what was obtained from transport undertakings which travelled all over the country.

He had mentioned accident rates calculated on a length of road, but, of course, accidents tended to congregate at "black spots," and were not usually distributed uniformly along the road. Here the important appropriate criterion was the number of vehicles involved in accidents at the points, divided by the number of vehicles that passed; mileage did not enter into the question.

Dr. J. O. IRWIN referred to a remark made by the author in discussing the distribution of accident proneness amongst the population. He had stated that it could be shown that the moments of the λ distribution were equal to the factorial moments of the accident distribution. Was that of any λ distribution?

Dr. SMEED: Yes.

Dr. IRWIN added that he only wished to enquire whether it was a general property. He could see that it happened in certain particular distributions.

The following contributions were received in writing after the meeting:

Dr. J. O. IRWIN: I have discovered a very simple short proof of the moment property to which I referred in speaking. If the number of accidents is x and

$$x = \lambda + \varepsilon,$$

where ε is a Poisson deviate from a mean λ, the factorial moment generating function of x

$$\mathrm{E}\{(1 + t)^x\} = \mathrm{E}\{(1 + t)^\lambda \, \mathrm{E}_\lambda(1 + t)^\varepsilon\}.$$

Now, $\mathrm{E}_\lambda(1 + t)^\varepsilon$ being the f.m.g.f. of the Poisson about its mean λ is equal to $(1 + t)^{-\lambda} e^{\lambda t}$, whence

$$\mathrm{E}\{(1 + t)^x\} = \mathrm{E}(e^{\lambda t}).$$

that is the f.m.g.f. of the accident distribution is equal to the m.g.f. of the λ distribution.

Mr. F. A. A. MENZLER: Most of the purely statistical points arising on the paper were fully dealt with in the discussion. There is, however, one major point to which attention should be directed, namely, the basic question in all statistical work, that of definition. The number of "accidents" can mean many different things, according to the administrative rules of thumb observed from time to time in counting them as well as the country in which the observations are made. It would be surprising if, for example, the "fatalities" ascribed to road accidents in different countries are as homogeneous as Table 4 might seem to imply. Statisticians—and actuaries—are prone to fit formulae to data, but when that operation has been performed, can it be said, with the author in his comments on Table 4: "It follows from the above formula that . . ."?

To turn to wider issues, it is a sobering fact that the numbers of persons killed and injured in road accidents (Table 6) exhibit some similarity, particularly in respect to deaths, with the experience in regard to diphtheria about 40 years ago. Something has been done about diphtheria. During the decennium 1901–1910 deaths averaged 6,000 per annum; last year we were content with no more than 250 deaths and fewer than 6,000 notifications. It is not generally realized that, in England and Wales, the number of motor vehicles free to take the road last year was about 10 per cent. higher than before the war. The number of licences current at the end of September, 1947, was a little over 3,000,000 compared with about 2,750,000 at the end of September, 1938. The number of private cars has remained stationary at about 1,800,000. Goods vehicles have

gone up by 160,000 to 610,000, motor cycles by about 70,000 to just over 500,000. The category of "hackneys," which includes taxis and motor buses, has increased by about 20,000 to 107,000. In the London Transport area the total number of licences, however, is about the same as before the war, viz. 725,000, private cars being down by about 50,000 to 420,000, this decline being offset by increase of 35,000 in goods vehicles to 165,000 and of 14,000 in motor cycles to 115,000. It is true that there has been a fall in the number of persons killed on the roads in Great Britain to below 5,000, but some part of this improvement must be attributed to the petrol restrictions. If and when these are relaxed, the congestion on the roads will be more in keeping with the material increase in the number of vehicles, and it may be difficult to hold the gain in safety implied by the figures in Table 6. Moreover, it must be assumed that as economic conditions improve there will be a considerable growth in the number of private cars, so increasing congestion and tending to increase accidents.

London Transport naturally possesses a good deal of information about the road accidents in which its vehicles are concerned, but the data are not capable of direct comparison with national figures. The definition of an "accident" reflects an outlook which *inter alia* looks at occurrences as possible sources of claims. The global figures of "accidents," therefore, include a considerable element of dilution in comparison with the police statistics though, of course, they should be comparable *inter se*, i.e. as between one form of transport and another. Then, again, London Transport drivers are a select class, but driving large vehicles with, in the case of the trams, restricted mobility.

London Transport runs roundly 9,000 motor transport vehicles, a little fewer before the war, a little more now. Of these, rather more than 5,000 are the red "Central Buses" and 2,600 are trams and trolleybuses. Before the war, however, the number of trams was over 1,500, while, as a result of conversions to trolleybuses, it is now below 1,000. As will be seen later, this change has an important influence on the number of "accidents." Whatever limitations may attach to these accident figures, we have the supreme advantage that we have a denominator in the shape of an exact figure of the car miles run on each form of transport. In 1947 we ran 406,000,000 car miles on the road services compared with 399,000,000 in the last pre-war year. Of these, the Central Buses accounted for 230,000,000 and the trams and trolleybuses roundly 110,000,000 in both periods; the balance was operated by the green Country Buses and Coaches. Subject to the qualification about the definition of "accident," the numbers of accidents per 100,000 car miles were as follows:

	1938/39	1947
Central buses	12	13
Trams	32	25
Trolleybuses	25	16
Country buses and coaches	7½	7½

These rates of accident reflect *inter alia* the varying densities of traffic in the areas served by the different forms of transport, but the disproportion of accidents concerned with the trams is striking. Although there has been a material fall since pre-war days, possibly owing to the decline in private car mileage, this feature runs through the whole of the statistics and is, of course, associated with the tram's lack of manoeuvrability and inability to draw up to the kerb. Doubtless some part of the excess over central buses is also due to the character of many of the tram routes which run over roads carrying very heavy traffic. Trolleybuses, too, as successors to the trams, also tend to serve routes with heavy traffic. This fact probably goes far to account for the excess of the trolleybuses over the central buses figure.

An unambiguous category in the group of accidents is that of "collisions," including those with pedestrians. The collisions per 100,000 car miles, in which London Transport vehicles were concerned, were as follows:

	1938/39	1947
Central buses	6	6
Trams	21	16
Trolleybuses	12	8
Country buses and coaches	5	4
Total	9	7

It will be seen that the trams produce about 2½ times as many collisions as the central buses for a given mileage run. The trolleybuses again show a marked improvement on their pre-war

record. Road users are presumably becoming "acclimatized" to their silence and rapid acceleration. While their collision rate still shows a 20–30 per cent. excess over central buses, this must in large measure be due to the heavier density of traffic on the roads served by trolleybuses in comparison with that in the wider area served by central buses.

The improvement in the trams and trolleybuses compared with pre-war must, in part, be associated with petrol rationing, and the consequent smaller mileage operated by the reduced number of private cars on the road in London. On the latter point, it may be observed that private cars now account for 2 collisions per 100,000 car miles over all forms of transport as compared with 3 per 100,000 car miles before the war. While, however, for other motor vehicles the figure is about the same as pre-war days, namely, 3 collisions per 100,000 car miles, pedal cycles are concerned in only half the proportionate number of collisions, the figure having fallen from 0·7 per 100,000 car miles to 0·35. Collisions with pedestrians remain at much the same figure, namely, ½ per 100,000 car miles. If, however, the pedestrian figures are broken down to forms of transport, we find that the trams and trolleybuses "collide," proportionately to car miles, with about twice as many people as the central buses, the trams being rather worse than the trolleybuses. In the case of collisions with other motor vehicles, the trams again top the bill with 4 times as many collisions, proportionately to mileage run, as the central buses. These figures demonstrate the disadvantages of the tram from the point of view of road safety. As already announced, the remaining trams are to be replaced in due course by buses.

It is customary in these discussions to look primarily at fatal accidents, although from the point of view of economic waste they represent a relatively small proportion of the total cost expressed in financial terms. So far as London Transport are concerned, the number of fatal accidents is of the order of about 140 a year, i.e. 3½ per ten million car miles, the figure in 1947 being much the same as before the war.

It is a depressing thought that only in 1949 will the collection of adequate statistics of road accidents begin. Dr. Smeed's paper demonstrates the need for further research, not only statistical research in the orthodox sense, but also operational research as that term is now understood to-day. No systematic progress will ever be achieved by the adoption of numbers of perhaps competing remedies without concurrent scientific observation of the effects of applying them. There is indeed scope for yet one more thesis, a sociological one, on the reasons why the community has tolerated the continuance of annual carnage on the roads on a scale equivalent to that of a fair-sized war.

Mr. E. van Rest: Dr. Smeed is to be thanked for having presented us with this comprehensive survey of the accident data available. The question of road safety has always been everybody's subject and much personal opinion has been voiced, not always with any sound foundation of observation of facts. We now have the prospect of a scientific study of the available data, and the scientific collection of further data.

Dr. Smeed makes a plea at the end of his paper that a large collection of facts should be built up; I would strongly support that plea. This is a subject so wide and so intimately concerned with our daily life that there is ample room for workers in this field to reinforce the good work recently begun by the Road Research Laboratory. I would emphasize that what is most strikingly lacking in this field is the "background" data to provide the denominators of the interesting fractions. As an example, only too often, in Great Britain at least, we have to be content with total population, or total number of cars as the divisor for our observed number of accidents. To get this background data the co-operation of many authorities may be needed, authorities only indirectly interested in Road Safety Research. It may mean collecting data which at first sight appear to have no immediate bearing, but this is a plea that it shall be done. If anyone is tempted to oppose the expenditure of time on the collection of such data I would ask him to consider first the cost to the country of the number of accidents we do in fact sustain, and second, that "knowledge is power."

The example that occurs to me as showing the need for denominators results from the observation that few drivers over 90 years of age are involved in accidents. Divorced from an observation on the number of drivers over 90, this informs us little about the liability to accident of drivers over 90. Such omissions, although elementary, are nevertheless responsible for not a little false reasoning on the subject of road accidents.

I am interested in the equation given in paragraph 2 as giving an over-all risk figure for the different types of accident—vehicle-vehicle, vehicle-pedestrian, vehicle-other. I would point out, however, that as it stands it is possible to get a negative value for the coefficient of N^2; this would result if c, the coefficient for vehicle-pedestrian, were greater than b, since, strictly speaking, the equation reads

$$D = aN + bN^2 + c\,N(P - N)$$

which is

$$= aN + (b - c)N^2 + cNP$$

Negative values of $(b - c)$ have, I believe, been observed, and are difficult to explain on the original equation.

In other words, as the number of cars increases, with fixed population, the number of accidents may not increase proportionately because more of the people are then *in* the vehicles, where they are relatively safer. The conclusion that everyone should have a car is not necessarily the logical one.

Mr. T. C. Foley: The discussion on Dr. Smeed's excellent paper will not be complete unless something is said about the effect of speed limits on the rate of accidents. A certain amount of information is available on this important aspect of the problem.

When a speed limit of 30 m.p.h. was introduced for roads in built-up areas in the year 1934 there was an immediate fall in the number of accidents. This was described by Mr. Hore-Belisha, then Minister of Transport, in the following terms:

> "During the 28 weeks the speed limit has been in operation the number of persons killed in towns where, generally speaking, it is in force, has fallen by 22 per cent. That is more than twice as much as the percentage in county areas, where it is not generally in force."

The efficacy of the limit was most strikingly demonstrated in the Metropolitan Police Area. The Commissioner of Police in his Report for the year 1935 wrote:

> "But the clearest single item of evdence in regard to the value of a limit on speed can be found by comparing the pedestrian fatalities caused by (1) vehicles of the private car class which were brought within the 30 m.p.h. speed limit on the 18th March and (2) commercial vehicles which were subject to speed limits before that date. The statistics for the second quarter of the year, when the general speed limit was in force and was generally respected, show that, whereas there was no appreciable change in regard to the pedestrian fatalities caused by vehicles of the commercial class, the corresponding fatalities caused by private cars were brought down by 50 per cent., as compared with the previous quarter."

In eight large municipal corporations Ministry of Transport enquiries were held to consider applications by motoring organizations to have certain roads "derestricted" or freed from the limit. Figures were given of the number of accidents on these roads for the same period before and after the reduction of the limit: the figures were 580 and 435 respectively.

In three other boroughs after roads had been "derestricted" against the wishes of the local authorities, there was such a sharp increase in the number of accidents that within a short period of time the Ministry of Transport was compelled to restore the limit.

The speed limit of 20 m.p.h. in the Royal Parks was established because of the startling increase in the number of motor accidents. Since the imposition of the speed limit, according to Sir Lionel Earle, former Secretary of H.M. Office of Works, the accidents have enormously diminished.

In the L.C.C. parks there is a speed limit of 12 m.p.h. and "traffic accidents to children in the parks are almost unknown; it would be extremely difficult to trace any record of one" (*Road Safety of the London Child*, published by the L.C.C.).

The most notable speed limit experiment abroad is that in the City of Providence, U.S.A., where a limit of 25 m.p.h. was introduced in 1938, combined with a propaganda campaign to all classes of road users. The number killed was reduced from 41 to 16 and the injured from 1,432 to 713 in the first year of the experiment, and Providence by comparative standards has remained the most accident-free city in the United States.

Dr. Smeed subsequently wrote as follows:

I regret that I puzzled a number of readers of my paper, including Sir George Maddex, by not explaining how the second empirical relationship between D, N and P was deduced from the first.

$$\text{We have } D = 0{\cdot}0000993N^{0{\cdot}3377}P^{0{\cdot}7323}$$
$$= 0{\cdot}0000993(NP^2)^{\frac{1}{3}} \,.\, N^{{\cdot}0044}P^{{\cdot}0656}.$$

Now, over the range of values of N and P considered, N varies from 54,000 to 29,853,000 so that $N^{{\cdot}0044}$ varies from 1·049 to 1·078. P varies from 1,286,000 to 129,825,000 so that $P^{{\cdot}0656}$ varies from 2·52 to 3·40. Similarly $N^{{\cdot}0044}P^{{\cdot}0656}$ varies from 2·640 to 3·674. Hence, over the range considered, estimates of D will not be greatly in proportionate error if it is assumed that $N^{{\cdot}0044}P^{{\cdot}0656}$ takes a constant value of, say, 3. We then get, approximately,

$$D = 0{\cdot}0003(NP^2)^{\frac{1}{3}}.$$

Sir George Maddex is, of course, quite right in believing that the two formulae cannot be identical unless there is an inverse relation between N and P. However, for practical values of

N and P, which are the only values that need be considered in an approximate empirical formula, the two formulae do both give approximately the same results. So, also, does the formula given by Sir George Maddex, which can be approximately deduced from the second one I gave.

$$\text{We have } D = \cdot 0003(NP^2)^{\frac{1}{3}} = \cdot 0003 \left(\frac{N}{P}\right)^{\frac{1}{3}}. \; P = \cdot 0003 \left[\frac{1}{27} + \left(\frac{N}{P} - \frac{1}{27}\right)\right]^{\frac{1}{3}}. \; P.$$

$$= \cdot 0001 \left[1 + 27\left(\frac{N}{P} - \frac{1}{27}\right)\right]^{\frac{1}{3}}. \; P.$$

Expanding the expression in the square bracket by the Binomial Theorem, we get:

$$D = \cdot 0001 \left[1 + 9\left(\frac{N}{P} - \frac{1}{27}\right)\right] P. \text{ if } \left(\frac{N}{P} - \frac{1}{27}\right) \text{ is not too large, i.e. if } 10^3 N/P \text{ is about 37.}$$

Fig. 1 shows that the values of N/P for a number of the countries considered are about this number.

Hence, $D = \cdot 0001 \left[\frac{2}{3} + 9\frac{N}{P}\right] P = \cdot 001 \left[\frac{P}{15} + \frac{9}{10}N\right]$, which is only slightly different from the formula of Sir George Maddex.

A formula is fitted to a set of data in order to summarize it and make it easier to deduce the properties of the data. Sir George Maddex's formula fulfils this requirement to some extent and mine does so rather better. However, the conclusions deduced, if legitimate, should all be the same. We can easily get the same qualitative conclusions from his as from mine. His formula can be expressed either as $1{,}000 \frac{D}{N} = \frac{4}{5} + \frac{1}{15} \cdot \frac{P}{N}$ or as $1{,}000 \frac{D}{P} = \frac{4}{5} \cdot \frac{N}{P} + \frac{1}{15}$, so that the fatality rate per registered vehicle decreases as the proportion of vehicles to the population increases whilst the fatality rate per unit of population increases.

Sir George Maddex says that it would be interesting to see how the formula $D = aN + bN^2 + cNP$, based on *a priori* assumptions, fitted the data given in Table 4. Since $\frac{D}{N}$ would then be $a + bN + cP$, and would, for constant values of P, increase with N, a glance at Fig. 1 shows that no satisfactory fit is possible since $\frac{D}{N}$ does not increase with N.

Sir George is, of course, quite right in believing that I did not imagine that any formula involving merely the population and number of vehicles would give a very good fit. However, the data do appear to show that there is an approximate relation, and we must be guided by the data and not by our preconceived ideas. Mr. Menzler comments in a similar vein, and says that the definition of a road accident can mean many different things in different countries. There are certainly differences in definition, but there is no reason to think that as far as fatalities are concerned these differences are sufficiently great to invalidate the conclusions reached.

Mr. Adams would like to see the effect of applying the formula to different counties in this country, to see whether one could find some indication of which were good and which were bad counties from the point of view of road accidents, and, if possible, to find out why. So many vehicles pass through some counties which do not belong to those counties that I doubt whether this could usefully be done. However, since writing the paper I have fitted the formula $D = \cdot 00030(NP^2)^{\frac{1}{3}}$ to all the sets of data which I could conveniently procure and compared the actual with the expected number of fatalities in each case. A summary of the results is given below, together with the fatality rates per unit of population and per registered vehicle.

Source of data	$10^5 \frac{D}{P}$		$10^3 \frac{D}{N}$		*Ratio of actual to expected fatalities*	
	Mean	*Coefficient of variation (per cent)*	*Mean*	*Coefficient of variation (per cent)*	*Mean*	*Coefficient of variation (per cent)*
Twenty countries of the world, 1938	11·1	·46	3·18	·87	1·01	·28
Great Britain, 1909–1946	10·3	·45	3·87	·40	1·05	·26
U.S.A., 1913–1946	22·6	·27	1·25	·38	1·29	·20
Police Districts of Gt. Britain, 1947*	10·8	·28	1·67	·39	0·88	·23
States of U.S.A., 1946	26·5	·36	1·04	·35	1·37	·36

* Police districts with populations less than 100,000, and for which the expected numbers of fatalities are less than 50 have been omitted from the calculation.

The mean value of $10^5D/P$ for the different sets of data varies from 10·3 to 26·5, i.e. by a factor of 2·57; the mean value of $10^3D/N$ varies from 1·04 to 3·87, i.e. by a factor of 3·72. The mean value of actual fatalities to expected number based on the formula varies from 0·88 to 1·37, i.e. by a factor of 1·56. In addition, the coefficient of variation within each set of data is generally much lower for the ratio of actual to expected fatalities than for either of the fatality rates considered. The formula based on both N and P therefore gives a fairly useful prediction of the number of fatalities over a much wider range of data than that on which it was first deduced, and this prediction is very much better than any that could be produced by assuming a constant value of either D/P or D/N.

Both Sir George Maddex and Mr. van Rest rightly draw attention to the necessity of obtaining exposure to risk data as well as accident data itself, and this is one of the most urgent problems of the Road Research Laboratory, but there is reason to believe that something can be done. The Americans claim to have succeeded in obtaining estimates of the vehicle-mileage per year in the U.S.A.

Sir Charles Goodeve is, of course, correct in saying that we could get more consistent data from vehicles, population, etc., inside a country rather than in a general comparison between countries. However, it is desirable to compare the accident position in this country with the position of others, and that is why I gave an international comparison. We are, of course, analysing Great Britain's data in various ways, one important division being the one suggested by Sir Charles Goodeve, namely, those accidents that would be expected to be proportional to the number of vehicles, those that would be expected to be proportional to the square of the number of vehicles, and those that would be expected to be proportional to the product of the number of vehicles and the number of pedestrians.

I wish to thank Professor Greenwood for drawing my attention to Miss Newbold's paper, *Journal of the Royal Statistical Society*, vol. xc, 1927, in which the relations between the first four moments about the observed means of the λ and accident distributions are obtained. Although the relations found by Miss Newbold are not nearly as elegant—owing to the moments being taken about the observed means instead of about their "natural" origins—as the unpublished theorem by Manning that I quoted, they do contain the essence of the practical results. I should perhaps have also cited a number of other papers on accident proneness, and especially the fundamental one by Greenwood and Woods, Report No. 4, of the Industrial Fatigue Research Board, instead of just saying that "the literature is too extensive to review in a general paper." But, in covering a very wide field, it was impossible to mention all relevant papers, and I confined myself to quoting only those papers actually used in the preparation of mine.

I cannot, however, see the justification for Professor Greenwood's statements that:

(i) The beginner on reading this paper might suppose that the power moments of the λ distribution could not be obtained from the power moments of the accident distribution.

(ii) The beginner might infer from the paper that the unmasking of accident proneness was a simple matter of fitting a negative binomial to such distributions as were given in Table 20.

So far as (i) is concerned, anybody who knows the definitions of factorial moments knows that they can be expanded in terms of power moments and so far as (ii) is concerned, there is no mention of the negative binomial in my paper.

Professor Greenwood mentions the efforts made in work on accident proneness to obtain homogeneous data. In road accident work this is probably impossible to achieve, and techniques are required to deal with the resulting situation. The λ used above must be regarded as the product of the proneness to accident and the exposure to risk.

I am glad that Mr. Adams has asked what is meant in Table 6 by persons killed by motor vehicles, as the question draws attention to one of the most unsatisfactory features of much of the published statistics on road accidents. Instead of giving the number of pedestrians, cyclists, etc., killed and injured in collisions with various types of vehicle, the number is given of persons killed in road accidents in which particular types of vehicle were primarily concerned. The figures can be most misleading. However, the explanation for Mr. Adams's difficulty is that the number of accidents involving horse-drawn vehicles has been decreasing more or less continuously since 1909, whilst the number of pedal cyclist fatalities has been increasing.

Unfortunately, I do not know of any details of the lighting in Hartford, Connecticut, about which Mr. Adams also asked, other than those given in the paper to which I referred. However, it is possible for a change from 0·18 to 0·43 lumens per square foot to make a definite change in the revealing power of a street lighting installation, even though the changes in the street lighting on a number of roads in this country to which Mr. Adams refers had no perceptible effects on the frequency of accidents. For background brightness over 0·1 equivalent foot candles, increasing the number of lumens per square foot has little effect on the revealing power of street lighting

installation.* If, however, the background brightness before improvement in Hartford was well below 0·1 equivalent foot candles, the improvement in revealing power might be considerable. However, I agree that many more data of this type must be obtained before we can be certain that great improvements in the accident rate can be obtained by improving street lighting installations.

Mr. Adams also asked what is meant by degree of curvature in Table 14. The meaning is the standard one, namely the angle at the centre subtended by an arc or chord of 100 ft.

Mr. Adams also questions the conclusion that increasing the width of a two-lane road to 23 ft. seems to have a most desirable effect on the accident rate. As he apparently accepts the figures at their face value, I cannot understand this, but if he means, as I suspect he does, that increasing the width of a 22-ft. wide carriageway to 23 ft. may not be the best way of spending a limited amount of money allocated to road safety, I might agree with him.

Mr. Foley has given some interesting information on accidents before and after the speed limit was imposed in 1935 on certain classes of vehicles. It is a pity that more adequate information is not available, but the Laboratory is trying to get some by observing the effects of speed limits recently imposed. The effect of speed limits on speeds is also being watched.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

Kenneth John Carpenter.
Samuel Lee Crump.
Daniel Gordon Davies.
Christopher Horace Devenport.
Roger Tatham Eddison.
F. E. Edwards.
Alec William Tyler Ellis.
Charles Arthur Ellis.
Alexander George MacGregor Fraser.
Nancy Goodman.
Raymond Hinks.
Hastings Dudley Huggins.
Layward Jayasundara.
Daniel Jones.
Ivor Rhys Jones.
Sidney Leggatt.
William Philip Dowie Logan.
Laurens van Lottom.
John Dudley North.
Raymond Emile Frederick Oggier.
Rasin Ward Orson.
Ivan Pedersen.
James Basil Holmes Pegler.
Ronald Fife Plane.
William Joseph Reiners.
H. Albert Rhee.
Sabet Kiddis Rizkalla.
Sidney Rosenbaum.
William John Ross.
Geoffrey Wadsworth Sears.
Patrick Walsh Seddon.
Rama Singh Srivastava.
Margaret Rennie Duguid Stephen.
Alan Carruth Stevenson.
Gordon Glyn Thomas.
John Francis Thompson.
Nandadeva Wijesekera.
John Lawrence Williams.
Kenneth James Wilson.
Conrad Payling Wright.

Corporate Representatives

Leo Thomas Frank Little, *representing* The Incorporated Accountants' Research Committee.
Eric Varcoe Roberts, *representing* Leeds College of Commerce.
Mark Hartland Thomas, *representing* The Council of Industrial Design.

* J. M. Waldron. *Trans. of the Illuminating Engineering Society* (*London*). Vol. 3, Dec. 1948. A. J. Harris and A. W. Christie, Road Research Laboratory, R.N. Note No. 1148.

Earnings and Working Hours of Manual Wage-Earners in the United Kingdom in October, 1938

By R. B. Ainsworth

[Read before the Royal Statistical Society, December 21st, 1948, The President, Dr. David Heron, in the Chair]

In October, 1938, the Ministry of Labour made an enquiry into earnings and hours of labour in the manufacturing industries and some of the principal non-manufacturing industries and services. In this enquiry information was obtained as to the weekly earnings of each individual worker in the last pay week in October, and as to the hours actually worked in that week. Employers were asked to indicate whether the workers were paid by time rates or by piece rates (including other forms of payment by results), and also whether they were men, youths and boys, women or girls. They were not asked to state the occupation of each worker.

A considerable amount of information derived from this enquiry has already been published in the *Ministry of Labour Gazette*, but the publication of the full results was prevented by circumstances arising out of the war. The information published relates to the average weekly and hourly earnings in the various industries and groups of industries, the average hours worked and the proportions of time-workers and piece-workers employed. Particulars have not been published, however, showing separate averages for time-workers and piece-workers respectively, or as to the distribution of earnings and hours worked. With the permission of the Ministry a summary of the information on these latter subjects is now made available in this Paper.

The principal industries and services not covered by the enquiry were agriculture, coalmining, dock labour, shipping, the distributive trades, the catering trades, the entertainment industries, commerce and banking, and domestic service. In the section of this Paper which relates to averages the figures also exclude railway employees, since the information was tabulated in order to provide comparisons with the results of enquiries made during the war, which did not cover the railway service. In the section dealing with the distribution of earnings, however, railway servants are included in the figures.

The information supplied by employers covered all wage earners, other than office staffs, shop assistants and out-workers working at home on materials supplied by the employers; managers, commercial travellers, clerks and typists and salaried workers generally were excluded. In cases where the works were stopped for the whole or part of the week as a result of a general or local holiday, breakdown, fire, or trade dispute, employers were asked to substitute particulars for the nearest week of an ordinary character. The earnings shown were totals before any deductions had been made in respect of workers' contributions to statutory insurance schemes.

The form of enquiry was sent to all employers with more than ten workers and to a random selection of one-fifth of those with ten or less workers, asking them to supply particulars of the earnings and hours of each of their wage-earners in the last pay week in October. Separate returns were asked for in respect of each establishment in the case of firms with several premises. Of about 113,000 establishments to which enquiries were sent (excluding those with no wage-earners within the scope of the enquiry), nearly 75,000 supplied returns suitable for tabulation. The number of employees covered was about 5½ million, representing over 70 per cent. of the total number employed in the industries concerned. The proportion of the total covered in the various industries differed, but on the whole the returns were regarded as a sufficient and reliable sample, except in the case of the linen industry, in which there was a deficiency of returns from Northern Ireland.

A very small proportion of the firms, employing 1·3 per cent. of the men covered by the returns and 1·0 per cent. of the women, did not supply information as to the hours worked. In computing the average hourly earnings used in this Paper the average weekly earnings of all the workers

covered, irrespective of whether information as to the hours worked was available, has been divided by the average hours worked by those whose hours were shown. The available tabulations do not show the average weekly earnings of those for whom the hours were given, but in any case the exclusion of the small proportion whose hours were not given would not affect the results to any significant extent. In the figures given as to the distribution of weekly earnings and hours, also, the earnings figures relate to all the workers covered irrespective of whether the hours worked were also available. Again, the refinement would have little significant effect.

It may be noted that the average earnings of both time-workers and piece-workers relate to one week only. In the case of piece-workers in some industries in which earnings of individuals vary appreciably from week to week, e.g. cotton weavers, information relating to one week only is not, of course, entirely satisfactory; it tends to exaggerate the range of the distribution of weekly earnings.

The average weekly earnings, the average hourly earnings, the average hours worked, and the proportions of time-workers and piece-workers in the industries covered by the enquiry were as follows:

	Men (21 *and over*)	*Youths and boys*	*Women* (18 *and over*)	*Girls*	*All workers*
Average weekly earnings	69s.	26s. 1d.	32s. 6d.	18s. 6d.	53s. 3d.
Average hourly earnings	17·4d.	6·8d.	9·0d.	5·0d.	13·7d.
Average hours worked	47·7	46·2	43·5	44·6	46·5
Percentage on—					
Time-work	82	79	54	73	75
Piece-work	18	21	46	27	25

Analyses of these figures by industry groups, as already published in the *Ministry of Labour Gazette*, are given in Appendices I to IV. Details as to the average weekly earnings and the average hours worked in respect of each industry were given in the *Ministry of Labour Gazette* for August, 1944, and February, 1945.

It should be noted, in connection with the figures contained in Appendices I and II, that the comparison of the averages for men and women, as between the different industry groups, is affected by the different proportions of skilled, semi-skilled and unskilled workers employed, while the averages for "all workers" are affected by the varying proportions of men, women and juveniles employed.

In calculating the figures used throughout this Paper for groups of industries, the earnings and hours in each industry have been weighted by the approximate total numbers employed. This has been necessary owing to the fact that the returns received represented varying samples of the numbers of firms in each industry, and figures derived directly from the aggregation of the returns would give undue weight to the industries, such as engineering, in which the proportion of returns was relatively high, and too little weight to others, such as building, in which the sample was smaller.

The weighting system has been adopted in compiling both the average earnings and hours and the figures relating to the distribution of earnings and hours. The weights were obtained by taking the estimated total numbers of men and women employed, and dividing these totals as between time-workers and piece-workers, and as between the 10s. groups of earnings and the four groups of working hours in the proportions shown by the returns. In effect, the weights thus represented the numbers shown by the returns in the various groups raised in the ratio of the total number employed to the total number covered by the returns.

Average Earnings of Time-Workers and Piece-Workers

The average weekly and hourly earnings shown in Appendices I and II for the industry groups, and in the *Ministry of Labour Gazette* for each industry, relate to all workers, time-workers and piece-workers combined. The returns, however, contain information from which it is possible to calculate the average earnings of time-workers and piece-workers separately. The term "piece-workers" is used throughout this Paper to include all persons remunerated on the basis

of results, including those paid partly by time and partly by piece rates. For all the industries combined the averages for men (21 and over) and for women (18 and over) were as follows:

	Men	*Women*
Average weekly earnings:		
Time-workers	66s. 6d.	30s. 6d.
Piece-workers	78s. 7d.	34s. 8d.
Average hourly earnings:		
Time-workers	16·5d.	8·4d.
Piece-workers	20·6d.	9·6d.
Average hours worked:		
Time-workers	48·3	43·6
Piece-workers	45·7	43·5

The calculations have been made only for men and women. For juveniles the figures have little practical value, since, generally, the time-rates of wages vary according to age, and it is probable that the majority of the piece-workers were at the higher ages.

It will be seen that the average weekly earnings of piece-workers exceeded those of time-workers by 18 per cent. in the case of men, and by 14 per cent. in the case of women. The average hours worked by men piece-workers, however, were lower than those worked by time-workers, with the result that the average hourly earnings of piece-workers exceeded those of time-workers by 25 per cent. The average hours of women piece-workers were about the same as those of time-workers, and the average hourly earnings of piece-workers exceeded those of time-workers by 14 per cent.

These averages, of course, conceal marked differences as between different industries, and an analysis has been made, firstly by industry groups, and secondly by individual industries. The average weekly earnings and the average hours worked in the principal industry groups are shown in Appendix V. In each of the groups in which a comparison is possible the average earnings of men on piece-work were higher than those of men from time-work, and in each group the average hours of piece-workers were lower than those of time-workers. The average earnings of women piece-workers exceeded those of time-workers, with one exception, but in three cases the hours of piece-workers were slightly longer than those of time-workers.

It will be seen that the average weekly earnings of men on time-work ranged from 56s. 1d. to 83s. in the different groups, and those of men from piece-work from 60s. to 108s. 5d. The average hours ranged from 46·2 to 50·6 for time-work and from 42·4 to 49·2 for piece-work. For women the range of average earnings was from 25s. 1d. to 35s. 4d. on time-work and from 30s. 5d. to 38s. 5d. on piece-work. The average hours for women ranged from 32·8 to 46·3 on time-work and from 40·6 to 47·1 on piece-work.

The amount of difference between the average time and piece-work earnings varied considerably. This partly arises from the fact that in some cases the groups included some industries in which time-work predominated, and others in which there were substantial numbers of piece-workers. The difference is minimized where a relatively highly-paid time-working industry is combined with a relatively low-paid industry in which piece-work prevails, or vice versa.

Appendix VI contains the average weekly earnings and the average hours worked by men and women on time-work and piece-work respectively in each of the individual industries, except those in which the proportion of piece-workers was relatively small. In the industries for which figures are given in this Appendix the earnings for both time-workers and piece-workers show a very wide variation. For men, the average time-work earnings in these industries ranged from 50s. 2d. to 89s. 9d., and the piece-work earnings from 51s. 10d. to 143s. 3d. For women, the time-work earnings ranged from 23s. 9d. to 40s. 4d., and the piece-work earnings from 30s. 2d. to 45s. 9d.

The conclusions to be drawn from these figures require careful consideration and qualification. The averages shown for time-workers and piece-workers in each industry do not, in most cases, relate to similar classes of workpeople. It is common, for instance, in many industries for the lower-paid unskilled workers to be employed wholly or mainly on time-work; the piece-workers being found mainly amongst the skilled operatives. For this reason the differences tend to be greater than those which would be found between time-work and piece-work earnings for the same type of work. Again, satisfactory conclusions cannot be drawn as to the relative earnings

on piece-work of men and women respectively, since the work performed was seldom similar. Even in the case of cotton weavers, although the piece-rates were generally the same, the average earnings of women were somewhat lower than those of men, since the proportion of weavers in charge of less than four looms was higher among women than among men, and the proportion on more than four looms was larger among men.

Perhaps the outstanding feature of the figures in Appendix VI is the fact that, with four exceptions, the average hours worked by men on piece-work were lower than those on time-work. Moreover, in many cases in which the time-workers were working hours, on average, in excess of the normal week, the piece-workers were working less than full-time. The differences are not so striking in the case of women. Although the average piece-work hours for women were less than those of men in the same industry, the hours of time-workers were considerably lower in many cases, reflecting more short-time working and absenteeism.

The differences in the hours of time-workers and piece-workers have, of course, an important bearing on the relative weekly earnings. A comparison is, therefore, given in Appendices VII and VIII of the average hourly earnings of time-workers and piece-workers respectively by industry groups and by separate industries. The piece-work hourly earnings have also been expressed as percentages of the time-work earnings.

It will be seen that the excess of piece-work hourly earnings over the time-work earnings varied considerably in different industries, and, in fact, in two industries for men, and one for women, the time earnings were the higher. In compiling these Tables some hesitation was felt as to the inclusion of the column showing the relationship of time and piece-work earnings, since the implication of such figures is that one is comparing like with like, which is not the case. The comparison, while possibly of interest, has little practical value. As stated above, in many industries the bulk of the lower-paid workers were on time-work, while the piece-workers were generally the higher paid skilled workers. Also, in many industries the piece-work occupations were different from the time-work occupations. Consequently, the average time and piece-earnings do not relate to workers of similar grades, and no conclusion is possible as to the relative wages from time-work and piece-work for the same occupations. It may also be noted that, as the average hours for men on time-work in some industries were in excess of the normal working week, the average time-work earnings in these cases contained an element of excess payments for over-time which was not present in the piece-work earnings.

Distribution of Earnings and Hours

The information contained in the returns enables the distribution of weekly earnings to be analysed in conjunction with the distribution of the hours worked. The distribution of the earnings and hours for men, women, boys and girls respectively in all the industries and services combined is shown in Appendix IX. In the case of railway traffic workers, full information as to the hours worked could not be supplied. In the Appendix, therefore, figures are given of the distribution of earnings in conjunction with hours in respect of other industries, while a second column of earnings is shown including the railways workers.

The median and quartile ranges for men and women were as follows:

	Lower quartile	*Median*	*Upper quartile*
Men	55s. 9d.	67s. 11d.	80s. 6d.
Women	25s.	32s. 3d.	38s. 7d.

The normal weekly hours varied in different industries. They were mostly 47 or 48 a week, but in some industries they were 44 in some districts. The proportion of men working 44 to 48 hours was 44 per cent. The proportion working less than 44 hours was 15·5 per cent., while 39·2 per cent. were working hours in excess of 48. In the case of 1·3 per cent. the hours were not shown on the return. Of the women, 57·2 per cent. were working 44 to 48 hours, 31·2 per cent. less than 44 hours, and 10·6 per cent. more than 48 hours. The proportion of men working more than 48 hours is rather surprising. In 1938 a very small number of workers had normal hours in excess of 48. The returns showed, therefore, that at least two-fifths of the men were working hours in excess of the normal number in a period when $1\frac{3}{4}$ million workers (about 12 per cent. of the insured population) were registered as unemployed.

It should be noted that the group 44 to 48 hours includes a considerable number of workers in the building, woodworking and printing industries whose normal hours were less than the more usual 47 or 48, but it also includes some workers on short time in the industries in which the normal week was 47 or 48 hours and some workers on overtime whose normal hours were 44. Although, therefore, the group does not consist solely of full-time workers, it is of some interest to see the median and quartile ranges of the earnings of workers in the three hourly groups—under 44, 44 to 48, and over 48. These are as follows:

	Lower quartile	*Median*	*Upper quartile*
Men:			
Under 44 hours	40s.	54s. 6d.	66s. 6d.
44 to 48 hours	55s. 6d.	66s. 6d.	77s. 9d.
Over 48 hours	62s. 6d.	73s. 9d.	87s. 4d.
Women:			
Under 44 hours	21s.	27s. 3d.	34s. 9d.
44 to 48 hours	27s.	33s. 7d.	39s. 3d.
Over 48 hours	31s. 3d.	36s. 9d.	44s. 6d.

The distribution of weekly earnings in conjunction with the hours worked for men and women in each of the industry groups is shown in Appendix X. It will be seen that there are marked differences in the distribution of the earnings of men in the various groups. The differences in the case of women are not so striking. Variations, as between industry groups, in the distribution of hours actually worked are affected partly by differences in the recognized normal full-time hours. For example, in the paper and printing and building groups, in which the normal hours were less than 47, the proportion of workers in the 44 and under 47 category was relatively high. In the case of women, some of the groups show relatively high proportions working less than 44 hours, and, compared with the men, relatively low proportions in the over 48 category.

The differences in the distribution of earnings in the various industry groups can be more clearly seen in Appendix XI, which shows the medians and quartile ranges. It will be seen that in round figures the lower quartiles for men fell within the range of 50s. to 60s., that of the medians in the range 60s. to 70s., and that of the upper quartiles between 70s. and 80s. There were, however, important exceptions. For women, the lower quartiles were within a range of 25s. to 30s., the medians 30s. to 35s., and the upper quartiles 35s. to 40s. It is worth noting that the broad ranges for women were about half those for men. The relatively low range of earnings for men in the textile industries was largely due to the fact that, in October, 1938, there was considerable under-employment in the cotton industry. The relatively low lower quartile for women in the public utility services was due mainly to the considerable proportion of part-time workers employed.

The medians and quartile ranges for time-workers and piece-workers respectively were as follows*:

	Lower quartile	*Median*	*Upper quartile*
Men:			
Time-workers	54s. 9d.	66s. 3d.	78s. 3d.
Piece-workers	62s. 6d.	76s. 6d.	93s. 3d.
Women:			
Time-workers	24s. 0d.	30s. 3d.	37s. 0d.
Piece-workers	27s. 0d.	34s. 3d.	40s. 6d.

* The figures in these tables are approximate (to the nearest 3d.) owing to lack of detail in the available tabulated material.

One feature of these figures is that, in the case of men, the gap between the earnings of time-workers and piece-workers gradually widens as the wage-scale rises. At the lower quartile level the gap is 7s. 9d., at the median 10s. 3d., and at the upper quartile 15s. This feature is not present in the case of women, where the differences are 3s., 4s., and 3s. 6d. Some comment on this is given below in relation to the figures in Appendix XI. It should be noted that the spread of

earnings is affected to some extent by the fact that the information as to piece-work earnings related to one week only. If information were available in respect of, say, four, consecutive weeks, the range might be somewhat narrower.

The corresponding figures for workers in the different hour-groups were as follows*:

	Lower quartile	*Median*	*Upper quartile*
Men:			
Time-workers:			
Under 44 hours	36s. 0d.	49s. 6d.	62s. 6d.
44 to 48 hours	55s. 0d.	65s. 0d.	76s. 0d.
Over 48 hours	61s. 3d.	71s. 6d.	84s. 6d.
Piece-workers:			
Under 44 hours	51s. 0d.	66s. 3d.	80s. 3d.
44 to 48 hours	60s. 9d.	73s. 3d.	90s. 3d.
Over 48 hours	73s. 6d.	87s. 0d.	102s. 6d.
Women:			
Time-workers:			
Under 44 hours	20s. 3d.	25s. 6d.	31s. 3d.
44 to 48 hours	25s. 6d.	31s. 9d.	37s. 9d.
Over 48 hours	30s. 0d.	35s. 0d.	40s. 0d.
Piece-workers:			
Under 44 hours	22s. 0d.	29s. 9d.	37s. 0d.
44 to 48 hours	30s. 3d.	35s. 6d.	42s. 0d.
Over 48 hours	32s. 6d.	38s. 3d.	49s. 3d.

* The figures in these tables are approximate (to the nearest 3d.) owing to lack of detail in the available tabulated material.

It will be seen that, in the case of men, the quartile range for piece-workers working less than 44 hours is much nearer to that of those working 44 to 48 hours than in the case of time-workers. This is partly accounted for by the fact that many of the time-workers working less than 44 hours were lower-paid men in industries in which time-work predominated, but the same feature is found in the industries in which both time-workers and piece-workers were employed. The figures suggest that the piece-workers made up for some loss of working-time by greater effort when at work. This factor is not apparent in the figures relating to the earnings of women.

Figures are given in Appendix XI showing the median and quartile ranges for time-workers and piece-workers separately in each of the industry groups. In most of the groups, as in the case of all the groups combined, the figures show that for men the gap between the medians for time-workers and piece-workers respectively is greater than that between the lower quartiles, and greater again between the upper quartiles. This feature is most marked in the clothing and paper, printing, etc., groups, in which it is due largely to the relatively very high earnings on piece-work of some of the skilled men. In the metal group the gap is relatively high at each of the points, but it does not show so marked a progressive increase. This suggests that there was a concentration of substantial blocks of piece-workers within the quartile range whose earnings were appreciably higher than those of time-workers, and that these blocks outweighed the effect of the very high earnings of a relatively small number of workers in such industries as iron and steel manufacture.

Comparison with 1906

The previous enquiry into the earnings of individual workers, made by the Labour Department of the Board of Trade, related to 1906, usually a week in September, and it is of interest to compare the results of the two enquiries. These enquiries covered practically the same industries, but figures relating to mining and quarrying were excluded from the available statistics for 1906 relating to all the industries combined. For the purpose of comparison, therefore, mining and quarrying has been excluded from the 1938 figures which follow. The 1906 figures also covered Eire, which was not covered by the 1938 figures. It is not practicable, however, to adjust

the 1906 figures to exclude Eire. Moreover, the 1906 figures are based on the straight aggregation of the returns without any re-weighting to adjust for differences in the size of the sample in different industries. The enquiry of 1906 was considerably more detailed than that of 1938, and the response of employers was not so satisfactory. The total number of workers covered by the returns was rather less than 3 million, but steps were taken to insure that the results were based on a representative sample. Taking all these facts into consideration, however, a rough comparison is possible between the figures for 1906 and 1938. The following are the main figures for men and women:

	1906		1938	
	Men	*Women*	*Men*	*Women*
Average	29s. 2d.	13s. 7d.	69s. 0d.	32s. 6d.
Lower quartile	22s. 0d.	10s. 0d.	55s. 11d.	25s. 0d.
Median	27s. 9d.	12s. 9d.	68s. 0d.	32s. 3d.
Upper quartile	35s. 3d.	16s. 8d.	80s. 9d.	38s. 7d.

The average hours actually worked were not ascertained in 1906. The average of the normal hours was 54·4 a week, but this figure is not comparable with the average of 46·5 hours actually worked in October, 1938. The normal hours were not ascertained in 1938.

The average earnings in April, 1948, were 134s. for men and 72s. 11d. for women, showing increases of 94 per cent. and 124 per cent. respectively in the 9½ years since October, 1938. The figures for 1906 and 1938, which as stated above are only roughly comparable, showed an increase in the 32 years of about 140 per cent. for both men and women.

The quartile and median earnings quoted above cover all workers, irrespective of the number of hours worked. For 1906 information is available in respect of those who worked full-time. For 1938 similar information is not directly available, but, for comparison, the figures for those who worked 44 to 48 hours inclusive, i.e. the range of the normal hours in the industries covered, have been extracted. The workers in this group included, however, not only those who worked a full week, but also some workers who were working short-time and overtime. Nevertheless a comparison between the range of earnings of full-time workers in 1906 with the range for those who worked 44 to 48 hours in 1938 is of interest. The figures are as follows:

	1906		1938	
	Men	*Women*	*Men*	*Women*
Average	28s. 8d.	14s. 2d.	67s. 6d.	33s. 9d.
Lower quartile	20s. 9d.	10s. 7d.	55s. 6d.	27s. 0d.
Median	26s. 7d.	13s. 4d.	66s. 6d.	33s. 7d.
Upper quartile	34s. 3d.	17s. 2d.	77s. 9d.	39s. 3d.

These figures provide evidence of the improvement in the 32 years in the position of the lower-paid workers. The percentage increases shown by these figures in the average and in the median and quartiles were as follows:

	Men (*per cent.*)	*Women* (*per cent.*)
Average	135	138
Lower quartile	168	155
Median	150	152
Upper quartile	127	128

While the comparison is only approximate, it is interesting to note that the lower quartile in the case of men was 60 per cent. of the upper quartile in 1906, and 71 per cent. in 1938. The corresponding figures for women were 61 per cent. and 69 per cent. This phenomenon was, of course, due to a variety of factors, the most notable of which were the increased standardization of wages in industries and localities where standardization was lacking in 1906, and the effect of the fixing of minimum rates of wages in various low-paid industries under the Trade Boards Acts.

Summary

1. In the industries covered by the enquiry the average weekly earnings were 69s. for men and 32s. 6d. for women. The hourly earnings were 17·4d. and 9·0d. respectively. The average hours worked by men were 47·7, and those worked by women were 43·5.

2. Of the men, 82 per cent. were paid at time rates, the proportion of women so paid being 54 per cent.

3. The average weekly earnings of piece-workers exceeded those of time-workers by 12s. 1d. (18 per cent.) in the case of men, and by 4s. 2d. (14 per cent.) in the case of women. Owing to the fact that the average hours worked by men on piece-work were lower than those of men on time-work, the average hourly earnings on piece-work exceeded those on time-work by 25 per cent.

4. The quartile range of earnings for time-workers and piece-workers combined was 55s. 9d. to 80s. 6d. in the case of men and 25s. to 38s. 7d. in the case of women. The corresponding range for men on time-work was 54s. 9d. to 78s. 3d., and for men on piece-work, 62s. 6d. to 93s. 3d. For women the ranges were 24s. to 37s. on time-work and 27s. to 40s. 6d. on piece-work.

5. A rough comparison with 1906 shows that the increase in average earnings was about 140 per cent. for men and women. The increase in the lower quartile, however, was considerably greater than that in the upper quartile, indicating a general levelling up of the wages of lower-paid categories of workers.

* * * * *

The main purpose of this Paper is to present, as concisely as possible, the material contained in the available tabulations. For this reason the Appendices are somewhat lengthy in proportion to the body of the Paper. In addition to the Appendices referred to in the text I have included one more (XII), in which are given the weights used in combining the separate industry figures for men and women into group and total figures. These details may be useful to those who wish to make further calculations.

Note.—The figures given throughout these Appendices relate to one week in October, 1938.

APPENDIX I

Average Weekly Earnings

Industry group	*Men (21 years and over)*	*Youths and boys*	*Women (18 years and over)*	*Girls*	*All workers*
	s. d.	s. d.	s. d.	s. d.	s. d.
Treatment of non-metalliferous mining products	66 5	31 1	29 8	17 11	61 0
Brick, pottery and glass	63 2	27 8	27 10	14 10	47 8
Chemical, paint, oil, etc.	69 3	29 5	32 8	18 2	55 0
Metal, engineering and shipbuilding	75 0	26 1	33 4	19 11	59 8
Textiles	57 3	24 0	31 9	19 8	37 10
Leather	64 1	25 4	34 11	17 6	46 9
Clothing	64 3	24 9	32 9	17 6	35 0
Food, drink and tobacco	65 3	28 1	32 11	19 0	47 0
Woodworking	66 3	23 4	33 8	17 5	51 10
Paper, printing, etc.	84 3	24 8	34 1	17 1	57 7
Other manufacturing industries	69 1	26 8	31 9	18 5	46 6
Building, contracting, etc.	66 0	25 8	*	*	61 2
Mining and quarrying (other than coal)	60 0	30 2	*	*	56 8
Transport, storage, etc. (excluding railways)	70 0	27 1	34 11	*	65 6
Public utility services	63 1	27 7	27 8	21 5	59 8
Government industrial establishments	75 3	32 7	44 9	*	70 6
All the above	69 0	26 1	32 6	18 6	53 3

* The numbers returned in this group were very small.

Appendix II

Average Hourly Earnings

Industry group	*Men (21 years and over)* d.	*Youths and boys* d.	*Women (18 years and over)* d.	*Girls* d.	*All workers* d.
Treatment of non-metalliferous mining products	16·0	7·9	7·8	4·8	14·8
Brick, pottery and glass	15·6	7·2	7·8	4·0	12·2
Chemical, paint, oil, etc.	17·2	7·6	8·9	4·9	14·0
Metal, engineering and shipbuilding	18·8	6·8	9·0	5·3	15·2
Textiles	14·4	6·3	8·6	5·1	9·9
Leather	16·2	6·5	9·2	4·5	12·0
Clothing	17·2	6·6	9·5	4·9	9·9
Food, drink and tobacco	15·9	7·1	8·6	5·0	11·8
Woodworking	17·0	6·1	9·1	4·6	13·4
Paper, printing, etc.	21·9	6·5	9·2	4·6	15·2
Other manufacturing industries	17·1	6·8	8·6	4·9	11·9
Building, contracting, etc.	17·1	6·6	*	*	15·9
Mining and quarrying (other than coal)	15·7	7·9	*	*	14·8
Transport, storage, etc. (excluding railways)	17·2	6·8	9·2	*	16·1
Public utility services	15·4	7·0	10·1	5·9	14·9
Government industrial establishments	18·2	8·3	12·0	*	17·2
All the above	17·4	6·8	9·0	5·0	13·7

* The numbers returned in this group were very small.

Appendix III

Average Hours Worked in the Week

Industry group	*Men (21 years and over)*	*Youths and boys*	*Women (18 years and over)*	*Girls*	*All workers*
Treatment of non-metalliferous mining products	49·8	47·3	45·5	45·2	49·3
Brick, pottery and glass	48·7	45·9	42·6	44·1	46·8
Chemical, paint, oil, etc.	48·4	46·7	44·0	44·6	47·2
Metal, engineering and shipbuilding	48·0	45·9	44·2	44·7	47·1
Textiles	47·7	45·6	44·5	45·9	45·8
Leather	47·4	46·8	45·7	46·5	46·8
Clothing	44·8	45·3	41·2	42·7	42·4
Food, drink and tobacco	49·4	47·3	45·8	45·9	47·8
Woodworking	46·9	46·2	44·3	45·0	46·4
Paper, printing, etc.	46·1	45·5	44·4	44·9	45·4
Other manufacturing industries	48·6	46·9	44·5	44·9	46·8
Building, contracting, etc.	46·3	46·5	*	*	46·3
Mining and quarrying (other than coal)	45·9	45·7	*	*	45·8
Transport, storage, etc. (excluding railways)	48·9	48·1	45·7	*	48·8
Public utility services	49·0	47·6	32·8	43·7	48·0
Government industrial establishments	49·5	47·1	44·9	*	49·1
All the above	47·7	46·2	43·5	44·6	46·5

* The numbers returned in this group were very small.

APPENDIX IV

Percentage of Workers on Time Work

Industry group	*Men* (21 *years and over*)	*Youths and boys*	*Women* (18 *years and over*)	*Girls*	*All workers*
Treatment of non-metalliferous mining products	85	91	70	60	85
Brick, pottery and glass	66	76	49	71	65
Chemical, paint, oil, etc.	95	95	76	85	91
Metal, engineering and shipbuilding	61	61	45	58	59
Textiles	71	86	36	65	55
Leather	66	88	77	86	73
Clothing	64	87	59	78	65
Food, drink and tobacco	92	95	73	83	86
Woodworking	87	91	74	81	85
Paper, printing, etc.	95	97	71	86	88
Other manufacturing industries	73	77	58	66	69
Building, contracting, etc.	100	99	*	*	99
Mining and quarrying (other than coal)	73	91	*	*	73
Transport, storage, etc. (excluding railways)	97	99	97	*	98
Public utility services	99	99	99	100	99
Government industrial establishments	81	72	76	*	80
All the above	82	79	54	73	75

* The numbers returned in this group were very small.

APPENDIX V

Average Weekly Earnings and Average Hours Worked Analysed for Time-workers and Piece-workers

A. *Industry Groups*

Industry group	*Average weekly earnings*				*Average hours worked in the week*			
	Men		*Women*		*Men*		*Women*	
	Time s. d.	*Piece* s. d.	*Time* s. d.	*Piece* s. d.	*Time*	*Piece*	*Time*	*Piece*
Treatment of non-metalliferous mining products	66 5	68 6	27 7	35 6	50·6	44·9	45·3	46·3
Brick, pottery and glass	60 10	67 10	25 1	30 5	50·3	45·4	43·5	41·8
Chemical, paint, oil, etc.	68 10	76 6	31 1	38 1	48·6	45·4	43·8	44·7
Metal, engineering and shipbuilding	69 1	84 3	30 0	36 2	49·3	46·2	44·7	43·9
Textiles	56 1	60 0	27 11	33 10	48·5	45·6	44·6	44·3
Leather	62 0	68 1	35 4	33 8	48·8	44·6	46·3	43·4
Clothing	62 6	67 1	31 8	34 3	46·2	42·4	41·8	40·6
Food, drink and tobacco	64 8	72 8	30 10	38 5	49·4	49·2	45·4	47·1
Woodworking	65 4	71 8	33 5	34 4	47·3	44·4	44·8	43·1
Paper, printing, etc.	83 0	108 5	33 3	35 9	46·3	44·4	44·9	43·5
Other manufacturing industries	66 3	75 9	29 11	34 1	49·3	47·1	45·1	43·9
Building, contracting, etc.	66 0	*	*	*	46·3	*	*	*
Mining and quarrying (other than coal)	58 1	64 6	*	*	47·2	42·4	*	*
Transport, storage, etc. (excluding railways)	70 0	*	34 11	*	48·9	*	45·7	*
Public utility services	63 1	*	27 8	*	49·0	*	32·8	*
Government industrial establishments	70 9	94 10	*	*	49·5	49·0	*	*
All the above	66 6	78 7	30 6	34 8	48·3	45·7	43·6	43·5

* The numbers returned in this group were very small.

Appendix VI

Average Weekly Earnings and Average Hours Worked Analysed for Time-workers and Piece-workers

B. *Separate Industries*

Industry	*Average weekly earnings*				*Average hours worked in the week*			
	Men		*Women*		*Men*		*Women*	
	Time s. d.	*Piece* s. d.	*Time* s. d.	*Piece* s. d.	*Time*	*Piece*	*Time*	*Piece*
Treatment of non-metalliferous mining products								
Coke ovens, etc.	70 3	79 5	*	*	50·6	48·2	*	*
Cement	72 0	71 3	*	*	52·4	48·2	*	*
Limekilns and whiting	54 1	59 1	*	*	48·3	42·3	*	*
Cast stone and cast concrete	52 2	67 6	*	*	49·2	48·3	*	*
Brick, pottery and glass								
Brick, tile, pipe, etc.	59 0	66 9	27 1	30 10	51·9	47·0	46·1	40·8
Pottery, etc.	57 7	65 1	24 8	30 2	47·6	43·5	42·8	41·7
Glass and glass containers	66 7	76 0	25 2	32 9	48·2	42·4	43·8	43·9
Chemical, etc.								
Chemicals and explosives	68 9	77 2	30 5	38 4	48·9	45·9	44·4	45·2
Metal, engineering and shipbuilding								
Pig iron manufacture	75 3	94 6	*	*	49·2	46·3	*	*
Iron puddling, steel melting, rolling, etc.	70 7	91 7	*	*	48·9	42·8	*	*
Non-ferrous metal manufacture	67 11	82 8	30 3	34 0	49·8	45·4	43·4	42·5
Tin plate and sheet manufacture	66 6	91 11	29 2	37 8	47·3	36·4	41·8	40·0
Iron and steel tube manufacture	67 0	77 1	24 10	34 3	51·1	42·0	43·5	46·0
Wire, wire netting, etc.	64 4	78 1	27 7	33 0	51·1	43·7	45·6	45·5
General engineering	67 11	82 10	28 9	35 2	49·7	47·5	44·3	43·4
Electrical engineering	69 5	80 6	29 4	34 1	50·1	48·4	45·0	44·7
Marine engineering	71 1	85 6	*	*	50·6	48·7	*	*
Constructional engineering	69 0	80 3	*	*	50·4	48·6	*	*
Motor vehicles, cycles and aircraft	75 10	89 2	29 11	45 9	49·0	47·2	43·7	41·3
Shipbuilding and repairing	65 9	84 0	*	*	45·8	42·4	*	*
Railway carriage and wagon building and repairing	61 9	77 3	*	*	48·9	47·9	*	*
Electrical apparatus, etc.	69 3	82 5	31 2	38 7	50·5	49·2	45·3	45·1
Hand tools, cutlery, etc.	67 7	69 9	26 3	30 11	48·8	42·7	44·2	43·0
Bolts, nuts, etc.	61 9	70 6	28 6	32 7	49·7	44·5	49·4	43·9
Brass and allied metal wares	66 2	78 2	27 7	33 9	49·2	47·1	45·1	44·8
Heating and ventilating engineering	72 9	83 9	*	*	50·6	47·9	*	*
Watches, clocks, etc.	69 8	75 9	29 9	33 1	49·4	44·5	45·4	44·5
Stoves, grates and general iron-founding	62 0	78 5	25 11	36 11	47·4	41·8	43·0	42·9
Other metal industries	67 3	75 10	23 9	33 4	49·3	45·3	44·7	43·4
Textiles								
Cotton	50 2	51 10	27 6	32 11	48·4	45·7	44·9	45·2
Woollen and worsted	56 10	60 1	27 9	33 9	48·7	45·5	44·9	42·3
Silk throwing, spinning and weaving	61 1	65 9	28 2	34 1	47·0	50·9	43·3	41·4
Hosiery	64 2	89 4	33 3	37 9	50·0	48·4	46·4	44·8
Lace	59 6	60 9	28 8	31 2	46·4	40·3	44·1	40·1
Carpets and rugs	57 3	65 11	29 2	38 11	50·2	44·9	48·3	45·8
Bleaching, dyeing, etc.	56 6	59 5	27 4	30 8	48·6	43·1	42·4	40·6

* The numbers returned in this group were very small, or the proportion of piece-workers was small.

APPENDIX VI—(*cont.*)

Industry	*Average weekly earnings*								*Average hours worked in the week*			
	Men				*Women*				*Men*		*Women*	
	Time		*Piece*		*Time*		*Piece*		*Time*	*Piece*	*Time*	*Piece*
	s.	d.	s.	d.	s.	d.	s.	d.				
Leather												
Tanning, etc.	59	1	67	6	28	10	32	4	49·0	44·5	44·8	40·6
Leather goods	61	3	62	11	30	7	32	0	48·4	46·1	46·7	45·1
Clothing												
Ready-made tailoring	67	0	75	5	32	1	33	11	46·2	43·9	43·3	39·3
Retail bespoke tailoring	71	6	68	9	35	5	42	6	45·5	42·1	43·0	45·3
Dressmaking and millinery	*		*		37	2	33	3	*	*	43·2	40·8
Hats and caps	62	0	68	1	33	2	31	8	47·2	38·7	43·0	36·4
Shirts, collars and underclothing	*		*		30	11	33	0	*	*	42·9	41·9
Boot and shoe making	60	5	67	7	35	8	40	4	44·9	42·5	43·7	42·7
Boot and shoe repairing	63	11	62	10	*		*		46·4	43·5	*	*
Food, drink and tobacco												
Bread, biscuits, etc.	64	3	64	11	29	10	38	11	49·4	51·1	46·5	48·9
Cocoa, chocolate, etc.	70	6	83	7	30	11	39	2	49·6	47·8	46·3	47·3
Tobacco	84	4	83	3	40	4	42	1	48·7	45·1	46·0	44·7
Woodworking												
Sawmilling	61	10	73	2	*		*		46·0	45·2	*	*
Wood box, etc.	65	1	82	4	*		*		48·4	55·3	*	*
Cabinet, furniture, etc., making	68	3	69	7	34	5	36	3	47·7	45·5	44·9	43·6
Carriage, cart, etc.	72	3	76	0	29	3	32	1	48·5	40·4	42·9	35·8
Paper, printing, etc.												
Paper and paper-board	65	8	71	9	30	11	32	8	50·6	46·3	45·2	42·8
Cardboard box	73	11	83	8	32	2	34	10	48·0	45·8	45·0	43·5
Printing, etc.	89	9	143	3	34	4	39	6	44·5	42·9	44·5	43·2
Other manufacturing industries												
Rubber	65	7	78	6	28	7	34	7	52·6	48·4	45·3	43·5
Oilcloth, etc.	59	2	73	11	*		*		49·8	49·6	*	*
Brushes, brooms, etc.	61	5	61	7	29	1	33	9	47·7	44·0	45·1	44·2
Scientific instruments	71	8	86	2	32	6	36	3	47·4	47·5	45·6	44·3
Musical instruments, toys, etc.	69	4	69	5	28	8	31	8	46·9	45·6	45·4	44·8
Mining and quarrying (other than coal)												
Iron mining	60	0	70	11	*		*		43·7	37·7	*	*
Stone quarrying	60	5	62	11	*		*		47·0	42·7	*	*
Clay, sand, gravel, etc.	55	2	63	1	*		*		49·2	43·3	*	*
Other	56	0	64	4	*		*		47·4	43·7	*	*

* The numbers returned in this group were very small, or the proportion of piece-workers was small.

Note.—Industries in which the proportion of men and women employed at piece-rates was relatively small have been omitted.

APPENDIX VII

Average Hourly Earnings of Time-workers and Piece-workers

A. *Industry Groups*

Industry group	*Men* — *Average hourly earnings* — *Time* d.	*Men* — *Average hourly earnings* — *Piece* d.	*Men* — *Piece earnings as a percentage of time earnings*	*Women* — *Average hourly earnings* — *Time* d.	*Women* — *Average hourly earnings* — *Piece* d.	*Women* — *Piece earnings as a percentage of time earnings*
Treatment of non-metalliferous mining products	15·8	18·3	116	7·3	9·2	126
Brick, pottery and glass	14·5	17·9	123	6·9	8·7	126
Chemical, paint, oil, etc.	17·0	20·2	119	8·5	10·3	121
Metal, engineering and shipbuilding	16·8	21·9	130	8·1	9·7	120
Textiles	13·9	15·8	114	7·5	9·2	123
Leather	15·2	18·3	120	9·2	9·3	101
Clothing	16·2	19·0	117	9·1	10·1	111
Food, drink and tobacco	15·7	17·5	111	8·2	9·8	120
Woodworking	16·6	19·4	117	9·0	9·6	107
Paper, printing, etc.	21·5	29·3	136	8·9	9·9	111
Other manufacturing industries	16·1	19·3	120	7·9	9·3	118
Building, contracting, etc.	17·1	*	—	*	*	—
Mining and quarrying (other than coal)	14·8	18·3	124	*	*	—
Transport, storage, etc. (excluding railways)	17·2	*	—	9·2	*	—
Public utility services	15·4	*	—	10·1	*	—
Government industrial establishments	17·2	23·2	135	11·1	*	—
All the above	16·5	20·6	125	8·4	9·6	114

* The numbers returned in this group were very small.

APPENDIX VIII

Average Hourly Earnings of Time-workers and Piece-workers

B. *Separate Industries*

Industry	*Men* — *Average hourly earnings* — *Time* d.	*Men* — *Average hourly earnings* — *Piece* d.	*Men* — *Piece earnings as a percentage of time earnings*	*Women* — *Average hourly earnings* — *Time* d.	*Women* — *Average hourly earnings* — *Piece* d.	*Women* — *Piece earnings as a percentage of time earnings*
Treatment of non-metalliferous mining products						
Coke ovens, etc.	16·7	19·8	119	*	*	—
Cement	16·5	17·7	108	*	*	—
Lime kilns and whiting	13·4	16·8	125	*	*	—
Cast stone and cast concrete	12·7	16·8	132	*	*	—
Brick, pottery and glass						
Brick, tile, pipe, etc.	13·6	17·0	125	7·3	9·1	125
Pottery, etc.	14·5	18·0	124	6·9	8·7	126
Glass and glass container	16·6	21·5	130	6·9	9·0	130
Chemical, etc.						
Chemicals and explosives	16·9	20·2	120	8·2	10·2	124
Metal, engineering and shipbuilding						
Pig iron manufacture	18·4	24·5	133	*	*	—
Iron puddling, steel melting, rolling, etc.	17·3	25·7	149	*	*	—
Non-ferrous metal manufacture	16·4	21·9	134	8·4	9·6	114
Tin plate and sheet manufacture	16·9	30·3	179	8·4	11·3	135
Iron and steel tube manufacture	15·7	22·0	140	6·9	8·9	129
Wire, wire netting, etc.	15·1	21·5	142	7·3	8·7	119
General engineering	16·4	20·9	128	7·8	9·7	124
Electrical engineering	16·6	20·0	120	7·8	9·1	117
Marine engineering	16·9	21·1	125	*	*	—
Constructional engineering	16·4	19·8	121	*	*	—

* The numbers returned in this group were very small, or the proportion of piece-workers was small.

APPENDIX VIII—(*cont.*).

Industry	*Men* — *Average hourly earnings* — *Time* d.	*Men* — *Average hourly earnings* — *Piece* d.	*Men* — *Piece earnings as a percentage of time earnings*	*Women* — *Average hourly earnings* — *Time* d.	*Women* — *Average hourly earnings* — *Piece* d.	*Women* — *Piece earnings as a percentage of time earnings*
Metal, engineering and shipbuilding (cont.)						
Motor vehicles, cycles and aircraft	18·6	22·7	122	8·2	13·3	162
Shipbuilding and repairing	17·2	23·8	138	*	*	—
Railway carriage and wagon building and repairing	15·2	19·4	128	*	*	—
Electrical apparatus, etc.	16·5	20·1	122	8·3	10·3	124
Hand tools, cutlery, etc.	16·6	19·6	118	7·1	8·6	121
Bolts, nuts, etc.	14·9	19·0	128	6·9	8·9	129
Brass and allied metal wares	16·1	19·9	124	7·3	9·0	124
Heating and ventilating engineering	17·3	21·0	121	*	*	—
Watches, clocks, etc.	16·9	20·4	121	7·9	8·9	113
Stoves, grates and general ironfounding	15·7	22·5	143	7·2	10·3	143
Other metal industries	16·4	20·1	123	6·4	9·2	144
Textiles						
Cotton	12·4	13·6	110	7·3	8·7	119
Woollen and worsted	14·0	15·9	113	7·4	9·6	130
Silk throwing, spinning and weaving	15·6	15·5	99	7·8	9·9	127
Hosiery	15·4	22·1	144	8·6	10·1	117
Lace	15·4	18·1	118	7·8	9·3	119
Carpets and rugs	13·7	17·6	128	7·2	10·2	142
Bleaching, dyeing, etc.	14·0	16·5	118	7·7	9·1	118
Leather						
Tanning, etc.	14·3	18·2	127	7·7	9·5	123
Leather goods	15·2	16·4	108	7·9	8·5	108
Clothing						
Ready-made tailoring	17·4	20·6	118	8·9	10·4	117
Retail bespoke tailoring	18·9	19·6	104	9·9	11·3	114
Dressmaking and millinery	*	*	—	10·3	9·8	95
Hats and caps	15·8	21·1	134	9·3	10·4	112
Shirts, collars and underwear	*	*	—	8·6	9·4	109
Boot and shoe making	16·1	19·1	118	9·8	11·3	115
Boot and shoe repairing	16·5	17·3	105	*	*	—
Food, drink and tobacco						
Bread, biscuits, etc.	15·6	15·2	97	7·8	9·6	123
Cocoa, chocolate, etc.	17·1	21·2	124	8·0	10·0	125
Tobacco	20·8	22·1	106	10·5	11·3	108
Woodworking						
Sawmilling	16·1	19·4	120	*	*	—
Wood box, etc.	16·1	17·9	111	*	*	—
Cabinet, furniture, etc., making	17·2	18·4	107	9·2	10·0	109
Carriage, cart, etc.	17·9	22·6	126	8·2	10·8	132
Paper, printing, etc.						
Paper and paper board	15·6	18·6	119	8·2	9·1	111
Cardboard box	18·5	21·9	118	8·6	9·6	112
Printing, etc.	24·2	40·1	166	9·3	11·0	118
Other manufacturing industries						
Rubber	15·0	19·5	130	7·6	9·5	125
Oilcloth, etc.	14·3	17·9	125	*	*	—
Brushes, brooms, etc.	15·5	16·8	108	7·7	9·2	120
Scientific instruments	18·1	21·8	120	8·6	9·8	114
Musical instruments, toys, etc.	17·7	18·3	103	7·6	8·5	112
Mining and quarrying (other than coal)						
Iron mining	16·5	22·6	137	*	*	—
Stone quarrying	15·4	17·8	116	*	*	—
Clay, sand, gravel, etc.	13·5	17·5	130	*	*	—
Other	14·2	17·7	125	*	*	—

* The numbers returned in this group were very small, or the proportion of piece-workers was small.

APPENDIX IX

Distribution of Weekly Earnings and Hours of Labour in all Industries Combined

Weekly earnings	*Percentages of persons working the following weekly hours* Under 44	44 and under 47	47 and up to and including 48	Over 48	Total* % (a)	Total* % (b)
	%	%	%	%	% (a)	% (b)
I. Men (21 years and over)						
Under 40s.	3·9	0·4	1·0	0·6	6·0	5·8
40s. and under 50s.	2·6	1·2	2·8	1·6	8·3	8·9
50s. ,, 60s.	2·8	3·6	6·5	5·4	18·5	18·6
60s. ,, 70s.	2·5	3·3	6·7	8·9	21·7	21·6
70s. ,, 80s.	1·7	4·8	4·9	8·2	19·8	19·5
80s. ,, 90s.	0·9	1·6	3·1	6·3	12·1	11·9
90s. ,, 100s.	0·5	0·7	1·5	3·7	6·5	6·6
100s. ,, 110s.	0·2	0·3	0·5	2·1	3·2	3·2
110s. ,, 120s.	0·1	0·2	0·3	1·2	1·8	1·7
120s. and over	0·3	0·3	0·3	1·2	2·1	2·2
Total	15·5	16·4	27·6	39·2	100·0	100·0
II. Women (18 years and over)						
Under 20s.	6·5	0·6	1·2	0·2	8·6	8·6
20s. and under 30s.	12·6	6·3	11·5	1·9	32·6	32·6
30s. ,, 40s.	9·1	10·0	15·3	4·8	39·5	39·5
40s. ,, 50s.	2·2	3·4	5·4	2·4	13·7	13·7
50s. and over	0·8	1·5	2·0	1·3	5·6	5·6
Total	31·2	21·8	35·4	10·6	100·0	100·0
III. Youths and boys						
Under 10s.	1·8	1·2	1·0	0·3	4·3	4·2
10s. and under 20s.	5·4	11·3	14·2	3·7	34·8	34·5
20s. ,, 30s.	4·4	9·0	11·2	6·3	31·1	31·0
30s. ,, 40s.	2·4	4·1	5·2	4·9	16·7	17·1
40s. and over	1·5	2·1	3·2	6·1	13·1	13·2
Total	15·5	27·7	34·8	21·3	100·0	100·0
IV. Girls						
Under 10s.	4·6	2·0	1·8	0·1	8·6	8·6
10s. and under 20s.	14·2	15·2	22·0	3·0	54·8	54·8
20s. ,, 30s.	6·1	7·5	12·2	3·7	29·7	29·7
30s. and over	1·0	1·6	2·8	1·4	6·9	6·9
Total	25·9	26·3	38·8	8·2	100·0	100·0

* Including a small number of people whose hours were not stated.

(*a*) Excluding railway traffic workers, in respect of whom full particulars as to hours worked are not available.

(*b*) Including railway traffic workers.

APPENDIX X

Distribution of Weekly Earnings and Hours of Labour in each Industry Group

A. *Men* (21 *Years and Over*)

Weekly earnings	*Percentage of persons working the following weekly hours*				*Total**
	Under 44	44 *and under* 47	47 *to* 48	*Over* 48	
Treatment of non-metalliferous mining products					
Under 40s.	4·8	0·2	0·4	0·4	5·9
40s. and under 50s.	2·8	1·6	2·1	1·7	8·4
50s. ,, 60s.	2·8	3·7	8·3	7·9	23·0
60s. ,, 70s.	1·7	2·1	6·6	12·6	23·2
70s. ,, 80s.	1·1	1·9	4·2	10·6	18·0
80s. ,, 90s.	0·4	0·6	1·8	7·6	10·6
90s. ,, 100s.	0·2	0·2	0·6	4·6	5·7
100s. ,, 110s.	0·1	0·1	0·5	2·2	2·9
110s. ,, 120s.	0·0	0·0	0·2	1·0	1·3
120s. and over	0·1	0 0	0·2	0·7	1·0
Total	14·0	10·4	24·9	49·3	100·0
Brick, glass, pottery, chemical, etc., industries					
Under 40s.	3·6	0·3	0·7	0·6	5·3
40s. and under 50s.	2·6	1·4	3·1	1·9	9·1
50s. ,, 60s.	3·6	2·7	9·7	7·0	23·2
60s. ,, 70s.	3·9	2·6	8·6	9·6	24·8
70s. ,, 80s.	2·0	2·0	5·2	8·0	17·4
80s. ,, 90s.	1·0	1·2	3·4	5·6	11·3
90s. ,, 100s.	0·4	0·4	1·1	2·7	4·7
100s. ,, 110s.	0·2	0·2	0·4	1·3	2·2
110s. ,, 120s.	0·1	0·1	0·2	0·6	1·0
120s. and over	0·1	0·1	0·2	0·6	1·0
Total	17·5	11·0	32·6	37·9	100·0
Metal, engineering and shipbuilding industries					
Under 40s.	2·7	0·2	0·4	0·2	3·6
40s. and under 50s.	2·4	0·7	2·3	0·7	6·1
50s. ,, 60s.	2·7	1·9	5·0	4·6	14·3
60s. ,, 70s.	3·6	2·9	6·2	8·4	21·1
70s. ,, 80s.	2·8	2·6	4·9	9·5	20·0
80s. ,, 90s.	1·9	1·6	3·0	8·1	14·7
90s. ,, 100s.	1·1	0·9	1·8	5·4	9·2
100s. ,, 110s.	0·5	0·4	0·7	3·4	5·0
110s. ,, 120s.	0·2	0·2	0·5	2·1	3·0
120s. and over	0·4	0·3	0·6	1·7	3·0
Total	18·3	11·7	25·4	44·1	100·0
Textile industries					
Under 40s.	8·6	0·7	7·9	1·6	18·9
40s. and under 50s.	4·1	1·4	10·5	4·0	20·1
50s. ,, 60s.	2·3	1·5	8·4	7·6	20·0
60s. ,, 70s.	1·5	1·2	6·7	7·2	16·7
70s. ,, 80s.	0·7	0·8	4·8	4·4	10·7
80s. ,, 90s.	0·3	0·4	3·0	3·1	6·8
90s. ,, 100s.	0·1	0·1	1·3	1·6	3·1
100s. ,, 110s.	0·1	0·1	0·6	1·0	1·8
110s. ,, 120s.	0·0	0·0	0·3	0·5	0·8
120s. and over	0·1	0·1	0·3	0·6	1·1
Total	17·8	6·3	43·8	31·6	100·0

* Including a small number of persons whose hours were not stated.

APPENDIX X—(*cont.*)

A. *Men* (21 *Years and Over*)

Weekly earnings	*Percentage of persons working the following weekly hours*: *Under 44*	*44 and under 47*	*47 to 48*	*Over 48*	*Total**
Leather industries					
Under 40s.	4·8	0·6	1·7	1·1	8·2
40s. and under 50s.	3·5	1·5	4·2	2·6	12·0
50s. ,, 60s.	3·2	2·4	9·7	7·0	22·5
60s. ,, 70s.	2·7	2·8	9·4	8·9	24·4
70s. ,, 80s.	1·3	1·7	6·2	6·3	15·8
80s. ,, 90s.	0·6	0·7	3·3	3·5	8·3
90s. ,, 100s.	0·2	0·3	1·5	1·7	3·8
100s. ,, 110s.	0·1	0·2	0·9	1·0	8·2
110s. ,, 120s.	0·0	0·1	0·3	0·4	0·9
120s. and over	0·1	0·3	0·7	0·7	1·9
Total	16·5	10·6	37·9	33·2	100·0
Clothing industries					
Under 40s.	5·8	1·2	1·1	0·6	9·1
40s. and under 50s.	4·2	2·1	2·1	1·1	9·8
50s. ,, 60s.	5·9	6·3	4·8	3·0	20·5
60s. ,, 70s.	4·2	7·6	8·5	5·8	26·8
70s. ,, 80s.	2·0	4·5	4·5	4·2	15·6
80s. ,, 90s.	1·0	2·8	2·5	2·4	8·9
90s. ,, 100s.	0·4	1·3	1·1	1·2	4·2
100s. ,, 110s.	0·2	0·6	0·7	0·8	2·3
110s. ,, 120s.	0·0	0·3	0·3	0·4	1·0
120s. and over	0·1	0·4	0·4	0·7	1·8
Total	23·8	27·1	26·0	20·2	100·0
Food, drink and tobacco industries					
Under 40s.	2·0	0·5	1·3	1·4	5·2
40s. and under 50s.	1·1	1·4	3·3	3·5	9·3
50s. ,, 60s.	1·7	3·1	8·4	8·5	22·0
60s. ,, 70s.	1·7	3·1	9·9	11·9	27·1
70s. ,, 80s.	0·6	2·6	6·0	8·9	18·4
80s. ,, 90s.	0·2	1·0	2·9	5·1	9·4
90s. ,, 100s.	0·1	0·4	1·3	2·6	4·4
100s. ,, 110s.	0·0	0·2	0·6	1·3	2·1
110s. ,, 120s.	0·0	0·1	0·2	0·7	1·1
120s. and over	0·0	0·1	0·3	0·5	1·0
Total	7·4	12·5	34·2	44·4	100·0
Woodworking industries					
Under 40s.	3·4	1·0	1·3	1·3	7·2
40s. and under 50s.	2·2	2·2	2·8	3·1	10·3
50s. ,, 60s.	2·5	3·7	5·3	5·3	16·9
60s. ,, 70s.	3·3	4·5	4·6	6·8	19·5
70s. ,, 80s.	2·7	9·6	6·4	5·9	24·8
80s. ,, 90s.	0·4	2·0	3·7	5·3	11·6
90s. ,, 100s.	0·2	0·6	1·0	2·8	4·7
100s. ,, 110s.	0·1	0·3	0·5	1·3	2·3
110s. ,, 120s.	0·0	0·2	0·2	1·0	1·4
120s. and over	0·0	0·1	0·2	0·8	1·3
Total	14·8	24·2	26·0	53·6	100·0

* Including a small number of persons whose hours were not stated.

APPENDIX X—(*cont.*)

A. *Men* (21 *Years and Over*)

Weekly earnings	*Percentage of persons working the following weekly hours*				*Total**
	Under 44	44 *and under* 47	47 *to* 48	*Over* 48	
Paper, printing, etc., industries					
Under 40s.	2·7	1·1	0·3	0·3	4·6
40s. and under 50s.	2·2	1·7	1·3	0·9	6·1
50s. ,, 60s.	2·0	3·1	2·4	2·5	10·1
60s. ,, 70s.	1·7	5·8	1·9	4·2	13·6
70s. ,, 80s.	1·6	9·1	1·6	4·0	16·5
80s. ,, 90s.	1·3	7·8	1·5	4·0	14·8
90s. ,, 100s.	1·2	4·5	1·1	2·8	10·0
100s. ,, 110s.	1·2	2·7	0·8	2·4	7·3
110s. ,, 120s.	1·0	1·4	0·5	1·8	4·8
120s. and over	3·2	3·1	1·0	4·5	12·2
Total	18·1	40·3	12·4	27·4	100·0
Other manufacturing industries					
Under 40s.	2·6	0·6	0·8	0·6	4·7
40s. and under 50s.	2·9	1·6	3·0	2·3	9·9
50s. ,, 60s.	2·6	3·2	6·4	7·1	19·5
60s. ,, 70s.	2·6	3·2	5·7	7·9	19·6
70s. ,, 80s.	2·4	3·0	5·0	7·1	17·6
80s. ,, 90s.	1·4	2·3	3·4	6·1	13·5
90s. ,, 100s.	0·6	1·1	1·4	3·9	7·1
100s. ,, 110s.	0·3	0·4	0·8	2·5	4·1
110s. ,, 120s.	0·2	0·2	0·3	1·2	1·9
120s. and over	0·2	0·2	0·4	1·3	2·1
Total	15·8	15·8	27·2	40·0	100·0
Building, contracting, etc.					
Under 40s.	6·8	0·5	0·2	0·4	7·9
40s. and under 50s.	3·7	1·5	0·5	1·0	6·8
50s. ,, 60s.	3·9	8·4	2·4	5·5	20·5
60s. ,, 70s.	2·5	4·6	1·9	10·7	20·0
70s. ,, 80s.	1·4	12·3	2·2	8·7	24·8
80s. ,, 90s.	0·1	2·0	1·2	7·7	11·1
90s. ,, 100s.	0·1	0·4	0·2	4·0	4·7
100s. ,, 110s.	0·0	0·1	0·1	1·8	2·0
110s. ,, 120s.	0·0	0·0	0·0	0·8	0·9
120s. and over	0·0	0·1	0·0	1·1	1·3
Total	18·5	29·9	8·7	41·7	100·0
Public utility services					
Under 40s.	1·9	0·3	1·1	1·9	5·5
40s. and under 50s.	1·5	1·4	5·3	3·3	11·6
50s. ,, 60s.	1·3	3·6	18·6	5·1	29·1
60s. ,, 70s.	0·7	2·0	14·5	7·5	25·2
70s. ,, 80s.	0·4	2·9	6·4	6·0	15·9
80s. ,, 90s.	0·1	0·5	2·5	3·8	7·1
90s. ,, 100s.	0·0	0·1	0·8	2·1	3·2
100s. ,, 110s.	0·0	0·1	0·3	0·9	1·3
110s. ,, 120s.	0·0	0·0	0·1	0·4	0·6
120s. and over	0·0	0·0	0·1	0·4	0·5
Total	5·9	10·9	49·7	31·4	100·0

* Including a small number of people whose hours were not stated.

APPENDIX X—(*cont.*)

A. *Men (21 Years and Over)*

Weekly earnings	*Percentage of persons working the following weekly hours*: Under 44	44 and under 47	47 to 48	Over 48	*Total**
Mining and quarrying (other than coal)					
Under 40s.	6·7	0·7	0·6	0·5	8·7
40s. and under 50s.	7·3	4·0	3·7	3·0	18·3
50s. ,, 60s.	5·5	5·9	6·9	6·9	25·4
60s. ,, 70s.	4·1	5·7	4·6	7·2	21·8
70s. ,, 80s.	1·7	4·0	2·7	5·5	14·0
80s. ,, 90s.	0·9	1·2	1·0	3·0	6·2
90s. ,, 100s.	0·3	0·5	0·4	1·5	2·8
100s. ,, 110s.	0·2	0·4	0·3	0·6	1·5
110s. ,, 120s.	0·0	0·1	0·1	0·3	0·6
120s. and over	0·0	0·1	0·1	0·2	0·7
Total	26·7	22·6	20·4	28·7	100·0
Transport, storage, etc.†					
Under 40s.	2·8	0·1	0·4	0·4	2·9
40s. and under 50s.	1·3	0·4	1·2	1·3	10·4
50s. ,, 60s.	1·7	1·3	4·2	5·1	15·5
60s. ,, 70s.	1·0	2·8	9·9	12·9	24·5
70s. ,, 80s.	0·5	2·1	7·1	13·3	20·1
80s. ,, 90s.	0·2	0·6	10·0	6·1	13·3
90s. ,, 100s.	0·1	0·2	5·9	2·5	8·7
100s. ,, 110s.	0·1	0·1	0·8	0·9	2·3
110s. ,, 120s.	0·0	0·1	0·1	0·3	1·1
120s. and over	0·0	0·1	0·2	0·4	1·2
Total	7·7	7·8	39·8	43·2	100·0
Government industrial establishments					
Under 40s.	0·7	0·0	0·1	0·0	1·0
40s. and under 50s.	0·6	0·2	3·9	0·6	5·3
50s. ,, 60s.	0·8	1·4	15·7	3·9	21·8
60s. ,, 70s.	0·8	0·7	10·8	4·8	17·3
70s. ,, 80s.	0·5	1·7	12·9	6·0	21·1
80s. ,, 90s.	0·9	1·0	5·0	6·4	13·3
90s. ,, 100s.	0·4	0·6	2·2	4·8	8·0
100s. ,, 110s.	0·1	0·4	0·7	3·6	4·8
110s. ,, 120s.	0·1	0·1	0·4	2·2	2·8
120s. and over	0·1	0·2	0·9	3·4	4·6
Total	5·0	6·3	52·6	35·7	100·0
Brick, glass, pottery, chemical, etc., industries					
Under 20s.	8·2	1·1	1·9	0·2	11·5
20s. and under 30s.	12·3	8·2	15·7	2·1	38·5
30s. ,, 40s.	11·7	7·0	14·3	3·4	36·7
40s. ,, 50s.	2·3	1·7	3·3	1·3	8·6
50s. and over	1·6	0·9	1·5	0·7	4·7
Total	36·1	18·9	36·7	7·7	100·0
Metal, engineering and shipbuilding industries					
Under 20s.	3·8	0·5	0·7	0·1	5·2
20s. and under 30s.	11·3	8·3	10·2	2·5	32·5
30s. ,, 40s.	15·5	16·3	13·8	5·1	50·7
40s. ,, 50s.	1·6	2·0	2·8	1·8	8·4
50s. and over	0·3	1·6	0·6	0·6	3·2
Total	32·5	28·7	28·1	10·1	100·0

* Including a small number of people whose hours were not stated.

† The percentages working the different hours exclude persons in the railway services, for whom the full analysis was not obtained. Such persons are, however, included in the figures shown in the "Total" column.

APPENDIX X—(*cont.*)

B. Women (18 Years and Over)

Weekly earnings	*Percentage of persons working the following weekly hours*				*Total**
	Under 44	*44 and under 47*	*47 to 48*	*Over 48*	
Textile industries					
Under 20s.	7·6	0·4	2·3	0·2	10·7
20s. and under 30s.	9·2	3·5	20·2	1·6	34·6
30s. ,, 40s.	5·3	2·8	24·5	2·7	35·5
40s. ,, 50s.	1·7	0·9	9·4	1·5	13·7
50s. and over	0·6	0·3	3·4	1·0	5·5
Total	24·4	7·9	59·8	7·0	100·0
Leather industries					
Under 20s.	4·6	1·0	1·5	0·3	7·5
20s. and under 30s.	8·8	6·1	11·2	4·2	31·1
30s. ,, 40s.	4·4	5·8	16·1	8·8	35·3
40s. ,, 50s.	1·1	2·9	4·6	3·8	12·3
50s. and over	0·9	4·5	3·4	4·9	13·8
Total	19·8	20·3	36·8	21·8	100·0
Clothing industries					
Under 20s.	7·5	0·5	0·3	0·1	8·5
20s. and under 30s.	20·6	6·0	3·0	1·0	31·1
30s. ,, 40s.	13·3	13·2	8·7	3·3	38·7
40s. ,, 50s.	4·0	5·7	3·1	2·2	15·0
50s. and over	1·3	2·5	1·4	1·4	6·7
Total	46·7	27·9	16·5	8·0	100·0
Food, drink and tobacco industries					
Under 20s.	4·1	0·5	1·3	0·3	6·3
20s. and under 30s.	8·7	6·8	13·7	2·9	32·3
30s. ,, 40s.	4·5	8·2	15·1	11·1	39·4
40s. ,, 50s.	1·0	3·8	6·0	5·2	16·3
50s. and over	0·2	0·8	1·7	2·9	5·7
Total	18·5	20·1	37·8	22·4	100·0
Woodworking industries					
Under 20s.	5·5	1·5	1·8	0·4	9·4
20s. and under 30s.	8·6	6·5	9·8	3·8	28·6
30s. ,, 40s.	7·0	9·9	13·0	5·5	35·6
40s. ,, 50s.	2·2	4·2	8·9	3·7	19·1
50s. and over	0·6	1·7	2·9	2·1	7·3
Total	23·9	23·8	36·2	15·5	100·0

* Including a small number of persons whose hours were not stated.

APPENDIX X—(*cont.*)

B. *Women* (18 *Years and Over*)

Weekly earnings	*Percentage of persons working the following weekly hours*				*Total**
	Under 44	44 *and under* 47	47 *to* 48	*Over* 48	
Paper, printing, etc., industries					
Under 20s.	3·1	1·4	0·3	0·2	5·1
20s. and under 30s.	6·7	12·9	3·4	1·8	25·0
30s. ,, 40s.	5·8	26·1	7·1	7·2	46·8
40s. ,, 50s.	1·5	9·7	2·0	3·2	17·4
50s. and over	0·5	3·1	0·7	1·1	5·7
Total	17·6	53·2	13·5	13·5	100·0
Other manufacturing industries					
Under 20s.	4·8	0·6	0·9	0·3	6·7
20s. and under 30s.	11·0	9·5	13·6	4·1	38·3
30s. ,, 40s.	8·0	9·4	13·4	6·1	37·1
40s. ,, 50s.	2·9	3·7	4·2	2·4	13·3
50s. and over	0·9	1·4	1·2	1·0	4·6
Total	27·6	24·6	33·3	13·9	100·0
Public utility services					
Under 20s.	22·5	0·1	0·3	0·1	24·3
20s. and under 30s.	23·3	1·6	1·7	0·7	28·1
30s. ,, 40s.	9·1	3·7	10·1	3·6	27·3
40s. ,, 50s.	1·5	2·3	7·8	2·1	14·0
50s. and over	0·8	0·7	3·3	1·2	6·3
Total	57·2	8·4	23·2	7·7	100·0
Transport, storage, etc.†					
Under 20s.	5·9	0·4	0·2	0·1	7·5
20s. and under 30s.	9·3	2·7	6·6	6·1	24·8
30s. ,, 40s.	5·2	5·6	9·8	20·7	42·1
40s. ,, 50s.	0·5	0·7	1·6	16·8	18·8
50s. and over	0·5	0·6	2·0	3·8	6·8
Total	21·4	10·0	20·2	47·5	100·0

* Including a small number of people whose hours were not stated.

† The percentages working the different hours exclude persons in the railway services, for whom the full analysis was not obtained. Such persons are, however, included in the figures shown in the "Total" column.

APPENDIX XI

Medians and Quartiles of Weekly Earnings in the Principal Industry Groups

A. *Men*

Industry group	*All Workers*			*Time-workers*			*Piece-workers*		
	Lower quartile s. d.	*Median* s. d.	*Upper quartile* s. d.	*Lower quartile* s. d.	*Median* s. d.	*Upper quartile* s. d.	*Lower quartile* s. d.	*Median* s. d.	*Upper quartile* s d.
Treatment of non-metalliferous mining products	54 8	64 5	78 1	54 5	64 6	77 6	57 0	68 7	80 10
Brick, glass, pottery, chemical, etc.	54 6	65 0	77 3	54 5	64 5	76 7	55 4	67 1	79 4
Metal industry	60 6	72 5	86 9	56 5	67 3	79 3	69 0	81 2	95 4
Textiles	43 0	55 6	69 7	43 4	54 11	68 4	41 11	57 4	73 9
Leather	52 6	63 0	75 0	51 1	60 2	70 5	56 11	68 10	78 9
Clothing	53 0	64 0	75 8	54 3	63 8	73 3	50 11	65 4	79 5
Food, drink and tobacco	54 9	65 0	76 2	54 6	64 7	75 6	58 10	70 8	884 9
Woodworking	54 5	68 1	78 6	54 1	67 5	77 10	58 2	71 8	85 2
Paper, printing, etc.	63 0	79 5	99 4	62 10	78 8	98 1	68 0	89 8	126 6
Other manufacturing industries	57 2	72 1	82 10	53 11	65 3	79 6	61 8	76 1	91 0
Building, contracting, etc.	55 0	67 5	78 0	55 0	67 5	78 0	*	*	*
Public utility services	52 9	61 6	72 4	52 9	61 6	72 4	*	*	*
Mining and quarrying (other than coal)	48 11	59 1	70 6	48 7	57 10	68 8	50 0	64 4	76 4
Transport, storage, etc.	57 6	70 6	81 3	57 6	70 6	81 3	*	*	*
Government industrial establishments	58 7	72 2	86 5	56 9	68 10	80 2	75 5	99 9	107 6
All the above	55 9	67 11	80 6	54 9	66 3	78 3	62 6	76 6	93 3

* The numbers returned in this group were very small.

APPENDIX XI—(*cont.*)

B. *Women*

Industry group	*All workers*			*Time-workers*			*Piece-workers*		
	Lower quartile s. d.	*Median* s. d.	*Upper quartile* s. d.	*Lower quartile* s. d.	*Median* s. d.	*Upper quartile* s. d.	*Lower quartile* s. d.	*Median* s. d.	*Upper quartile* s. d.
Treatment of non-metalliferous mining products	*	*	*	*	*	*	*	*	*
Brick, glass, pottery, chemical, etc.	23 6	29 11	36 10	23 2	29 1	35 8	24 2	31 9	38 8
Metal industries	26 1	32 5	37 4	23 5	28 4	34 7	30 7	34 7	38 7
Textiles	24 2	31 4	38 4	22 4	27 2	33 8	26 3	34 0	40 10
Leather	25 10	33 4	40 10	25 11	33 6	42 3	24 9	32 3	39 0
Clothing	25 4	32 8	39 2	25 4	32 3	38 3	25 6	33 8	41 6
Food, drink and tobacco	25 9	32 11	39 3	24 7	31 1	37 5	31 9	38 2	45 11
Woodworking	25 5	33 3	40 10	25 5	33 1	40 0	25 11	35 1	42 5
Paper, printing, etc.	28 0	34 3	39 7	26 9	33 3	38 7	30 2	35 10	43 1
Other manufacturing industries	24 9	31 4	38 1	23 9	29 1	36 3	27 5	34 2	40 0
Building, contracting, etc.	*	*	*	*	*	*	*	*	*
Public utility services	20 3	29 3	38 3	20 3	29 3	38 3	*	*	*
Mining and quarrying (other than coal)	*	*	*	*	*	*	*	*	*
Transport, storage, etc.	27 3	34 3	40 8	27 3	34 3	40 8	*	*	*
Government industrial establishments	32 11	38 4	45 6	32 11	38 4	45 6	*	*	*
All the above	25 0	32 3	38 7	24 0	30 3	37 0	27 0	34 3	40 6

* The numbers returned in this group were very small.

APPENDIX XII

Weights used in Combining Industry Figures

Industry	*Men*	*Women*
Treatment of non-metalliferous mining products		
Coke ovens, etc.	10	0
Cement, whiting, etc.	14	0
Cast stone and cast concrete products, etc.	20	1
	44	1
Brick, pottery and glass		
Brick, tile, pipe, etc.	71½	4½
Pottery, etc.	24	27½
Glass and glass container	28½	4½
	124	36½
Chemicals, etc.		
Chemicals and explosives	72½	23½
Paint, varnish, etc.	12	2
Oil, glue, soap, ink, etc.	41	9
	125½	34½
Metal, engineering and ship-building		
Pig iron manufacture	12	0
Iron puddling, steel melting, rolling, etc.	128	1
Non-ferrous metals	35	2½
Tinplate and sheet	17	1½
Iron and steel tubes	23	1
Wire, wire netting, etc.	16	2½
General engineering	383	21
Electrical engineering	51	21
Marine engineering	30	0
Constructional engineering	30½	0
Motor vehicles, cycles and aircraft	265	16
Shipbuilding and repairing	100	1
Railway carriage and wagon	20½	½
Electrical apparatus, etc.	66	52
Hand tools, cutlery, etc.	15	5½
Bolts, nuts, etc.	11½	8
Brass and allied metal wares	14	5
Heating and ventilating engineering	14½	½
Watches, clocks, etc.	18	11
Stoves, grates, etc., and other	169	73½
	1,419	223½

Industry	*Men*	*Women*
Textiles		
Cotton	99	179½
Woollen and worsted	69½	92½
Silk throwing, spinning and weaving	15	21
Rayon, nylon yarn, etc.	13	7
Flax	13	31
Jute	7	12½
Hemp, rope, twine, etc.	4	8½
Hosiery	18	68
Lace	5½	6
Carpets	9	14
Other	13	27½
Bleaching, dyeing, etc.	50	16
	316	483½
Leather		
Tanning, currying, etc., and fur	27	7½
Leather goods	8	9½
	35	17
Clothing		
Tailoring	37	107
Dressmaking and millinery	3	75
Hats and caps	8	12½
Shirts, collars, etc.	6	73½
Other	3½	20½
Boot and shoe making and repairing	57½	37
Laundries	20	87
Dyeing and dry cleaning	7½	13½
	142½	426
Food, drink and tobacco		
Bread, biscuits and cakes	80	31
Grain milling	21½	1½
Cocoa, chocolate and sugar confectionery	16½	33
Other food	35½	32½
Drink	84½	16½
Tobacco	10	20
	248	134½
Woodworking		
Sawmilling, etc.	41	½
Wood box and packing case	6	1½
Cabinet making, furniture, etc.	85	15
Carriage, cart, etc.	7½	1
Other	14	2
	153½	20

APPENDIX XII—(*cont.*)

Weights used in Combining Industry Figures

Industry	*Men*	*Women*
Paper, printing, etc.		
Paper making	41	10
Cardboard box, paper bag, etc.	21	33½
Wallpaper	4	1½
Stationery requisites	3½	4
Printing, publishing, etc.	146	52½
	215½	101½
Other manufacturing industries		
Rubber	27	15½
Linoleum, etc.	9	½
Brushes and brooms	3½	3½
Scientific instruments	17½	8
Musical instruments, toys, etc.	11	8
Other	65	46½
	133	82
Building, etc.		
Building	743	3
Civil Engineering	188	0
Electrical wiring, etc.	26	1
	957	4
Transport, storage, etc.		
Tram and omnibus	222	8
Goods transport by road	119½	1
Dock, harbour, etc., service	25½	½
Warehousing	29	5
	396	14½
Mining and quarrying (other than coal)		
Iron mining	9	0
Stone quarrying	39½	0
Clay, sand, etc.	15½	0
Other	19½	½
	83½	½
Public utility services		
Gas, water and electricity	207	2
Local authority services	263½	27½
	470½	29½
Government industrial establishments	118	2½
All above	4,981	1,611

DISCUSSION ON MR. AINSWORTH'S PAPER

Professor A. L. BOWLEY: I have great pleasure in moving a vote of thanks to the author, not only for this valuable paper, but for the great kindness shown to me by his Department of the Ministry of Labour in furnishing me with information they possess, in helping me in difficulties, and in not being unduly annoyed if I made mistakes in quoting their figures, or suggested that more figures would be acceptable.

My acquaintance with Mr. Ainsworth's organization dates back, I suppose, to the year 1893 or 1894. Then, or very soon afterwards, I was acquainted with the personnel, and in my very early work on wage statistics I was in the appropriate "cupboard" of what was then a department of the Board of Trade. From that date to this, with some intermission, I have had contacts of various kinds with this very important branch of the public service.

Perhaps it might be useful if I gave some figures prior to the date covered by the present paper. A wage census was taken under the auspices of Sir Robert Giffen in 1886, and somehow or other I managed to compute the average and median wages and earnings—my memory does not serve as to which it was—at that date, and these, I think, can be compared roughly with the table which the author gives in his paper, setting out the main figures for 1906 and 1938. I had also studied the 1906 census, and for a reason I have not been able to ascertain my averages run about 1*s.* 0*d.* higher than Mr. Ainsworth gives. There is difficulty in knowing what age and industrial groups are included unless one has all the material in front of one.

In 1886, for men, the average earnings were 24*s.* 11*d.*, the lower quartile was 20*s.* 0*d.*, the median 24*s.* 2*d.*, and the upper quartile 29*s.* 5*d.* In 1906, by my reckoning, the average was 30*s.* 6*d.*, the lower quartile 23*s.* 4*d.*, the median 29*s.* 4*d.*, and the upper quartile 37*s.* 2*d.**

* Wages and Income since 1860, p. 42.

Those figures show the following rather important results: In that period of 20 years, from 1886 to 1906, the lower quartile rose 16 per cent., on my reckoning, the median 21 per cent., and the upper quartile 26 per cent. In other words, the upper quartile was running away from the lower quartile perceptibly, though perhaps not very fast. In the figures which Mr. Ainsworth gives, the opposite happens. In his first table, comparing 1906 with 1938, the lower quartile goes up 154 per cent., the median 145 per cent., and the upper quartile 129 per cent., that is to say, the lower quartile is catching up with the upper.

What is the position now in 1948? Perhaps Mr. Ainsworth can give us a guess, though he may not think it proper to do so. I am pleased to see the quartiles coming into their own, but I want to see the deciles also. There is importance in knowing how large the largest wage-earnings are, or what proportion the lowest rate of wage-earnings bears to the whole. It is also interesting, though not so important, to know about the upper tenth, but I want particularly to know about the lowest tenth. Those in the upper ranges are, perhaps, in no particular need of salvation; those in the lowest are very poor. That is the kind of reason why I have always striven to get deciles. I have not been satisfied with the current interpretations; they are not fine enough. If Mr. Ainsworth has the figures and can put them in it would be quite useful. I have already published them, in my reckoning, for the earlier census of 1906, and it would be very useful to have them for 1938.

Still speaking of the distribution of earnings—in fact that is the thing that has interested me most—while we know very many of the averages, we know almost nothing about the distribution. We have the little blue book called *Guides to Official Sources: No.* 1, *Labour Statistics*, issued three weeks ago, and I notice that the individual wages had been obtained in 1938, and had not been published. I did not then know that I could get them, here and now, in the present paper. I do not remember what was published in that way for the census of 1906. I have had all the volumes, but they have not been accessible to me during the last few weeks. The distribution seems to me much more interesting, in a way, than the average. It is true that we must know the averages for further computations, and they show a very great deal.

But people, apart from statisticians, are not interested so much in averages as in the fact that so and so is not getting the average, but so much more or so much less. Not nearly enough attention has been paid to the distribution of wages, nor, for that matter, to the distribution of expenditure. Therefore I welcome this part of the paper especially. Naturally, I tried the normal curve on the distribution. There is a resemblance, though it does not fit, and I have not had time to find out exactly the reason. I hope some mathematical statistician will assign an equation for the distributions shown in the Appendix, and tell us how far they are normal. When I first found that wage distribution was approximately normal I was working with the late Professor Edgeworth, and the results were published in the *Journal* early in this century. I was interested then to observe how the distribution does shake down, as it were, into normal. If there is continuity, there is a certain law of wages which has its own importance.

All that I have said so far is highly appreciative of the present paper. I come now to the omissions. I believe that it is not in any way the responsibility of the Ministry of Labour to know what agricultural labourers or coal-miners are earning. Nobody knows what agricultural labourers earn in general, so far as I can ascertain. This is a very important omission from national statistics. I do not know whether anybody knows the distribution of earnings in the coal industry. If that knowledge is not available that again is a very important omission. As regards dock labour and one or two other industries, these are numerically less important. I do not know whether the central statistical body can take any steps whatever to introduce the departments to each other and get them to repair the two important omissions of which I have spoken, so that we do not always, when we look at the *Labour Gazette*, have to bear in mind that coal-mining and agriculture are not referred to.

These are very serious omissions. It may be said that they are inevitable in a way, but are they really inevitable? Can nothing be done to treat the wage-earning body as a whole, and not in portions as it comes under a particular department?

Finally, when are we going to get a post-war wage distribution census? Will Mr. Ainsworth come to the Society next year and complete this paper? I hope we shall not have to wait until 1958 for the 1948 figures.

Professor J. H. RICHARDSON: I am happy to join with Professor Bowley in congratulating Mr. Ainsworth on his paper. I, too, have had opportunities of working with Mr. Ainsworth and his colleagues at the Ministry of Labour, and I know something about the high standard of the Ministry's statistical work.

My criticisms are few, and they are not of the paper itself but of the enquiry, and are along the lines already indicated by Professor Bowley. It would be useful if similar information in future

could be compiled more frequently than once in a generation, and if departmental difficulties could be overcome so that the enquiry could be made wide enough to cover the whole of British industry, without omitting the very substantial parts already mentioned in the paper and referred to by Professor Bowley. It is true that there are data for some of the omitted branches of industry, but they are not comparable, either in time or in kind, with those of the enquiry. Again, in the 1938 enquiry the earnings of clerks, typists and other salaried employees, even in the industries covered, were not obtained, and although there were, no doubt, good reasons for their exclusion, it would be desirable to include them in future enquiries, perhaps for separate tabulation.

Now that we have become a nation of form-fillers it is interesting to note that while the 1938 returns were better than those of 1906, one-third of the establishments failed to make returns suitable for tabulation.

However, it is more with the results of the enquiry than with its scope and method that I want to deal. The paper enables rough comparisons to be made between average money earnings in 1906, 1938, and 1948. If these statistics are related in an even rougher comparison to changes in cost of living, it appears that the purchasing power of money wages has more than doubled during the past 42 years, and this increase has been achieved notwithstanding two world wars, and substantial reductions in hours of work.

In 1938 the upper quartile of average weekly earnings was about 55 per cent. above the lower quartile, but this margin was less than in 1906. In other words, the earnings of the lower-paid workers had increased relatively more than those of the higher-paid. I believe that this tendency has continued since 1938, and there is danger in trends which reduce too much the margin for skill, as fewer youths will be induced to train for skilled occupations.

Then there is the relationship between the earnings of men and women. Notwithstanding the qualifications which Mr. Ainsworth made, particularly about under-employment of certain groups of women workers, the level of earnings of women seems unduly low relatively to that of men, being 47 per cent. of the men's level of weekly earnings, and 52 per cent. of hourly earnings. Between 1906 and 1938 the earnings of women did not improve relatively to those of men, though both the lower paid men and the lower paid women had improved their position relatively to the more highly paid men and women respectively. Although the position of women relatively to men in that period seems to have remained unchanged, the rise in women's wages during the recent war was somewhat greater than that of men, and it would seem desirable that the relative rise should continue further.

The paper reveals the small proportion of workpeople who are on incentive wage methods. For all workpeople only 25 per cent. were paid by such methods in 1938. The percentage for men was much lower, being only 18 per cent.; that for women was higher, at 46 per cent. For men the proportion must be one of the lowest in any of the advanced industrial countries. Although the use of incentive methods has increased since 1938 in Great Britain, there is scope for greater application both of individual and group incentives.

The paper shows a significant relationship between earnings and hours. Amongst the lower-paid workers, those who worked more than 48 hours earned over 50 per cent. more than those who worked less than 44 hours, while among the higher paid the difference was 30 per cent. This helps to measure the cost of leisure.

The wide range of earnings between the different industries reveals anomalies due to tradition and custom, to variations in the strength of trade unions, and to differences in economic conditions —anomalies quite incompatible with a scientific wage relationship. Such differences show how difficult to would be to devise and apply a national wage policy; yet in these days of nationalized industries and controls there is likely to be a persistent effort to remove the main anomalies.

I have great pleasure in seconding the resolution that our thanks be given to Mr. Ainsworth for a paper which is so rich in social and economic data.

The vote of thanks was put to the meeting and carried unanimously.

Mr. Ramsbottom said that the Royal Statistical Society had a special share of the responsibility for the collection of these statistics of the distribution of earnings in 1938; for, as the *Journal* for 1936 recorded, early in that year a memorial was addressed to the Ministry of Labour by the Society, drawing attention to certain deficiencies in the official statistics of wages and cost of living. One point on which that memorial laid special emphasis was the fact that although statistics of average earnings were frequently collected, there was an absence of any statistics showing the distribution of earnings, from which information coud be obtained as to the numbers and proportions of workpeople earning more or less than certain specified amounts. The Minister of Labour in a sympathetic reply promised that when certain enquiries then in progress were completed, special consideration should be given to the Society's representations.

About the same time demands were made for such statistics from other quarters, and the

outcome was the enquiry of October, 1938. The Society might congratulate itself, not only on its initiative in pressing for these figures of the distribution of earnings, but also on the fact that the first publication of the statistics was to be made in its *Journal*.

While expressing appreciation for the contribution which Mr. Ainsworth had made to the Society's records in this way, the speaker urged that it was highly desirable that the Ministry of Labour should officially publish these statistics, in considerably greater detail than was possible within the limited scope of a paper for the Society.

Professor Bowley had referred to the 1906 enquiry. The results of that enquiry were published in eight volumes, covering over 100 separate industries, and the distribution of earnings was given in one-shilling groups. In this paper Mr. Ainsworth had provided figures for certain broad groups of industries, in ten-shilling groups, but the speaker wished to reinforce Professor Bowley's plea that more details should be published by the Ministry. This enquiry had cost a large amount of taxpayers' money, and had put 70,000 employers to trouble and expense in supplying the information. During the war the Ministry of Labour had other preoccupations which, of course, made it impossible to compile elaborate statistics of wages, but the Ministry should now publish the figures in detail. Fifty to a hundred years hence these statistics would be part of the social and economic history of this country, and anyone wanting to consult them should not have to rely solely upon copies of the Society's *Journal* to find them.

In his paper Mr. Ainsworth had pointed out that it seemed surprising that in 1938, when there were over a million unemployed, two-fifths of all the men were shown as working more than the 48 hours constituting a normal week. In view of that remarkably high figure it seemed desirable that an extended tabulation should be provided, giving particulars of the distribution of the actual working hours amongst those two-fifths of the men. It would be of particular interest to know whether those men were mostly working overtime of only half-an-hour or one hour, or a much more substantial amount of overtime. Further, did any considerable number of employers, despite the instructions which were given to them, show the hours paid for—including hours paid at premium rates—rather than the hours actually worked, in their returns?

As Professor Richardson had pointed out, 82 per cent. of the men in 1938 were found to be paid at time rates of wages. The speaker had looked up the 1906 figures, and found that the percentage of men on time rates, among the two million men whose earnings were returned, was 82 per cent. in that year also. It seemed remarkable that there was no increase between those two dates in the proportion of men paid by results. It would be of special interest to see the figures, not only for groups of industries, but for some of the larger individual industries, and he hoped the Ministry would now supplement Mr. Ainsworth's paper by publishing separate figures for a considerable number of the principal industries.

Professor R. G. D. ALLEN said that he had time only to make a few comments that came to his mind. A small point, but one of some importance, was that it would be useful to have attached to this paper a copy of the blank form which was used in the 1938 enquiry. In the pamphlet that Professor Bowley had mentioned (*Guides to Official Sources:* No. 1, *Labour Statistics*) there was a reproduction of the form used in current earnings enquiries, and Mr. Ainsworth's paper would be even more valuable if he could append the blank form used in 1938.

The next point on which he wished to say a word was one on which Professor Bowley had already spoken, namely, the distribution of earnings. The distribution in 1938 seemed to him to be fairly symmetrical. The symmetry was, of course, more clear in the case of the time workers than in the case of the piece workers. In 1906 the distribution was certainly not symmetrical. As Mr. Ainsworth had said, between 1906 and 1938 the earnings of the lower paid workers had advanced more than those of the upper paid workers; they had advanced in such a way as to convert an unsymmetrical distribution to a more symmetrical one. Presumably the distribution in 1886 was more symmetrical and more like 1938. He liked to think of this as a kind of caterpillar movement. From 1886 to 1906 the head ran away from the tail; from 1906 to 1938 the tail was catching up with the head. Professor Richardson had expressed the opinion that, since 1938, the tail was catching up still further, but it was highly important that they should know the precise position with regard to that. The Ministry of Labour, as far as he was aware, had no plans for including in the current enquiries information as to earnings of individual workers. He would urge that at intervals, say every two years, the information should be extended to include individual workers' earnings.

Finally, he wished to mention one or two other ways in which the extension of the present regular earning enquiries would be most useful. He would mention them without comment. One of the most important things was to have not only six monthly, but quarterly, information on earnings. Secondly, the industrial scope could be extended in various ways, particularly to cover distribution trades. Thirdly, information could be obtained concerning absenteeism. It struck

one right away that the average earnings were the result of a division of total earnings by the number of employees actually at work, and that the denominator excluded those who were away for at least a week. No information was forthcoming from this source concerning workers absent for a week or more, and little from any source.

But he could not conclude these requests for information without stressing again that the method now adopted by the Ministry of Labour of relying upon voluntary co-operation from firms was one to be encouraged and developed. He hoped that they would never have compulsory powers applied in this field. At the same time, the voluntary principle did limit the information which could reasonably be obtained.

Professor BRADFORD HILL said that the advantages of the voluntary basis of the returns had been stressed, though by this method replies were received from only about two thirds of the employers. Clearly this might introduce some bias. For example, comments had been made on the small proportion of people who were paid on piece rates. Was it possible that it was easier to fill in the forms when time rates were paid? If so, the piece-rate group would be understated.

Professor J. H. JONES said that he was glad of the opportunity, not of contributing to the discussion, but of referring to one omission from Professor Bowley's remarks. Professor Bowley had not said that he himself was associated with the 1906 enquiry. That enquiry was but added evidence of the influence which Professor Bowley had exercised on public statistics.

With regard to the paper, he was afraid that other speakers had already covered the ground he would have liked to have covered. He would, however, ask one question. Mr. Ainsworth had said, in effect, that the wages of piece-workers were about 18 per cent. above those of time-workers, and as time-workers worked longer hours, he said that the hourly rate of the piece-workers was about 25 per cent. above that of the time-workers. He then went on to say that the piece-workers were, on the whole, more skilled than the time-workers, and therefore the 25 per cent. could be said to exaggerate the difference between the two. He himself had been a little surprised at that statement. He did not wish to contest it for one moment, because he had only his own observation to go upon, whereas Mr. Ainsworth and his associates had their departmental records. But he wondered whether piece-workers were, in general, more skilled than time-workers.

In an engineering factory the piece-workers were the process workers and very often they were unskilled, while a very large proportion of the workers who were highly skilled were on day work, such, for example, as those employed in tool setting, those employed on maintenance work, and so forth. Altogether they often exceeded the number of the people in the plant who were unskilled workers. He thought that the same thing was true in the heavier industries where there were large numbers of maintenance workers who were themselves highly skilled and who were necessarily paid by time. He had an impression that the piece-worker was not a worker of the highest skill. Though often a skilled worker, he was rarely a highly skilled one, and was usually employed on repetition work. Our knowledge would be very much increased if the occupation rather than the industry were adopted as the unit of investigation. He felt that generalizations about industries, most of which contained a great variety of occupations, were apt to be less reliable on the whole than generalizations based on occupations.

He had been interested in what Professor Bowley had said on the wage grading of different workers in the 1886 census, and he wondered whether they were not apt to be misled by the state of industrial prosperity or depression in the year in which the investigation took place. Thus, for example, in 1886 the country was passing through a very severe depression and the figures were bound to be affected by such depression. On the other hand, in 1906 there was a period of great industrial prosperity, and the figures again would be influenced by the degree of prosperity enjoyed. In 1938 we were again entering upon a period of depression which was bound to be reflected to some extent in the distribution of wages.

The relation between the different grades of workers was very largely determined by such factors as whether or not the people were members of strong trade unions and whether the skilled workers, being members of craft unions, had shown less or more determination in the field of wages. He hoped that some day the Ministry of Labour would find it easy to recast its material not according to industry but according to occupation, and he felt confident that the information on the subject-matter of this paper would be considerably increased if that were done.

Mr. J. L. NICHOLSON said that Appendix XII showed the weights used in calculating the average earnings of men and women, and asked if corresponding figures could be given for youths and girls. If they could not be given for individual industries, he would like to have, at least, the figures for all industries combined.

Mr. BLACKLOCK said that the paper gave separate average rates for piece-work and time-work employees. Was this analysis reliable, and was it worth the cost? In many industries there were numerous incentive wage systems, and they ranged from an equivalent of time-work with a small bonus computed on the time saved by a group of operatives, whose bonus rate might also apply to machine setters, toolroom workers, and cleaners. In such industries classification was wasted effort.

He doubted if changes in the lowest decile rate would help the administrator. Labour dilution and short-time work might fluctuate so greatly that even the quartile rates might lack significance.

Personnel administration was our basic problem, and on the success of labour administrators depended much of the success of our policies for food, coal, steel, exports and dollars. The variants of hourly rates, weekly earnings and unemployment rates should be collected and published without regard to cost, and it might well be worth while to plan a "decimal" classification of regions, industries and occupation.

The PRESIDENT said that he wished to make three comments. In the first place, he would follow the fashion of asking for a little more. He thought the paper would be more helpful to many of the younger Fellows if it included references to the various articles on the subject, particularly in the *Ministry of Labour Gazette*. There should also be appended a short bibliography of the subject.

Secondly, he wanted to suggest that something might be omitted. There was a warning at the beginning of the paper about the danger of using the averages for *all* workers, because they were affected by the varying proportions of men, women and juveniles, but he felt that it would be well to omit altogether the figures relating to "all workers," as he did not think any deduction could be drawn from them.

His third point was perhaps rather more important. From a statistical point of view we were living in an age of small samples, and it was only too often forgotten that these small samples were sometimes drawn from other samples which were not so small, and these in turn were drawn from larger samples, so that the total population was thrown into the almost infinite distance, and the question of randomness was paid only lip-service, if, indeed, it was ever mentioned. Small samples were quickly transformed into millions, and it was only when the question of the size of the small samples was approached that our "Gallup-ers"—as he called them—were inclined to be a little coy.

The present paper dealt with some very large numbers, but the weights given in the last table indicated that there were very large variations in the sizes of the groups, and, indeed, that some had to be rejected because they were too small. Every paper presented to the Society should give some idea of the absolute numbers on which its deductions were based. This was, in fact, done by the Ministry of Labour in the 1945 investigation, and the data appeared in the *Ministry of Labour Gazette* for February, 1946. A table of that kind could, with great advantage, be added to the present paper.

Mr. R. B. AINSWORTH, in reply, said he would prefer to answer most of the points in writing, and would make only two positive replies at the moment. He would try to put in the deciles as Professor Bowley had suggested, and he would also see that a copy of the form of enquiry in 1938 was appended. Unfortunately, he was only a statistician of the Ministry of Labour, and was not in a position to commit any Ministry to produce statistics about the earnings of coal-workers, agricultural workers and dock labourers, nor, indeed, could he commit his own Ministry to undertake an elaboration of the enquiry in the way some speakers had suggested.

But in the Ministry they had been considering the recommendations of the Interdepartmental Committee on Social and Economic Research, and were considering what information they could make available to research workers from that which was in their possession and had not been published. The results of the 1938 enquiry were amongst the subjects which they were considering in order to discover what information they could make available to research workers, not necessarily in a published form, but in some widespread form which would be available to the Universities and others.

Mr. AINSWORTH subsequently replied in writing, as follows:

I am very grateful to Professor Bowley for coming to the meeting and proposing the vote of thanks, and I should like to thank him for his remarks about my paper and the work of my Department. In response to his request for the decile ranges for October, 1938, I give these in the following table:

Industry Group	*Men*		*Women*	
	Upper decile	*Lower decile*	*Upper decile*	*Lower decile*
	s. d.	*s. d.*	*s. d.*	*s. d.*
Non-metalliferous mining products	46 3	91 4	—	—
Brick, pottery, glass and chemical trades	46 5	83 8	18 11	42 9
Metal industries	50 8	100 0	22 5	40 8
Textiles	32 4	90 0	19 7	45 9
Leather	42 2	88 3	20 11	53 9
Clothing	41 7	89 6	21 0	47 5
Food	46 8	88 7	22 4	46 7
Woodworking	43 4	89 5	20 4	48 1
Paper, printing, etc.	48 8	124 9	20 3	45 5
Other manufacturing industries	46 6	97 6	21 5	44 4
Building, contracting, etc.	44 6	88 3	—	—
Public utility services	43 10	82 10	13 6	47 3
Mining and quarrying (other than coal)	40 10	82 6	—	—
Transport, storage, etc.	42 6	93 5	21 0	46 1
Government industrial establishments	51 8	103 6	—	—
All the above	46 0	95 0	20 9	45 6

I am unable to account for the difference between Professor Bowley's calculation of the average and quartile ranges in 1906 and those which I have calculated. It is possible that the working papers at my disposal are more detailed than the information available to him.

With regard to the point raised by Professor Richardson as to the proportions of workers employed on time-work and piece-work respectively, the figures for 1906, 1938 and October, 1947, are as follows:

	Proportion on piece-work				
	Men	*Youths*	*Women*	*Girls*	*All workers*
September, 1906	18	16	59	31	25
October, 1938	18	21	46	27	25
October, 1947	24	20	39	35	28

It will be seen that the proportion of men on piece-work, which was unchanged between 1906 and 1938, has since increased, but not to any great extent. These figures, however, conceal much greater changes in individual industries. In the metal industries and boot and shoe manufacture, for instance, the proportion on piece-work has risen appreciably, whereas in textiles, clothing and pottery there has been a marked decline. The proportion of women on piece-work has decreased considerably since 1906. This change is partly due to an actual drop in many industries, including textiles and clothing, and partly to the fact that there has been an expansion in the number of women employed in certain industries in which time-work prevails.

I attach a specimen of the blank form used in the 1938 enquiry, as suggested by Professor Allen.

I do not think that there is any support for the suggestion made by Professor Bradford Hill that firms employing only time-workers responded more readily than those employing piece-workers. The form simply required firms to enter the amount paid to each individual in the selected week. The entry of piece-work earnings would normally present no greater difficulty than would those of time-workers.

I have looked into the question raised by Mr. Ramsbottom as to whether the high proportion of men working more than 48 hours was due to the fact that employers quoted the hours paid for at overtime rates rather than the time actually worked. Two-thirds of the men in this category were reported as working over 48 and under 56 hours. Employers were particularly asked to quote the hours worked, and there is no means of checking the information supplied.

I think that the point made by Dr. Heron with regard to statistics of earnings based on a relatively small number of quotations is met by the fact that, in the detailed appendices, I have

omitted figures in respect of piece-workers and women workers in the case of industries in which the number of such persons was small. As regards references in the *Ministry of Labour Gazette* to the subject of this paper, the only figures published in the *Gazette* were in the issues for August, 1944 and February, 1945.

As regards Mr. Nicholson's suggestion, the weights used were 4,981 for men, 1,611 for women, 956 for boys and 559 for girls.

Specimen Form of Enquiry

EARNINGS AND HOURS OF WAGE-EARNERS

Name of Firm...

Address of Works covered by this Return...................................
(So far as possible, a separate Return should be made for each Works)

Branch of Industry carried on at above Works.............................

Actual earnings of each wage-earner in the last pay-week of October, 1938 (*See note* *)	*Number of hours worked by each wage-earner in that pay-week* (*See note* †)	*State whether Man (M), Woman (W), Boy (B) or Girl (G)* (*See note* ‡)	*State whether paid at Time rates (T) or Piece rates (P)* (*See note* §)

NOTES

* A separate line should be used for each wage-earner, except workpeople doing work at home on material supplied by the employer, who should be excluded. Foremen, carters, packers, etc., casual workers and part-time workers should be included, but managers, clerks, typists, commercial travellers, shop assistants, and salaried persons generally should be excluded. If any workers are provided with board and lodging in addition to cash wages, those workers should also be excluded.

The earnings shown should include the workers' contributions to National Health, Old-Age and Unemployment Insurances, but should exclude the employers' contributions.

If any workers employ helpers, the amounts shown should be the net earnings, after deducting the amounts paid to the helpers, and the earnings of each helper should be shown on a separate line.

If employment in the last pay-week of October was affected by a holiday, fire, breakdown, strike or other exceptional circumstances, the nearest week of an ordinary character should be substituted.

† If in any cases the exact number of hours worked is not known, please give an approximate estimate.

‡ The age-grouping should be as follows : Men, 21 years and over; Women, 18 years and over; Boys, under 21 years; Girls, under 18 years.

§ Work-people paid under any system of payment by results should be included under piece-workers, and marked P.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

Artur Apfel.
Frederick Charles Douglas Back
Nripendra Bhattacharya.
Basil Bradlaugh Bonner.
Karl Henrik Borch.
Alan Frank Brazier.
William Alexander Brown.
Robert Laurence Caldwell.
Ronald Arthur Cooper.
Brian Charles Coote.
Brian Dalton Copland.
Harry Crossland.
Michael Patrick Curwen.
František Egermayer.
Eric Evans.
Geoffrey Hudson Faulding.
Harry Forshaw.
Jack Donald Hewitt.
Alec Lovell Hodges.
Ian Alexander Marshall Hustwick.
Niels Kay Jerne.
Kenneth Guy Jack Charles Knowles.
Eardley Greville Lyon Lieversz.
Frederick John Lloyd.
Francis Henry Charles Marriott.
William Henry Mason.
Joan Doris May.
Eric Morley.
Montague William Mulready Jones.
John Oswald Newsam.
Ernest James Francis Oldmeadow.
Charles George Paradine.
Ruth Florence Parker.
Dil Mohammad Qureshi.
S. Raghavachari.
A. A. Rau.
Ray St. Clair Sandall.
Henry Scheffé.
Kenneth Ainsworth Stott.
Gerhard Stuvel.
Walter Laws Smith.
John Curnow Tanner.
Peter Frank George Twinn.
Walter Brian Vane.
Leslie Edward Webster.
James Stanley Weightman.
David Robson Westgarth.
George Denby Whiteside.
Kennedy Williams.
Sam Wood.

Corporate Representative

Sir William Goodenough, *representing* Barclays Bank Ltd.

The Sources and Nature of Statistical Information in Special Fields of Statistics*

Statistics of the Censuses of Production and Distribution

By H. Leak, C.B.E.

This note deals primarily with the Census of Production, which is the responsibility of the Board of Trade; agricultural production comes within the province of the Ministry of Agriculture and Fisheries, and is not covered by the Census of Production or dealt with in this note. The note relates also to the Census of Distribution, powers to take which have recently been given to the Board of Trade by the Statistics of Trade Act, 1947. The powers of the Board of Trade to take a census of production are likewise derived from legislation, and to appreciate the reasons why the information collected at successive censuses has not always been similar, it is necessary to know what powers to obtain such information existed at the time.

The original Census of Production Act was passed in 1906, and arose out of the tariff reform controversy at the beginning of the century. The first extension of the powers of the Government to obtain compulsory information from industry also arose in connection with tariffs, the Import Duties Act, 1932, giving power to hold inquiries in respect of the production of goods of kinds covered by the new tariff. This power was extended in the Finance Act of the following year to relate also to goods covered by the Silk Duties and the McKenna Duties.

Section 3 of the 1906 Act specified the matters about which information could be obtained as follows: "the nature of the trade or business, and particulars relating to the output, the number of days on which work was carried on, the number of persons employed, and the power used or generated and relating to such other matters of a like nature, except the amount of wages, as may be found necessary for the purpose of enabling the quantity and value of production to be ascertained," together with "the aggregate estimated value of the materials used and the total amount paid to contractors for work given out to them," with the proviso that quantitative information about output could only be obtained in the detail set out in the Import and Export List. The Census of Production Act, 1939, removed the limitations on the detail that could be required for materials and output, extending in effect to all products the powers given by the Import Duties Act; but owing to the outbreak of war no census has been taken under the Act as thus amended. The Statistics of Trade Act, 1947, extended the powers to cover practically any matter that might usefully be investigated at a Census. This Act has, in effect, changed the character of a Census of Production from a summary of output and employment into an instrument enabling an analysis to be made of the economic factors of production.

When the Census of Production Bill was introduced in 1906, it contained powers as wide as those given by the Statistics of Trade Act, 1947, but, as pointed out by Yule (1907), these powers were substantially reduced by the House of Commons and the original design of the Census was thereby altered in a marked degree. The limitation of powers was, perhaps, only natural in those days when something new was being tried in this country for the first time. The United States had had periodical Censuses of Manufactures since 1870, and Yule compared the potential scope of our census with the actual scope of those taken in the United States and Canada. In addition, he furnished information about the censuses that had been taken in all countries, and international comparisons were also made by Flux in 1924.

Censuses have been taken in respect of the years 1907, 1912, 1924, 1930 and 1935. Import Duties Act Inquiries were held for 1933, 1934, 1937 and 1938, and a Partial Census was taken in respect of 1946. The final report on each census gives comprehensive definitions of the terms used. This note can only refer to some of the changes that have taken place as the census has been developed over the past 40 years; for more precise information reference must be made to the notes on each report.

* For previous articles in this Series, see *Journal, Series A*, 1948, Parts II and III.

SCOPE OF THE CENSUS OF PRODUCTION

The first question that needs consideration is the precise meaning to be attached to the term "census of production." The list of persons required to make returns is set out in the schedule to the 1906 Act as follows:

(*a*) The occupier of every factory or workshop within the meaning of the Factory and Workshop Act, 1901.

(*b*) The owner, agent, or manager of every mine and quarry.

(*c*) Every builder, that is to say, a person who, by way of trade or business, undertakes the construction or alteration of a building or any part thereof.

(*d*) Every person who by way of trade or business executes works of construction, alteration, or repair of railroads, tramroads, harbours, docks, canals, sewers, roads, embankments, reservoirs or wells, or of laying or altering gas or water pipes, or telegraphic, telephonic or electric lines or works, or any other prescribed works.

(*e*) Every person who by way of trade or business gives out work to be done elsewhere than on his own premises.

(*f*) Every person carrying on any other trade or business which may be prescribed.

The Census does not cover agriculture, though this is not explicitly stated in the Act, and the word "industrial" could well have been inserted before "production." If, however, industrial production is understood to include, as it should, the transformation of material goods by manufacturing processes, one should cause enquiry to be made of farmers who manufacture butter or cheese on their premises, and not take account only of the production of such goods in factories. It is, however, more appropriate to regard the farm dairy as associated with agriculture, and to use in conjunction with the record of factory production the results of such enquiry as may be made into agricultural production in an endeavour to arrive at the total quantity of dairy products made in this country. Another illustration is that of the scouring of wool. While the washing of the fleece on the sheep's back before shearing is quite clearly a part of agricultural industry, the scouring of wool imported in the greasy state is a manufacturing process, as is the removal of wool from the skin of a dead sheep, though not of a live one. The sawing of timber may be a process of adaptation of portions of trees for use as fencing, or as fuel, on the estate where the trees were grown, in which case it is rather an element in forestry production, but if timber in logs is sawn into planks for builders and workers in wood, it is regarded as coming within the scope of the Census of Production.

The borderline between industrial production and services is likewise very narrow, and in this respect the United Kingdom practice has not been absolutely uniform in that the laundry, cleaning and dyeing trades were included before the 1930 Census. They were then and at the 1935 Census regarded as being more akin to services than to production, but were specifically exempted from the census. The 1948 Census reverts to the original view, and particulars are being obtained concerning these trades. Laundry and other receiving offices come, however, within the scope of the Census of Distribution and Services rather than within that of the Census of Production. Repair work, whether carried out at establishments wholly devoted to that form of activity or not, has always been included within the scope of the census in this country, though not in the United States, where it was included in their Census of Business. The pre-war rule has been modified as a result of our now taking a Census of Distribution, and it has been decided that firms mainly engaged in repairing goods direct for members of the public should be included in the Census of Distribution, while those undertaking repairs for the trade remain in the Census of Production. Important cases are motor garages, and boot and shoe repairers of whom many more thousands than those recorded at pre-war censuses were found to exist when they had, during the war, to apply for a licence to obtain leather. A number of these will escape record at either post-war census since they only do repairs during their spare time, but the pre-war Census of Production figures are clearly defective. The packing of goods is, in the case of textiles destined for distant

markets, a specialized business, and is regarded as a separate industry for the purpose of the Census of Production, but the blending and packing of tea, though adding proportionately more to the value of the goods, was not taken into account before the war, though it will be at the 1948 Census. In so far as packing, whether for retail sale or for export, is carried out in the establishment where the goods are produced, the recorded value of the goods is enhanced by the work done, but not if done elsewhere, with the exception just mentioned; and the growing practice of manufacturers of putting up their preparations in convenient packages may interfere to some extent with comparability of the value figures for successive censuses. The bottling of alcoholic and other liquors by wholesalers is regarded as industrial production, but no account is taken of the work of a similar nature carried out in retail establishments. The production of the extractive industries is in this country included as industrial production, but this is not invariably the case elsewhere. Building and other work of construction is likewise included here, though not in the corresponding United States census.

The reason for the inclusion of class (*e*) of the list of persons who may be required to make returns may not be obvious. But a firm can carry on a very extensive business by purchasing materials and arranging for work to be done on them by other firms. By such action a firm adds to the work done by other firms the work done by its own employees, and so increases the value of the goods produced. Merchant converters in the textile trade are a case in point; mostly they purchase cloth from manufacturers and have it finished to customers' requirements, whether as cloth or as made-up household textiles, etc., but they have other activities, notably wholesaling. Their activities have not been covered at previous censuses, when they were regarded as primarily wholesalers, but they are being included at the 1948 Census of Production, while their wholesaling activities will have to be brought within the scope of the Census of Distribution reports. An advantage of collecting returns from them is that they can provide information as to the production of different kinds of cloth. The selling value as well as the quantity will be recorded. This will be the first publication of such details, though quantitative returns have been made in somewhat similar detail during and since the war. Previously, for example, it was only possible to obtain by calculation, done at each census, the aggregate value of all cotton piece goods in their finished condition. Now one can distinguish between shirtings and overall cloths, towelling and handkerchief cloth, etc. etc.

The precise scope of each pre-war census is set out at the commencement of the volumes relating to it. Exemptions generally made, by rules under the Census of Production Act, have been the work of Parish Councils and Parish Meetings, taxidermy work, the making of wigs, flax scutching, and portrait and trade photography. Only the first and last of these are being excluded from the scope of the 1948 Census. The output of Philanthropic Institutions is of importance in some industries, but the aggregate value of their output is small, and has quite a different relationship to the numbers employed to that existing in commercial establishments. Apart from the 1907 Census, these institutions were not required to furnish information before the war (though they will be for 1948) except in so far as the work was carried out on a commercial basis, e.g. the printing work of the Salvation Army.

Treatment of the smaller firms has varied. The First Census in respect of the year 1907 and the Third in respect of 1924 included production at all establishments irrespective of size, but the smaller establishments have been exempted from the intermediate Censuses. At the 1912 Census the exemption applied to establishments at which not more than five persons were employed. At the 1930 Census firms employing not more than ten persons were exempted, except in Northern Ireland, where a separate Census was taken and the limit was five instead of ten persons. The same limit applied for the 1935 Census, information being supplied, however, for Northern Ireland on the Great Britain basis as well as on their own basis. This was necessary in order to ensure comparability of results with those for 1933 and 1934 for the trades covered by the Import Duties Act Inquiries.

All firms claiming exemption had to provide information about the total number of persons employed, including the proprietor. An experiment was made in 1935 with a very simplified form for the small firms in the Baking Trade who, in addition to information about numbers employed, had to furnish either the number of 4-lb. loaves baked or the number of sacks of flour used in that year. The response was good, and enabled a reliable estimate to be made of the

total quantity of bread baked. The technique thus initiated is being applied for 1948 to a number of other trades where the output of the small firms is of importance.

There were 204,151 small firms in 1935, which employed about 826,700 persons, or just over 10 per cent. of the total. For the factory trades, Table 1 summarizes the particulars about distribution by size of establishment contained in the Final Summary Tables for 1935.

TABLE 1.—*Size of Establishments in the Factory Trades in* 1935

Size of establishment	*Number of establishments*	*Number of persons employed* (thousands)
1– 10	132,338	536·6
11– 49	29,032	716·7
50– 99	8,582	601·6
100– 299	8,002	1,352·4
300– 999	2,795	1,381·0
1,000–1,499	267	323·7
1,500 and over	266	782·2
Total	181,282	5,694·2

Similar information is not available for the non-factory trades, but defining a firm as covering all businesses in one trade group that were operated under the same trading name, there were 12,490 firms in 1935 employing 2,147,900 persons in the non-factory trades against 41,164 firms employing 5,157,600 persons in the factory trades, not counting those employing fewer than 11 persons. The distribution of these firms is given in the 1935 Summary Tables. Leak and Maizels (1945) gave a table showing the distribution of all firms in the census when the whole of the establishments operated under a single trading name were taken together, and a further table for the single firms or combines employing 500 or more persons, the largest firms in each table being two employing more than 50,000 persons.

Period of Returns

Firms have always been given the option provided for in the original Act of making returns for their business year instead of the calendar year, if different. To start with, firms made returns for periods most nearly coinciding with the calendar year, and firms whose year ended on June 30th sometimes included the first half of the calendar year and sometimes the second. At the 1930 Census the ending date had to lie within the year ending March 31st, 1931, and the Census report shows that the mean terminal date was about the middle of the last week of December. Most trades had the ending date within the last fortnight of December, but in a few trades there was a very different general ending date—the brewers, for example, most often used a September year. At subsequent censuses the ending date was advanced so as to include the firms using the income tax year, so that the mean terminal date should approximate still more closely to the end of December. The small firms required to give only summary particulars of employment, output and materials are asked for information relating to the calendar year.

At the First Census of Distribution a similar option will be given; for the Pilot Census the final ending date was March 31st, 1948.

Output and Materials

Net output, or net value of production, may be defined as the difference between the gross value of the products and the cost of the materials used in producing them, payment for work given out to be done by other firms being also deducted. Excise duty, if included in the value of the output but not in that of the materials, is also deducted; the values recorded are inclusive

of subsidy but exclusive of purchase tax. This conception of value added by the industrial processes has not always been adopted in other countries. In Canada, for example, added value or net output was regarded, prior to the Conference of British Commonwealth Statisticians at Ottawa in 1935, as the difference between the value of the gross output and the cost of (*a*) the materials transformed in the industrial processes, and (*b*) the packing materials. The value of the fuel used was not deducted unless, as in a blast furnace, it was transformed.

Net output is the fund from which wages, salaries, rent, royalties, rates and taxes, advertisement and selling expenses and all other similar charges have to be paid, as well as profits and any amount set aside to cover the depreciation of plant and machinery. It contains no duplication within itself, but it will duplicate to some extent the values returned at a census of distribution. There was no power under the Census of Production Act to collect information about the various important factors of production mentioned above, but that power is given by the Statistics of Trade Act, and it is being used, for example, to discover the various kinds of payments made to other firms for services rendered.

Gross output is the aggregate of the net selling value of the goods produced and the amount received for work done. The value is net of trade discounts, agents' commissions, etc. Before the war firms were instructed to add to the value of goods sold the value of their stocks of such goods at the end of the year and to deduct that at the beginning, thus giving a value of the goods produced in the year. They were also instructed to deduct any sum paid for carriage outwards. That was not an easy instruction to apply to each individual product, and it is probable that in many cases it was not complied with. The wider powers now available enable the value of gross output to be ascertained with greater precision and furnished by the manufacturer with greater ease. The value of sales is asked for, net of trade discounts, etc., as before, and also the aggregate value of stocks and the value of work in progress at the beginning and end of the year, together with the total amount paid to other firms (or credited to the transport department of the same firm) for carriage outwards. For certain important products the quantity and value of stocks at the beginning and end of the year is also asked for. For these products, therefore, the quantity made during the year is available to match the pre-war figures, and the effect of rising or falling prices on valuation can be seen. The fall in prices in 1930 was very severe, and resulted in a fictitiously low value being put upon gross output and also net output for that year. The textile trades were particularly affected. The quantity of goods sold adjusted for stocks does not always give the quantity of goods produced. In many cases intermediate products are made and some are further processed in the same works; for a number of these the total quantity made was ascertained at pre-war censuses, and the list is being extended at the 1948 census. This enables, *inter alia*, a true comparison to be made between production and exports, and it also saves incorrect conclusions being drawn from reduced sales (or production) which may have resulted solely from a greater degree of integration in an industry.

The Census tables show the value of the gross output of the firms classified in the industry, and how this is divided between the goods regarded as products of the industry and the goods appropriate to other industries. The total quantity and value of the products of the industry made by firms in all industries covered by the census is also tabulated in the Final Report published as separate volumes, but the preliminary reports published in the *Board of Trade Journal* are incomplete in this respect, since the output of firms in other industries may not have been summarized, and they may also be incomplete owing to the whole of the returns for the industry not having been received at the time of publication.

In Table 2 particulars are given of the aggregates recorded at the 1907, 1924, 1930 and 1935 Censuses. The 1907 Census included firms in the territory which is now Eire, the net output there being estimated at about £11 million. It also included firms of all sizes that made returns, whereas the particulars for the later censuses relate only to firms employing over ten persons. The aggregate output of the small firms is some 8 per cent. of the total, but the particulars recorded for 1907 were deficient owing to returns not being received from all firms, and the extent of the deficiency has been estimated at between 6 and 7 per cent. The table also shows the distribution in 1935 between England and Wales, Scotland and Northern Ireland. The reports go into greater detail about production in the principal industrial areas, of which 13 are distinguished in England and Wales and 4 in Scotland.

TABLE 2.—*Summary Particulars for the 1st, 3rd, 4th and 5th Censuses of Production*

Area		*Gross output (selling value of goods made and value of work done)* (£ mill.)	*Cost of materials and fuel used and amount paid for work given out* (£ mill.)	*Net output (excess of Col. 2 over Col. 3 after deducting Excise duties)* (£ mill.)	*Average number of persons employed (excluding outworkers)* (Thous.)	*Net output per person employed* (£)
United Kingdom,	1907	1,765·3	1,053·2	696·8	6,985	100
	1924	3,747·0	2,105·5	1,548·0	7,298	212
	1930	3,370·5	1,789·4	1,503·5	7,141	211
	1935	3,564·6	1,876·0	1,625·1	7,306	222
England and Wales,	1935	3,205·5	1,683·9	1,462·0	6,495	225
Scotland,	1935	300·3	153·4	143·0	671	213
Northern Ireland,	1935	58·8	38·7	20·1	140	143

Note.—Similar information is not available for the 1912 Census.

Particulars are given below to show the changes between 1924 and 1935 in the distribution of industry between the principal trade groups. Figures are shown for all groups the net output of which was £100 million or more in 1935. There are many reasons for differences between various trades in net output per head, on which more light will be thrown by the collection of new kinds of information at the 1948 Census, but one cause of difference for the same trade as between one year and another is the extent of short time worked, which will be reflected in the statistics of wages paid.

TABLE 3.—*Distribution between Main Trade Groups in* 1924 *and* 1935

Trade group	*Gross output*		*Net output*		*Average number of persons employed**		*Net output per person employed*	
	1924 (£ mill.)	1935 (£ mill.)	1924 (£ mill.)	1935 (£ mill.)	1924 (Thous.)	1935 (Thous.)	1924 (£)	1935 (£)
Engineering, Shipbuilding and Vehicles	402·2	491·4	198·4	249·3	985·6	1,104·4	201	226
Food, Drink and Tobacco	669·6	663·7	172·5	201·5	439·8	520·6	392	387
Textiles	762·9	446·0	221·8	157·5	1,262·1	1,054·9	176	149
Iron and Steel	295·4	280·6	98·6	116·5	498·9	539·3	198	216
Paper, Printing and Stationery	161·6	184·2	93·9	111·7	343·6	409·0	274	273
Other Factory Trades	734·3	771·2	290·7	345·1	1,326·7	1,529·4	219	226
TOTAL—Factory Trades	3,026·0	2,837·1	1,075·9	1,181·6	4,856·7	5,157·6	222	229
Public Utility Services	255·1	312·6	145·4	184·6	644·7	698·1	226	264
Mines and Quarries	273·0	166·8	226·4	137·0	1,281·0	845·1	177	162
Building and Contracting	162·7	216·0	80·6	100·0	419·1	502·3	192	199
TOTAL—Non-Factory Trades†	721·0	727·5	472·1	443·5	2,441·5	2,147·9	193	206
TOTAL—ALL TRADES	3,747·0	3,564·6	1,548·0	1,625·1	7,298·2	7,305·5	212	222

* Excluding outworkers.
† Including Government Departments.

Under the Census of Production Act the only information obtainable about materials was their total value, but as power was specified as a subject about which information could be obtained, the quantity of coal, coke and electricity used for power was obtainable and was asked for at the

1930 Census, with a voluntary question about the quantity used for other purposes. At the 1935 Census the powers given by the Import Duties Act enabled the information to be asked for compulsorily from most industries, and the other industries gave information about the total quantity and value of electricity and the various fuels used in preference to supplying information about the amount used for power only. By the collection in this country for the first time of particulars of the principal materials used by each industry, it was possible to see the inter-connections between industries, and the 1935 Final Report contains tables showing the distribution of products of the iron and steel industry, and also of cotton yarn. This is the material connection between industries. The inter-connection between firms in different industries forms the subject of a paper by Leak and Maizels on the Structure of British Industry, which contains the results of an enquiry made on this subject in connection with the 1935 Census.

As the finished products of one firm frequently form the materials of another, there is a large measure of duplication in the results of the census as a whole, but in so far as goods are sold by one firm to another in the same trade there is duplication there too. Detailed estimates of the gross output of each trade free from duplication will be found in the volumes containing the results of the 1907, 1924 and 1935 Censuses, and for the more important cases of duplication approximate figures for 1930 are given in Part V of the Final Report on the census. The post-war estimates should be much more precise than those made before the war, as the list of materials can now include the products which it is anticipated may be duplicated.

The duplication in gross output for the census as a whole prevents any direct calculation of the total value of all industrial goods made. It is therefore necessary to proceed indirectly, using the figures for imports and agricultural output. The latest calculation made relates to 1930, and a full description of the method employed is contained in Part V of the Final Report on that census. The calculation also covers the total value of all goods made or produced for sale and the proportion exported. This forms an important part of the national income. The figures for 1924 will be found in *The National Income* by Flux, and for 1907 in the Final Report on the Census for that year.

Volume of Production

The output of an industry or of a number of products can only be added together usefully in terms of value, and between one census and the next this is subject to variation as a result of changes in price as well as in quantity. A measure of the quantitative changes can be arrived at by eliminating the effect of price changes, the result being called the change in the volume of production.

The Census reports for 1924, 1930 and 1935 and the reports on the Import Duties Act Inquiries for 1933 and 1934 give the results of a calculation of the volume of production of the goods classified as principal products of each trade (not the gross output of the firms classified in the trade) in the year in question compared with that in an earlier year. The general method has been to value at the prices of the base year each of the kinds of goods produced for which quantity figures are available, and to compare the aggregate of such values with the value recorded in the base year. The value of other goods produced for which quantity figures were not available for both years has generally been deflated in like proportion,* and the total value when compared with the corresponding total value in the base year gives a measure of the change in the volume of production between the two years. The Board of Trade state that in industries where quantity figures were not available a calculation has been made based upon changes in the prices of materials and other manufacturing costs. If the years are close together, or if price changes have not been large, the results are generally reliable, but it must be noted that if the calculation was made the other way round, the estimated change in volume would not be identical. A factor which comes into play particularly if there is a considerable interval between the two years is the change in the amount of duplication.

In *Studies in the National Income*, 1924–1938, there is a general discussion of the subject, and the volume of production is estimated for the years 1924, 1930 and 1935. The method finally

* This method has not been followed invariably, since the item for which quantitative information is not available may be more akin to one or more products of a trade than to the aggregate of all products for which quantities are recorded.

used was open to criticism, as pointed out in a note by Leak (1943), in which reference is also made to work on the subject in the London and Cambridge Economic Service Special Memorandum No. 47, and *The Manchester School of Economics and Social Studies*, Vol. 10.

A discussion took place in the *Journal* (Miscellanea) 1944, III–IV, on the possibility of arriving at a volume of net output, thus eliminating the duplication mentioned in connection with gross output. Given adequate information about the materials used it is clear that there is little more difficulty in arriving at a volume of materials purchased and used than there is for gross output. The coverage for materials and fuel is, however, never complete, as it normally is for output, and there is no means of knowing whether prices of the unspecified materials have moved in a similar manner to those of the specified. Except in a few trades the unspecified materials at the 1935 Census were one-third or more of the total. The pre-war tables of principal products and materials cannot, however, be compared. The materials table relates to the materials purchased and used by the firms classified in the industry, whereas the principal products table relates to the output of those products by firms in all industries. To enable a comparison to be made, the principal products made by firms in other industries have to be deducted and the other descriptions of goods made by firms in the industry have to be added, each being revalued for a volume comparison.

For information about productivity, the factors affecting it and some international comparisons reference should be made to Flux (1933) and to "Comparative Productivity in British and American Industry" by Rostas, which contains a comprehensive bibliography.

Power Equipment

Particulars of the prime movers, electric generators and electric motors ordinarily in use and in reserve or idle were obtained at the 1924 and 1930 Censuses. At the 1907 Census, particulars of electric motors were not asked for, and at the 1907 and 1912 Censuses prime movers and electric generators were not divided according to whether they were ordinarily in use or in reserve or idle. Prime movers have always been shown by kinds and generators according to the kind of prime mover by which they were driven. From the later data it is possible to calculate the total power available, though it must be borne in mind that the division into equipment "ordinarily in use" and "in reserve or idle" may not be absolutely precise.

The quantity of electricity generated and the amount purchased was obtained at the 1907 Census but not again till 1930, when the quantity used was asked for, the balance of the amount generated being recorded as output.

Classification of Industries

Before the war there were three major sources of information about industrial employment, and in many cases these did not cover the same field, so that it was difficult to move from one set of statistics to another. The sources of information were the Census of Population, the Census of Production and the Employment and Unemployment Statistics of the Ministry of Labour. A fourth set of statistics existed in the Factory Inspectors' List, but these were necessarily compiled on a somewhat different basis for purposes of accidents, etc., and need not here be considered.

All firms in the Census of Production were classified into one or other of the 123 trades on which separate reports were published. The Census of Production classification has no such indeterminate headings as occur in the other two classification—metal industries, not separately specified, other manufactures, etc. etc.—which are unnecessary owing to detailed information as to a firm's output being available. The Census of Population classification has to rely on information given by individuals, who are asked to state the nature of the business of their employer, and they do not always do this in the same way. Hence some employees of a firm may be entered under one Census of Population heading and some under another. The Ministry of Labour employment statistics were derived from the annual exchange of unemployment books in July, most of which was done by the firm by whom the person was employed at the time, but those out of work exchanged their own. The Census of Production data relate to the whole year, but the other two sets to spot dates, the one generally in April and the other in July—seasonal influences may thus have an effect on comparability.

An attempt was made by Clark (1932) to match the numbers employed in 1924 in the metal industries according to the Census of Production, the Unemployment Insurance statistics and the Ministry of Labour Wage Inquiry. The fact that this was not a very good effort illustrates the difficulty in using the Census of Production data. A lot of information is now published in the census reports about employment in smaller industries included within a main census trade. Some of these are industries too small by themselves to form the subject of a separate report—coir fibre, horsehair and feather, for example—while others are sections of a main industry, such as the 23 specific divisions of the Mechanical Engineering Trade, for which separate information was published at the 1935 Census. The question of specialization is discussed in Part V of the 1930 report.

With relatively minor differences it is possible to match the Census of Production classification with groupings of the more elaborate Census of Population classification, and this is true also of the Ministry of Labour statistics; but the two sets of groupings differ, and a comparison between the Census of Production and the Ministry of Labour classifications shows some major differences, some of which can be reconciled by the use of the supplementary information about employment referred to above.

The above relates to the position as it existed prior to 1948. Then, a standard industrial classification was published, with the object of securing uniformity and comparability in the statistics provided by Government Departments. It comprises a minimum list with a number of recommended optional sub-headings. The minimum list of 124 industries within the scope of the Census of Production is being used by the Ministry of Labour. The Census of Population will probably use all the optional subdivisions, while the Census of Production may be expected to occupy an intermediate position.

Employment and Wages

The census divides workers into operatives and administrative, technical and clerical staff, the dividing line between the two being before the war slightly different from that used by the Ministry of Labour, though brought into agreement at the 1946 Partial Census. The main difference lay in the treatment of works foremen, regarded as operatives in the census but not by the Ministry of Labour. The censuses have now changed their definition. The census has always included working proprietors, but as these were not, until recently, insured against unemployment they were not included in the Ministry of Labour statistics. Some minor differences in the definition of "employed" remain. Canteen staff at a factory are non-effective workers for Census of Production purposes, as are persons engaged in purchasing and selling again the same goods, but the pre-war and existing statistics of the Ministry of Labour will include these if employed on the same premises. These would be counted by the Board of Trade at a Census of Distribution among those engaged in the catering trades and wholesaling respectively. The Census is concerned with counting as employed all those whose labours contribute to the sale value of the goods as recorded. Hence bakers' roundsmen are included, and shop assistants selling the baker's own goods, if, as is usual, the sale value is that paid by the consumer. The post-war census forms have been so designed as to enable these various classes of workers, who may be recorded differently by the Registrar-General and the Ministry of Labour, to be identified.

The basis of the census is the establishment, but head office staff situated away from the factory or factories is included, being spread among the various establishments if more than one. Equally the staff of a holding company is included in so far as it performs the functions of a head office.

At the first two censuses particulars about the numbers employed were obtained at quarterly intervals, but greater accuracy has been secured since then by asking for information concerning numbers of operatives employed in one week in each month and numbers of other staff in October only, the latter not being subject to the same seasonal influences as operatives. The 1935 report gives, for the trades covered by the Import Duties Act Inquiries of 1933 and 1934, index numbers of operatives employed in each month of the three years by firms furnishing returns in respect of the twelve months ended December.

The exemption from making detailed returns granted after 1924 to firms employing not more than ten persons has already been referred to. These small firms have always been asked for

information about the number of persons employed, and from the information thus provided, coupled with the number of firms on the register who failed to furnish any information, it has been practicable to estimate the average number of persons employed in each industry in each censal year except 1912, for which information was incomplete.

Wages was specifically excluded as a subject of enquiry by the original Census of Production Act, and it is only at the post-war censuses that information is being obtained on the census forms. The Ministry of Labour have, however, conducted an enquiry into earnings and hours of work at about the time of each census, except 1907 and 1912, and included in it a question about the sums paid annually to operatives. The enquiry was voluntary, and the sample furnishing information was clearly not representative. Part V of the final report on the 1930 Census contains an estimate of the total wages bill for 1924 and 1930, based on the results of the Ministry of Labour enquiries, and includes estimates also for the larger industries; similar estimates have been made in the Final Summary Tables for 1935.

The importance of the field potentially within the scope of a Census of Distribution and Other Services (Census of Distribution for short) in relation to the Census of Production may be seen from the following table classifying employed persons into broad groups. The figures relate to males aged 14–64 and females aged 14–59, including employers and self-employed.

TABLE 4.—*Employment in* 1939 *and* 1948

	1939 (*Million*)		1948 (*Million*)	
I Covered by Census of Production:				
Manufactures (including gas, water and electricity)	7·1		7·5	
Mining and quarrying	·9		·8	
Building and contracting	1·3		1·4	
		9·3		9·7
II Covered by Agricultural Statistics, etc.:				
Agriculture, Forestry	·9		1·1	
		·9		1·1
III Not covered or not adequately covered:				
Distributive Trades	2·9		2·4	
Transport	1·2		1·5	
Other "Service" Trades including catering*	2·2		2·1	
		6·3		6·0
Grand Total*		16·5		16·7

* Excluding Government Service, and private domestic service and private gardening.

CENSUS OF DISTRIBUTION

A Census of Distribution has not yet been taken in this country. One is planned for 1950 and a Pilot Census was taken in respect of 1947, as recommended by the Census of Distribution Committee. No information obtained at the Pilot Census has been published at the time of writing, and the only detailed statistics available are those obtained at a "Trial Census" carried out in 1936 by a group of business firms. This census was taken in six towns, geographically and characteristically diverse, but of similar size, so that comparisons could more easily be made. Retailing only was covered, and information was recorded relating to—

(1) trade description of the shop (85 descriptions used);
(2) commodities sold (127 separate groups);
(3) economic type of shop; and
(4) rateable value,

the last mentioned being considered to be the best available alternative to turnover, details of which could not be obtained at a voluntary enquiry. The report stated that information about employment, operating expenses, stocks and stock turnover and the operations of wholesalers had also to be omitted because its collection was impossible in an unofficial census.

The Census of Distribution Committee, in their report published in 1946, considered the uses of a census of distribution and made recommendations as to its scope and the kind of questions that might be asked. Each of the matters about which it was suggested enquiry might be made was included in one or other of the schedules at the Pilot Census for 1947. This was on a voluntary basis, and was designed to test out these questions and to discover the difficulties likely to be met with in taking a full census rather than to obtain publishable results on a sample basis.

The first difficulty is that of compiling a register of the shops and other premises to be sent forms for completion. As a result of rationing and other controls, with the Local Authority rating lists and the list of persons licensed to sell tobacco, a lot of information is now available to Government Departments which may assist in checking lists of premises furnished by local enumerators, but it is unlikely that on the first occasion at any rate the register compiled by these means will be 100 per cent. complete, and the Statistics of Trade Act makes special provision for calling attention by advertisement to the traders' obligation to apply for a form if he had not received one.

The question of what constitutes a shop (or a retail outlet about which separate information should be supplied) is a problem. Should the census attempt to enumerate all the mail order houses, coal order offices, laundry receiving offices, street traders, hawkers, slot machines, sales in cinemas, sports grounds, etc.? Is a department store one shop or many? If there are two different businesses carried on in adjacent buildings by one owner, do these constitute one shop or two? These are some of the matters on which the report of the first census will show the decisions arrived at.

The tables to be published should distinguish in broad outline between the co-operative societies, the multiple shops and the independent retailers. Difficulty may arise in giving local information about the co-operative societies for comparison with the other two types owing to the shops coming under one ownership, and the risk of disclosure may also prevent information being given in some instances about multiple shops. Apart from the co-operative societies there is no clear-cut line of division. When does an independent shopkeeper who acquires a number of branches become a multiple shop for census purposes? The preliminary census forms ask for the number of branches, and it may be presumed that shops are to be classified by this method; multiples may then be defined arbitrarily, in accordance with the present practice, as belonging to a trading concern with 10 or more branches.

As the large multiple organizations perform wholesale as well as retail functions, the forms prepared for the Pilot Census attempted to split these two sides of the business, so that the reports on wholesaling and retailing could include the respective parts of the business of the multiples, and this applied also to the co-operative societies. This involved the calculation of a transfer value, and it is evident that the on-cost (difference between purchase price and transfer value) at the wholesale stage might be calculated differently by various concerns, leading to somewhat arbitrary results. This same problem of transfer value has arisen in connection with the census of production, but is there more tractable, though it has led to some anomalous results owing to the census instruction about valuation not always being adhered to. For the First Census of Distribution the calculation of transfer values for multiples has been abandoned; it remains to be seen how these organizations will be treated in the census tables.

At the Pilot Census there were 20 kinds of forms, which mostly went in pairs, the one set being for the larger establishments, and the other (much simplified) for the small businesses with a turnover of less than £4,000 in the year. The preliminary forms for the full census show that the small businesses will be asked fewer questions. The dividing line is fixed at a turnover of £5,000. The preliminary forms for the 1950 Census so far prepared number 16 as follows:

Retail shops—small, large, multiple and co-operative societies.
Stall holders, street traders, etc.
Pawnbrokers.
Motor traders—small and large.
Restaurants, snack bars, etc.—small, large and multiple.
Staff canteens.
Service trades.
Repairers—boot and shoe, and other.
Wholesalers.

The preliminary form for wholesalers indicates how wide a field is covered. Thirty different types of business are distinguished, ranging from coal merchants to distributors from lorries, from import merchants to confirming houses, from brokers to wholesale auctioneers, and distinguishing warehouses with or without cold storage facilities.

Residential hotels and public houses are not being covered, but otherwise the scope is similar to that of the Pilot Census. The scope of the First Census follows closely the minimum list recommended by the Census of Distribution Committee, the changes being the exclusion of public houses and the inclusion of importers of industrial materials, export merchants and warehouse keepers. The additional list given by the Committee, not as exhaustive, but as indicating roughly the extent of the field that will require to be surveyed, related to hotels; advertising agencies; entertainment; ticket agencies; estate agents; transport—rail, road and coastwise shipping, taxis, hire service, bus and coach; stevedoring.

The Standard Industrial Classification published in 1948 gives some idea as to the minimum classification that will be adopted for the census of distribution. It divides the distributive trades into—

(1) Dealing in coal, builders' materials, grain and agricultural supplies (wholesale or retail), with 4 subdivisions.

(2) Dealing in other industrial materials and machinery, with 7 subdivisions.

(3) Wholesale distribution of food and drink, with 7 subdivisions.

(4) Retail distribution of food and drink (excluding catering), with 8 subdivisions, the last being general shops, defined as shops dealing in both food and non-food goods and whose trade is so mixed that they cannot be classified in any of the preceding subdivisions.

(5) Wholesale distribution of non-food goods, with 8 subdivisions, including one for general export merchants.

(6) Retail distribution of non-food goods, with 11 subdivisions, the last being department stores and variety chain stores, including department stores operated by co-operative societies, but excluding, where possible, restaurants attached to department stores.

(7) Retail distribution of confectionery, tobacco and newspapers, with 5 subdivisions.

Estimates that have been made of the number of retail outlets vary from 500,000 to 1,000,000, the variations to some extent being accounted for by the definition adopted—the inclusion or exclusion of caterers, for example. Smith (1948) and Levy (1947) together mention all the estimates made and such statistical information as is available bearing on the subject, and the latter gives a comprehensive bibliography. It seems probable that the number of retail outlets in the United Kingdom is not less than 750,000 and the census schedules to be sent out will probably exceed 1,000,000. The two books give estimates of the distribution of the outlets according to the description of goods sold, and some estimates of the type of control and size of shop. The most comprehensive analysis of retailers according to the kind of goods sold is, however, given in an article in the issue of *The Economist* for April 17th, 1946, in which a number of the possible pitfalls are mentioned. The method of operation of the various types of retailers is described by Neal (1932), though with the exception of mail order houses he does not mention the shopless retailers, such as market stalls, slot machines, selling in cinemas, etc. The compilation of the register for the first census will give the first reliable information as to overall numbers, but the amount of detail available will depend on the instructions given to those responsible for preparing it and the ability of the enumerator to distinguish different kinds of business. It may be hoped that some information on this subject will become available in 1950 before the census is taken.

The preliminary census forms ask for the value of goods sold and services rendered and also for the cost of the goods purchased. From these figures, coupled with the value of goods in stock at the beginning and end of the trading year, the gross margin will be arrived at. This is a figure of great value, being analogous to net output in the census of production (though not identical, since it may include payments made for transport), but it has not so far been published as a result of a census in any country. Elsewhere the censuses have asked for the trading receipts and a number of expense items. Though more questions about expenses were included in the

Pilot Census, retailers at the first full census will only be asked for information about wages and salaries. For wholesalers, however, transport is a very important item in many instances, and questions about the expenditure incurred in operating vehicles owned and about the payments made to contractors for the delivery of goods to customers are included. Export merchants raise some special problems, such as the treatment of marine freight and insurance, etc., where goods are sold c.i.f., and the purchase and re-sale of goods which never pass through the United Kingdom, and specific questions are to be asked on these subjects.

The questions asked will enable tables to be prepared for each kind of business showing what proportion of sales is represented by the gross margin and how much of this is paid out for labour, and the value of end-year stocks in relation to sales. These figures can be broken down by showing under each kind of business the size of the firms engaged as measured by turnover (or possibly by labour or gross margin for the multitudinous varieties of wholesalers), and particulars can be given for regions and large towns as well as for the country as a whole in so far as disclosure will allow. Turnover in each kind of business can also be split up between sales of goods, sales of meals and refreshments, and receipts from services, and can be related to the employment afforded, separate particulars being given of male and female full and part time working proprietors and paid employees, employees being divided so as to distinguish those aged under 18.

Sales of goods in broad categories are being asked for, but these are required for the purpose of correct classification of businesses and will not provide outlet information, which was one of the subjects of enquiry at the Pilot Census. The information provided will enable the retail trade statistics published monthly in the *Board of Trade Journal* to be made much more accurate, as the sales in the different groups can be weighted.

At the 1948 Census of Production firms are being asked to analyse their sales according to the class of purchaser—manufacturers, wholesalers, retailers, etc., distinguishing the manufacturer's own organizations from those not under his control. The census schedule emphasizes that firms should make the best estimates they can, since records may not be kept in the kind of detail required for census purposes. This information, with similar information to be obtained from distributors, is essential for showing the channels of distribution of goods, one of the most useful results that may be expected from a census of distribution. The preliminary form for wholesalers at the 1950 Census asks similarly for an analysis of their sales, and all but the small retailers are being asked to distinguish their sales to other retailers for re-sale and to trade and industrial users on trade terms. The census of distribution questions for wholesalers are not so detailed as those for manufacturers at the census of production, but when the broad picture is obtained, sample enquiries may enable sales to be estimated more precisely as between the various classes of purchaser, the details then matching those for manufacturers. Expenditure on goods by consumers will be able to be analysed into sales by retailers, by wholesalers, and direct by manufacturers, and it will be possible to see to what extent sales by manufacturers, by agents, by wholesalers, etc., contribute to our exports.

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PROCEEDINGS OF SECTIONS

INDUSTRIAL APPLICATIONS SECTION

PROCEEDINGS OF THE LONDON GROUP

TWENTY-EIGHTH ORDINARY MEETING, APRIL 1ST, 1949

Subject: "Statistical Principles in Quality Specification for Plastics."

Principal Speaker, Mr. C. Wainwright; Chairman, Mr. A. Blackwell; Attendance, 18.

Mr. Wainwright showed samples of three forms of the phenolic resin under discussion, together with examples of mouldings made from them, including a telephone hand microphone set.

A British Standard had been set up (B.S. 771 (1948) Synthetic Resin Phenolic Moulding Materials), and the paper was concerned with the statistical argument used in setting up that standard, a process which had been going on for several years, and in which they had had the advice of Mr. Jennett.

The specification dealt with eleven physical properties measured usually on specially moulded test pieces, and gave the worst limits for those properties on a specified sequence of tests. The material was in powder form, and the very thorough blending it underwent made a special sampling technique unnecessary. Because of the variability within test pieces a large number of test pieces was required to give confidence. This was overcome in the Certification Scheme of the B.S.I., by which the manufacturer certified each half year that he had made the tests demanded by the British Standard and that the quality was as specified on the label. (Eight type classifications of quality were used.) Thus the continuous information which the manufacturer naturally had was turned to the benefit of the purchasers.

Miss Keen, Mr. Jennett, Mr. Blackwell and Mr. van Rest took part in the discussion.

TWENTY-NINTH ORDINARY MEETING, MAY 6TH, 1949

Chairman, Mr. D. Newman; Attendance, 60.

After the business of the Annual General Meeting a Forum was held; the speakers were Professor E. S. Pearson, Dr. B. P. Dudding, Mr. Philip Lyle and Mr. E. C. Fieller. The following topics, in the form of questions, were discussed:

1. Is it true that quality control is suited only to long runs of manufacture on one product? Why are 1 in 40 limits regarded as "significant" and 1 in 1,000 limits as "highly significant"?
2. There would seem to be a danger of too closely identifying the significance test of the statistician with real practical risks, the statistician's part thus appearing to be an artificial one in practical problems. Can the probabilities residing in data be combined numerically with the judgment resulting from experience to enable a quantitative measure to be obtained of the actual risk involved in making a decision leading to action?
3. What is the best way of approaching management for approval in starting statistical experimentation?
4. What significant results could be obtained from a study of serial correlations, for example, of business and economic data over a period of twenty years?
5. In the initial stages of an investigation involving many factors of uncertain importance, there is a risk that experiments will be wasted because of an unfortunate choice of the levels of the factors. What is the most efficient way of finding the correct ranges for the main experimental effort which is to follow these preliminary investigations? (There is assumed to be some definition of "correct" in terms of the statistics under investigation.)
6. Can any useful information be obtained from curve fitting if the extrapolation of the curve beyond the data gives values that are clearly impossible? (E.g., data lying closely on the bend in a cubic, the ends of which sweep away from any possible values.)
7. When dealing with two correlated variates there are two regression lines which can be fitted to the data; what relation have these lines to the true underlying physical relationship between the pairs of observations?

PROCEEDINGS OF THE NORTH-EASTERN GROUP

TWENTIETH ORDINARY MEETING, MAY 25TH, 1949

Subject: The Organization of Statistical Control in a Heavy Industry.

Principal speaker, Mr. A. W. Swan; Chairman, Mr. N. J. Squirrell.

Mr. Swan recounted the various difficulties he had experienced in starting a statistical control section in the United Steel Companies after the war. Many statisticians believed that statistics should be instituted at the lower management levels. He himself maintained that unless the statistician had access to the manager in each department and to all relevant material it was impossible, or very difficult, to produce useful results. Psychology should also be brought to bear in a statistician's work; to obtain the necessary co-operation one must "sell" the idea that statistical technique could help. The man whose job it was to collect the data from the factory workers and, if possible, obtain their interest and co-operation, must have the right personality. Mr. Swan's advice to his assistants was to avoid, whenever possible, the use of statistical terms when in the factory, so that the layman could understand what the statistician was doing and would thus be more willing to help.

In conclusion, Mr. Swan said that such methods had produced satisfactory results; most of the executives now came to him with their problems, while formerly his section had to ask if they could help in any way. He thought that two of the reasons for this were that before a report was made on any problem the technician was consulted, so that he would know he had not been by-passed, and that no report was made without including some recommendation.

FORMATION OF THE SOUTH WALES GROUP

A provisional group of the Section has been successfully operating in South Wales, and meetings have been held in Crumlin, Cardiff and Swansea during the last year. The support for the meetings and the enthusiasm of the local organizing committee made it possible for the Group to be formally admitted to the Section in May, 1949, and it will be known as the South Wales Group. Dr. T. V. Starkey and Mr. E. Lloyd have consented to act as Chairman and Honorary Secretary respectively.

Further information and forms for application for membership can be obtained from Mr. Lloyd, or from the Assistant Secretary of the Society.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*The Measurement of Colonial National Incomes: An Experiment.* By Phyllis Deane. National Institute of Social and Economic Research. Occasional Papers. Cambridge University Press, 1948. 9½". vii + 153 pp. 12*s.* 6*d.*

This book contains detailed estimates of the incomes of three colonies: Northern Rhodesia, and Nyasaland in 1938 and Jamaica for each of the ten years 1929 38. It is of equal interest to students of national accounting and of colonial economies. By bringing together the scattered information available for these territories Miss Deane makes an important contribution to the library of national income studies, while her analysis of the unusual methodological difficulties arising in colonial economies is of great value.

Miss Deane frequently reminds us that this enquiry into colonial national incomes is an experiment. Even in Great Britain it was only during the war that the construction of national income estimates got beyond the experimental stage, but the ground had been well prepared by pioneers. For colonial economies no previous estimates had ever been made, unless one counts those of Dr. Rao for India. The income of India, however, does not really present problems different in kind from those of Britain or America. Nor, it appears from the book, does that of Jamaica. Unlike an African territory, these economies are highly diversified and specialized, with the greater part of production being for exchange. In Nyasaland, on the other hand, close on 50 per cent. of the income of residents is produced by families for their own consumption, and not for sale or exchange.

This feature of colonial economies is one of the major difficulties in the construction of their social accounts. The problem is twofold: of estimating the physical quantities produced or consumed and saved (e.g. for seed), and of assessing the "prices" at which the income or output is to be evaluated. In the absence of any overall records of area under cultivation for each crop the estimates of physical output have to be pieced together from those fragments of the country covered by recent surveys. This leaves two potential sources of error: firstly that the areas surveyed may not be in fact representative of the whole region for which their averages are to be generalized; and secondly, that the methods of enumeration or sampling used in any such survey may not in any case lead to averages representative even of the surveyed area. Both these sources of error are all too frequently encountered in economic statistics for Africa.

The "pricing" of subsistence output is a fundamental economic problem of valuation rather than a statistical one. Miss Deane points out that "the act of consumption is also the act of production for which the most comprehensive statistical reporting system would not find two different sources of evidence." Indeed, there may be no source of evidence as to prices. Where part of the output is traded some indication of this price is available, but the proportion traded is often low; for example, in Northern Rhodesia Miss Deane puts it at less than one-fifteenth. This suggests that in many areas the whole of output must be for subsistence only, owing to distance from a market, or for some other reason. What prices shall be held to obtain in such an area? Clearly this problem cannot be dealt with as in more advanced economies merely by exclusion, and Miss Deane's discussion of the fundamental issues should be of interest to the student of national income problems: to which he may add, perhaps, the query as to the dangers of adding together estimates of such different validity.

Miss Deane is not diverted from her experiment by the intricacy of problems such as these in the logic of social accounting. She draws attention throughout to the character of the assumptions she is making and to the probable magnitude of the errors involved, and she makes her case for the estimate finally adopted. She is thus able to proceed to the completion of the models both for Jamaica and for each of the African colonies analysed, i.e. Northern Rhodesia and Nyasaland.

The completed estimates throw the problem into a proper perspective. For one thing they show that even in the simplest economies such as Nyasaland this poorly documented part of national output is not more important than the remainder, which is more adequately recorded and in which the margins of error are smaller. The error in the total is thus correspondingly smaller than that of the subsistence sector alone. Secondly, the comparison of these three territories shows the declining importance of this sector as economic diversification occurs.

These estimates of colonial national incomes are experimental also in another sense, for they incorporate the most recent advances in technique, and in particular the use of cross-checks as far as possible, by evaluating each sector separately by the income, output and expenditure methods. This not only reduces the confidence-limits of the estimates, but provides a more general picture of the economy than the use of only one formula.

The analysis follows a familiar form in Jamaica, which is almost wholly an exchange economy, but Miss Deane is careful in her presentation of the African estimates to enable the reader to distinguish the subsistence contribution from the rest in every item. In the African colonies she makes two other useful distinctions: evaluating separately the share of income and outlay due to the different racial groups, and summing separately the "taxable income" and the "income of residents." This latter distinction does not, as might be imagined at first sight, denote the national income excluding and including subsistence output. The first includes "income accruing to the foreign owners of capital" (p. 22), the second excludes this item. Miss Deane argues, "If the calculation is intended to throw some light on the standard of living enjoyed in Northern Rhodesia, or on the purchasing power at the disposal of its people, it is inappropriate to include the income of non-residents. If the calculation is intended to reveal the total yield of Northern Rhodesia's physical resources, or the value of the economic activity carried on within this political area, it is essential to include the activities of non-residents." The difference between the aggregates in Northern Rhodesia is considerable: Taxable Income £13 million, Residents' Income £8 million. The student of social accounting must consider again the definition of income to be adopted, a problem which, though fundamental, lies on the fringe of such studies in developed societies, since in practice the various possible definitions lead to roughly similar aggregates. In colonial economies the distinction is vital, and this, together with the many other detailed analyses of this book, provide the first real insight into the economic workings of these plural societies.

P. A.

2.—*Analysis of Wisconsin Income.* By Frank A. Hanna, Joseph A. Pechman and Sidney M. Lerner. New York: National Bureau of Economic Research Inc., 1948. 9¼". xviii + 261 pp. $3.50.

This book consists of three studies of the distribution of income in the State of Wisconsin in the years 1929–1936. The first study is a co-operative attempt by all three authors to estimate the total income received in Wisconsin in 1936, and its distribution among individuals. It is intended to provide a background for, and to supplement, the two later studies, which deal only with incomes subject to income tax. Since the object of making the estimate was limited in this way, its interest will be mainly for American workers attempting to make estimates of income received in individual States in non-census years who may expect to encounter similar difficulties in obtaining data to those surmounted by these workers. The results of this attempt appear to agree well, allowing for differences of concept and definition, with the Department of Commerce estimate of "income payments" in Wisconsin in 1936. It is not clear how far this coincidence represents a genuine check on the accuracy of the two estimates.

The two studies by Hanna and Pechman are of much greater general interest than the co-operative study of total income received. They are based on the unusually detailed and extensive income-tax returns of the State of Wisconsin in the years 1929–1936, and contain far more new material than can be properly summarized in a short review. According to the foreword, the writers have deliberately refrained from using this material "to answer questions," but have preferred to raise new questions and to re-open old, supposedly settled ones. In practice, as might be expected, both writers do attempt to answer some at any rate of the many direct questions with which they introduce their respective contributions.

Pechman, in Part II of the book, discusses the functional sources of incomes of various sizes. His object is to test the widely used generalization that knowledge of changes in the functional distribution of the National Income provides a sufficient basis for statements as to changes in its size distribution. In order to do this, he compares the Lorenz curve representing the distribution of income actually reported with that resulting from hypothetical reductions by arbitrary percentages of each receipt of various functional types. He finds that "there is no inherent relation between changes in (functional) composition and in the relative distribution of income (among persons). In only special cases, when (1) both National Income and the share of wages increase

concurrently or (2) both decrease, is there a more or less direct relation. In case (1) the relative distribution becomes more equal; in case (2) less equal. Even this conclusion is not entirely certain, because it assumes that shifts in the Lorenz curves utilized to describe the relative income distribution are uniform at all points." Pechman has combined wages and salaries and treated them as one "functional type" of income. It is not clear whether this seriously affects his conclusions, but in view of the institutional differences between salaries and wages, an explicit justification of this treatment seems to be required, particularly in a study specifically concerned with the functional sources of income. Certainly, when the words "and salaries" are elided in reference to the compound group, some odd statements appear, as that "wages are 10 per cent. (of incomes) in the group above $100,000."

In Part III Hanna analyses the returns of a sample of 13,000 identical taxpayers who filed returns in seven consecutive years, 1929–1936. His objects are to study the distribution of incomes in consecutive years. In particular, he has studied the effect on the apparent equality of income distribution of lengthening the accounting period beyond the conventionally adopted period of one year. He finds that the effect of this is "to increase the equality of distribution of each income item sufficiently to make it yield a Lorenz curve distinct from the Lorenz curve that represents the average of several accounting periods." Hanna uses correlation methods to measure the extent of re-ranking among individuals from year to year. These methods are not very satisfactory, since the distributions are so skewed that a few very large items are able to exert an appreciable effect on the absolute value of the correlation coefficients. Hanna allows for this tendency by omitting the extreme items.

This point is connected with the general problem of graduating income distributions, a problem which receives very little attention in this book. The writers rely almost exclusively on the Lorenz curve as an indication of the degree of inequality of income distribution. This reviewer finds this a rather clumsy instrument, even when the Lorenz curves of two income distributions being compared do not intersect. Pareto receives a passing mention, but there is no reference to Gibrat.

It remains true that this book is a very important contribution to the literature of the subject, and contains much data which will materially add to our knowledge of the dynamics of income distribution, and increase the realism of some macro-economic models. E. J. M. B.

3.—*Principles of National Income Analysis.* By Carl S. Shoup. Boston, New York, etc.: Houghton Mifflin Company, 1947. xii + 405 pp. $5.00.

At the risk of seeming pedantic, a review of this book must begin with the suggestion that it would have been more accurately entitled "The Practice of National Income Measurement in the U.S.A." Its scope is, in fact, far less general than the title implies. The book consists of two distinct sections. The first and shorter of the two is intended to serve the pedagogical end of explaining the principles of National Income accounting to university students. The treatment of the subject here seems to this reviewer at once excessively detailed and insufficiently complete. Shoup's method is to discuss the accounting relations among the individual members of a hypothetical economic system. His starting-point is the transactions entered into by the individual entities, and not the accounting aggregates of the whole economy. This approach has the disadvantage, as compared with the "Social accounting" approach, that the fundamental concepts are not so readily identifiable with the concepts of the General Theory of Employment. Other shortcomings of Shoup's treatment are the omission of explicit treatment of the Balance of Payments in relation to the national accounts, and the allocation of a mere six and a half pages to the "Government Sector." These shortcomings may, however, be less important in an American text-book than they would be in an English one. This reviewer is doubtful whether the two paragraphs headed "The business firm items in terms of accounting," which include the sentence, "Investment is not evidenced by those credits to cash or vouchers payable that are accompanied by charges to accounts like vouchers payable, labor, selling and distribution cost, or financial cost, or to asset accounts that represent financial claims or claims to future performance, for example, loans receivable or prepaid insurance," will be of much use to students.

The remainder of the book consists mainly of useful descriptions of the U.S. National Income estimates. There are chapters entitled "National Income as Factor Payments," "National Income as Net Product," "The Government Section," and "Saving and Investment," as well as shorter ones on the industrial and regional break-downs of the National Income in the U.S.A. There is also a chapter on the National Income of the U.K. The treatment is generally descriptive rather than analytical. The tables bringing together the estimates made by Kuznets and the Department of Commerce of the various aggregates are useful. No reference to the Balance of Payments appears in the Index. There is a massive bibliography.

It is not possible to commend this book wholeheartedly as what it claims to be—a treatment of the Principles of the subject. As such, it must inevitably challenge comparison with Stone's *Appendix to the U.N. Report on the Measurement of National Income*, and the comparison is unfavourable to the work before us, which is inferior in generality, lucidity and information per unit price. E. J. M. B.

4.—*Studies in War Economics*. Prepared at the Oxford University Institute of Statistics. Oxford: Basil Blackwell, 1947. vii + 410. 8½". 25*s*.

Throughout the last war the research staff of the Oxford Institute of Statistics performed the highly valuable function of analyzing and interpreting the economic problems which were directly or indirectly connected with the war. Under the stimulus of Professor Bowley's immense statistical experience, and Mr. Kalecki's and others' economic thinking, the Institute developed a successful technique of applying modern economic theory to contemporary problems. A selection of these articles, most of them printed in the Bulletin of the Institute, has been published in the above volume. Some of the articles were brought up to date, but most of them were not revised, neither was the statistical material supplemented by the wealth of data published after the end of the war.

The book is not a systematic treatise on war economics, nor is it an analysis of the economic lessons of the war by economists who saw the system work from the inside. All the essays are by authors who were observers from the outside of the Government machine, and who discussed the various problems of war economics as and when they arose. This gave to the articles a certain freshness which is preserved in this volume.

The wide range of problems covered can be seen from the main headings of groups of articles: I. Economic mobilization and general controls. II. War finance. III. Consumer's rationing and price control. IV. Wages and national income. V. Consumption and prices. VI. Industrial organization. VII. War contracts and efficiency. All but three of the fifty-odd articles relate to our own experience. As may be expected, not all of them are of equal value or merit. The most valuable are the essays dealing with war finance, rationing and prices. Those by Mr. Kalecki on "What is inflation?", "General rationing," "Rationing and Price control" and "The Problem of Profit Margins" are well worth re-reading, especially as many of the problems which we have had to face during the war are still with us. For the statistician, the paper by Mr. Daniel and two papers by Miss Schulz on war-time family budgets and on the minimum cost of a satisfactory diet, all three based on original collection of data, will be of interest, as well as the methodological article by Mr. Nicholson on index number problems raised by rationing.

The weakest part of the book is that relating to industrial organization and industrial policy, both in the civilian, and even more in the defence sector. One or two problems, e.g. concentration and manpower mobilization, are adequately discussed (Mr. Burchardt's article on "Output and Employment policy" is still very topical reading), but such important subjects as the principles and practice of price control, the economics of utility production, the way in which individual industries were controlled, etc., still await their economic historian. This weakness in the book is, in a way, natural, because very little worth-while information was released during the war on these subjects, and, for lack of facts, it was difficult for an outsider to discuss such problems adequately. Altogether this is a valuable work, in many parts still topical, and so far the only study discussing the economic aspects of our war experience. L. R.

5.—*Towards a Dynamic Economics. Some Recent Developments of Economic Theory and their Application to Policy.* By R. F. Harrod. London: Macmillan. 1948. x+170 pp. 7*s*. 6*d*.

Though, perhaps, the over-enthusiastic reader should be warned that this new work is by no means easy to read, its main pattern is clear. It contains a wealth of original thoughts. What makes Mr. Harrod's book so scholarly as compared with the avalanche of recent writing on "applications" of theory to policy is the elegant way in which the level of abstraction is preserved right through, and the clear cut between matters of theory and problems of application. The book consists of five eminently successful lectures delivered at the invitation of the University of London in 1947, in the midst of the greatest fuel crisis in the history of England. They deal with long-term problems and the interplay of the three dynamic variables, viz., population, productivity per head and the quantity of capital. The first two are taken as independent dynamic variables.

Having to deal with definitions, polemical matters are occasionally introduced. Of these, the two most interesting digressions are on the comparison with Professor Hicks's neutral inventions (pp. 24–27) and on Professor Hayek's credit policy and technical progress, being a refrain of an old controversy which used to enthuse economists in the early thirties. For Mr. Harrod the field of static economics is confined to a state of rest, or to an active but unchanging process (p. 4).

In dynamics the fundamental conditions will themselves be changing, and the unknowns of the equations to be solved are not the rates, but the increases or decreases in the rates of output per annum. Thus, for instance, the acceleration principle is essentially dynamic, since it makes the demand for new capital a function of the rate of increase in the economy. The trade cycle is a border-line case. It is Mr. Harrod's aim to build a dynamic theory which would lend itself to econometric verification. He is interested in how capital growth (and hence the demand for savings) should behave to be consistent with the growth of population and changes in technical progress—he is thus searching for a "warranted rate of growth" of income (G_w), absorbing saving at full employment level. The question of risk-bearing is for the time being ignored and the rate of interest assumed constant.

The central interest of the book is in the demand for savings in a growing economy. First, we have the case of population growth with constant technical knowledge. With constant interest rate, Mr. Harrod declares that the requirement for capital will grow at the same rate as population. The appropriate fraction of total income which would have to be saved will be constant. This fraction will depend on the ratio of all capital *in use* (my italics) to the value of income during a period. This ratio Mr. Harrod calls "capital coefficient." Two difficulties may, perhaps, be mentioned in this connection: one is the relevance of constant interest rate (unless by interest rate we are to understand productivity), and the second is the somewhat imprecise formulation of the capital "in use." Is it the employed capacity, or all capital installed? In the case of prime factors, the factor income relation is, of course, unambiguous. The results obtained by Mr. Harrod are that with constant technique and constant rate of interest, the capital/income (or capital coefficient) will remain constant. Thus the requisite fraction of income which should be saved to feed the population expansion can be postulated as equal to the per cent. rise in population times the "capital coefficient." Secondly, we have the parallel case of steady population with advancing technique and constant rate of interest. Advancing technical progress can be of various types. Neutral progress of technique is defined in such a way that it leaves the "capital coefficient" constant. Productivity of labour in all stages of production is changed in equal measure on the assumption that the rate of interest does not change. In such circumstances the distribution of the (now enlarged) national product between labour and capital remains unchanged. With neutral technical progress, new capital requirement will rise in the same proportion as the advance of technical progress. The new capital required will be a constant fraction of income, and thus no fresh saving would be called for. Not all technical progressess are nuetral—some may require a good deal of saving. Finally, we have changing population and changing technique. The result of the two changes is that in the most general case we must save a fraction of income, $a + b + ab$. Where a is the fraction of income where technology is constant and population increases at a given rate x, and b is a fraction where population is constant and the progress is causing income to rise at a rate y.

The next problem is the supply of saving. The discussion here is fresh and stimulating. All saving is made up of three groups: (*a*) "hump" saving (for old age spending), (*b*) saving for posterity, and (*c*) corporate saving. In a stationary population with constant technology "hump" saving will be zero: saving of the under-retirement age will be equal to dissaving of pensioners: corporate savings will also be zero. There may be some saving for posterity, thus driving down the rate of interest and calling forth a "deepening" of capital. Growing population will cause "Hump" savings to rise, corporate savings will respond positively, but there may not be enough national income generated to allow for more saving for posterity. Technical progress will have the same effect on corporation saving as population increases; little can be said about saving for posterity; "hump" savings will rise more rapidly than income. "Hump" saving would, probably, rise at an equal rate with increase if time-preference $\left(\frac{1}{T}\right)$ and elasticity of income utility (e) were likely to remain the same with growing output per head. But are T and e likely to be constant? We know little about probable movement of (e), but time-preference will fall (T will rise), thus inducing more abstention now.

The hard core of the problem which is to be solved in dynamic economics, in which attention is focused on the rate of saving, is the fact that the forces which determine saving may easily be such that the rate of saving may not be what is warranted to sustain a steady advance of production with the rate of interest constant. One may be constantly compelled to keep on adjusting the rate of interest. The obvious danger is also that constant populations with advancing progress may suffer from oversaving.

The balance of the theoretical section is confined to the relationship between the actual rate of growth, the warranted rate of growth of investment and saving as compared with their so-called natural rates at the desired level of employment or capacity working. The tendency of the system to relapse into depression before it achieved the state of full employment receives attention. Is

saving keeping in step with capital requirements, or is it too big? The discussion goes along Keynesian lines. Dynamic equilibrium of a steady advance in a community is expressed by the ex-ante equation $G_w C_r = s$, where G_w is the warranted rate of growth of total production, C_r the required capital coefficient, which is equal to requirement for new capital divided by the increment of output to which this capital was instrumental, s the saving expressed as a fraction of income.

Some interesting pages devoted to the foreign balance of trade and the contra-cyclical policy arising out of the fear of possible chronic insufficiency of demand are written freshly, though possibly somewhat heterogeneously. Finally, the volume closes with a discourse on the future of interest, and the limitations of the low rates of interest as a weapon for bringing into balance the Fundamental Equation. When we have reached the end of Mr. Harrod's book, we close it with a feeling that we have been intellectually enriched and well rewarded for having read it.

A. M. DE N.

STATISTICAL NOTES

British Official Statistics

The Ministry of Agriculture has recently issued a volume entitled *Agricultural Statistics, 1940–44, England and Wales*, Part II (price 2*s.*), recording in tabular form *the prices and index numbers of agricultural produce* over the above period. These figures are now mainly of historical interest, but their publication maintains continuity with the pre-war series of agricultural statistics and the aim is gradually to bring them up to date. It should be noted that this publication relates to England and Wales only, and is distinct from a similar and to some extent supplementary series relating to the United Kingdom.

The normal difficulties met with in arriving at reliable price quotations are much aggravated in war time, when commodity prices may not only vary according to grade, season, area and stage of marketing, but may be associated with subsidy or acreage payments, or if controlled may be fixed, minimum, or maximum prices. The changes due to these various causes are as far as possible explained, and tables given of the prices prescribed by the Ministry of Food or other Departments for commodities under control, as well as prices collected by Market Reporters or obtained from other sources. Though much of the information, especially that relating to the index numbers, is of value, the volume as a whole seems to be overloaded with detail likely to be of interest to an extremely limited number of persons.

The annual *index number of agricultural prices* calculated by the Ministry of Agriculture for the calendar year 1948 shows a further increase over previous years, the general index having risen to 220, as against 213 in 1947 and 183 in 1946. This is the highest figure recorded since these index numbers were started more than 40 years ago, and is to be compared with a pre-war (1937–39) figure of 90½. The rise in 1948 is largely attributable to adjustments necessary to cover higher wages for agricultural workers and larger profits for farmers, consequent on a need to stimulate production.

The changes in the past five years are shown below for the three groups of commodities which make up the index number. Allowance is made for Government subsidies, including payments in respect of acreage under wheat and potatoes based in the case of wheat on estimated quantities entering into sale, and in the case of potatoes on estimated total production. The acreage payments were equivalent in the case of wheat to 2*s.* 11*d.* per cwt. in 1947 and 2*s.* 3*d.* in 1948; for potatoes they were respectively 31*s.* 5*d.* and 32*s.* 1*d.* per ton.

(Base 1927–29 = 100)

	1944	1945	1946	1947	1948
Cereals and farm crops	177·5	183	183	197	222
Live stock and live stock products	158·5	164	178	199·5	215·5
Fruit, vegetables and glasshouse produce	204	193·5	201	288	231·5
General index	169	173	183	213	220

Increases were general in the commodities making up both the cereal and the live stock groups, but fruit and vegetables averaged lower figures than in 1947. Among important items, the index for wheat rose from 191 in 1947 to 231 in 1948; potatoes from 181 to 202; and milk from 222 to 235.

The interim index of retail prices compiled by the Ministry of Labour and National Service remained unchanged from November, 1948, to April, 1949, but rose by two points in May. The food figure rose slightly in February, but fell again temporarily in March. In May it rose by six points as a result of increases in the price of meat announced in the Budget speech. The clothing

figure continued to show a slight upward tendency. The end of the winter charge for electricity was reflected in the fuel and light figure for March. The figure for miscellaneous foods rose in April, mainly due to an increase in the price of matches. The drink and tobacco figure fell in May consequent on the Budget reduction in the price of beer. The figures for other groups of items showed little change in the period. The detailed figures for January to May were as follows:

(Prices at June 17th, 1947 = 100)

Date	*Food*	*Rent and Rates*	*Cloth-ing*	*Fuel and Light*	*House-hold Durable Goods*	*Miscel-laneous Goods*	*Services*	*Drink and Tobacco*	*Total*
Weights . .	348	88	97	65	71	35	79	217	1,000
Jan. 18th, 1949 .	108·2	99·6	116·1	113·8	108·8	109·3	105·1	110·8	109
Feb. 15th, ,, .	108·6	99·6	117·0	113·8	108·9	109·3	105·1	110·8	109
Mar. 15th, ,, .	108·0	99·6	117·4	111·8	108·9	109·3	105·2	110·8	109
April 12th, ,, .	108·5	100·1	117·8	111·5	109·0	112·6	105·2	107·6	109
May 15th, ,, .	114·3	100·1	118·1	111·5	108·4	113·1	105·2	107·5	111

In publishing the figures the Ministry of Labour states that they are in the form in which they are used in the procedure adopted for calculating the index for all the groups combined, i.e. to the nearest first place of decimals. The decimals are shown only in order that, if desired, calculations can be made of the effect of combining particular groups and excluding others. The information available as to price changes, however, is such that no precise significance can be attributed to the decimals, and for any other purposes, therefore, the figures should be used to the nearest whole number.

The Ministry of Labour index of weekly wage rates, which rose to 108 in January, remained at that figure till May. The following is a summary of the figures since June, 1947, when the present series was instituted:

(Wage Rates at end of June, 1947 = 100)

Date (end of month)	*Men*	*Women*	*Juveniles*	*All Workers*
June, 1947	100	100	100	100
Sept., ,,	101	101	102	101
Dec., ,,	103	103	106	103
Mar., 1948	105	107	107	105
June, ,,	105	107	108	106
Sept., ,,	106	108	109	106
Dec., ,,	107	109	110	107
Jan., 1949	107	109	110	108
Feb., ,,	107	109	110	108
Mar., ,,	108	110	111	108
April, ,,	108	110	111	108
May, ,,	108	110	111	108

Early in 1949 a new series of manpower estimates became available, based on the administration of the national insurance schemes. There was a consequent widening of the scope of the figures, which now include private domestic servants and persons over insurable age under the old scheme. The new schemes also provide material for more reliable estimates of the numbers of persons who were formerly outside the scope of the unemployment insurance scheme, mainly non-manual workers with a rate of remuneration exceeding £420 a year. Also, part-time women workers are now counted as units, in place of the former estimates of such workers counted as half-units.

The total working population and the numbers in civil employment shown by the new series of figures are as follows:

Thousands

	Total Working Population			*Numbers in Civil Employment*		
	Males	*Females*	*Total*	*Males*	*Females*	*Total*
Mid-1948	16,057	7,089	23,146	14,945	6,981	21,926
End ,,	16,069	7,116	23,185	15,019	6,992	22,011
Jan., 1949	16,093	7,121	23,214	15,020	6,989	22,009
Feb., ,,	16,079	7,115	23,194	15,014	6,986	22,000
Mar., ,,	16,053	7,111	23,164	15,019	6,989	22,008
April, ,,	16,056	7,151	23,207	15,059	7,038	22,097

It will be seen that between mid-1948 and January, 1949, the total working population rose by 68,000, and fell by 20,000 in February. In April it was 13,000 higher. The number in civil employment was 74,000 higher in February than at mid-1948, and it rose again by 97,000 by April.

The level of unemployment, which had risen by 48,500 in November, fell by 15,430 in February and by 79,877 in March. In April and May it fell by a further 36,241.

Number of Unemployed Persons on the Registers of the Employment Exchanges of the Ministry of Labour and National Service

Date	*Men and Boys*	*Women and Girls*	*Total*
Dec. 6th, 1948	244,015	83,198	327,213
Jan. 10th, 1949	279,524	96,189	375,713
Feb. 14th, ,,	264,318	95,965	360,283
Mar. 14th, ,,	250,733	89,673	340,406
April 11th, ,,	242,389	82,484	324,873
May 9th, ,,	227,943	76,222	304,165

The total for May, 1949, includes 35,708 married women. The totals do not include registered disabled persons who were classified as suitable only for employment under sheltered conditions.

CURRENT NOTES

An Exhibition of Calculating Machines and Equipment Held at the University of Sheffield.—Calculating machines are now an essential part of the equipment of any establishment in Science, Industry or Commerce with much numerical work to do, and there are few research laboratories, works accounts departments, or banks that do not use a mechanical calculator of some kind. The kind of equipment used will depend on the nature of the job, the training of the user, and either the equipment or funds available for its purchase. With the dual aims of educating the general public in the subject, and of placing before the specialist all the alternatives so that he may be able to choose the most suitable equipment for his purpose, an exhibition of calculating machines and equipment was held in the Firth Hall of the University of Sheffield on Friday and Saturday, April 29th and 30th. It was attended by about two thousand people, some mainly scientific in their interests, some mainly commercial, with schools and the general public also well represented. The exhibition included almost all the machines and equipment which can be bought at the present time, together with non-commercial machines built for special purposes. This display was supplemented by three lectures, of which summaries are given below, by a film about punched card equipment, and by organized visits to four local punched card installations, the owners of which kindly extended hospitality to visitors to the exhibition.

Exhibits were classified into groups, according to the basic principles of operation. The first group consisted of analogue machines, and included a wide variety of slide rules, circular and cylindrical logarithmic calculators, nomograms, statistical calculators (designed to evaluate $\sqrt{\Sigma(x \quad \bar{x})^2/n}$), planimeters, a ball and disc integrator, a large electrical heat-flow computer, and exhibits relating to differential analysers. The second group was of digital machines, and included abacus-type calculators, key-driven calculators, adding-listing machines, general purpose hand- and electrically-operated desk calculators, accounting machines, punched card machines, and exhibits relating to large electronic and relay machines. The third group consisted of a wide selection of mathematical tables, including many of historical interest, while the fourth included such varied items as counters, slotted cards, scales, examples of computations and a relay-operated noughts and crosses machine which provided a little light relief and formed a centre of popular attention.

In this report it is possible to mention only a few points which are likely to be of particular interest to statisticians. The amateur statistician, or the industrial worker experimenting with statistics, is often dismayed by the amount of arithmetic required in even the simplest statistical calculations. He finds that an ordinary slide rule gives him little help, firstly because he cannot obtain enough significant figures, and secondly, because much of the arithmetical labour occurs in adding squares and products, for which a slide rule is unsuitable; further, because of its cost, a desk calculator is not available to him. The labours of such a man would be made much lighter if he could expend three or four pounds on a combination of equipment such as the following, all of which were shown:

(1) *Barlow's Tables of Squares, etc.*, edited by L. J. Comrie (Spon.).

(2) A Pocket Adding Machine. This has the same working principles as the abacus, but requires less practice for efficient use.

(3) A Dial Calculator giving four significant figures' accuracy in the results of calculations.

With data coded to two-figure numbers in analysis of variance and regression problems these modestly-priced aids to computation will serve the worker well.

In regression problems where many variables are involved, the sums of squares and products are most conveniently calculated on punched card machines by the method of progressive digiting. More slowly, but still efficiently, the same technique may be applied using Cope-Chat cards in conjunction with a number of adding machines (or even an abacus!), which take the place of the counters of the punched card tabulator. A set of cards to demonstrate this was shown at the exhibition.

The question "What is the best machine?" which is asked so frequently cannot be answered as it stands; in any class there are usually machines made by different firms, each with its own special characteristics, each "best" in certain kinds of calculations. For example, in many commercial uses of adding-listing machines, one of their most important features is that they must necessarily list every number added into the total; to scientists, however, this feature can be a nuisance, and for certain jobs they may prefer a machine in which the listing of any number can be suppressed if desired. Many individual features of this kind, which are not easily found out, are mentioned in the exhibition catalogue, copies of which may be obtained free of charge on application to Mr. G. H. Jowett at the University of Sheffield.

Summaries of Lectures

"Recent Developments in Calculating Machines," by Professor D. R. Hartree.

This lecture was intended mainly to give an introduction to those types of calculating equipment represented in the exhibition only by photographs. It is convenient to divide calculating equipment into two main classes: (1) *instruments* (or analogue machines) in which the numbers on which the calculations are to be performed are represented by physical quantities (lengths, rotations, electrical potentials, for example) of which the numbers are measures; and (2) *machines* (or digital machines) in which the individual digits of a number are represented by the groups of discrete objects (such as teeth of a gear wheel) or of events, and operations are carried out directly with these representations of the digits of the numbers. Examples of instruments are the slide-rule, most forms of planimeters and harmonic analysers; of machines, the ordinary desk calculating machines.

The differential analyser is an instrument for the mechanical integration of differential equations. The quantities in the equation are represented by rotations of shafts which can be connected through units of various kinds such as integrators, adding units, input tables, so that the relations between the rotations of the shafts satisfy the equation to be solved. Another kind of instrument, for the solution of the partial differential equations of non-steady heat flow by an electrical analogue, has been developed and is represented in the exhibition.

The main recent developments in digital machines have been in the direction of large machines capable of carrying out automatically long sequences of arithmetical operation, and so designed that the sequence of operations can readily be changed from that required for one calculation to that for another. In addition to having a unit for carrying out arithmetical operations such a machine must have a store for intermediate results, for tabular data, and for the operating instructions, and a control system to take the place of the human computer in a hand calculation. Storage and control are aspects of such machines at least as important as the means of doing arithmetical operations.

The original conception of a large automatic general-purpose machine is over a hundred years old and is due to Charles Babbage, but it has been realized in practical form only in the last ten years. The first such machine was the I.B.M. Automatic sequence-controlled calculator, installed at the Computation Laboratory at Harvard University. In this machine, arithmetical operations are carried out by mechanical counters driven through electromagnetic clutches controlled by relays. Control is carried out through a punched paper tape; electrical contacts made through the holes actuate relays which cause the operation expressed in coded form on the punched tape to be carried out.

Another machine, the first to make use of electronic circuits, is the E.N.I.A.C. (Electronic Numerical Integrator and Calculator). In this machine, electrical pulses, generated at 100,000 per second, are routed to the different units of the machine by electronic gate circuits and counted by electronic counting circuits. In the E.N.I.A.C. the capacity of the store is inconveniently small for a general-purpose machine. An important advance has been the development of a simple, reliable store with rapid access and not requiring a large amount of electronic equipment. Three forms have been developed for this purpose; one uses delay lines in which information is stored in the form of strains of supersonic acoustic pulses; another uses different magnetic states of magnetic material in the form of a wire, tape or drum; the third uses the distribution of electric charge on an insulating screen.

"Applications of Punched Card Machines in Accounting and Scientific Computing," by Mr. G. B. Hey.

The lecture commenced with a survey of the principal machines which combine to form a complete punched card installation. Mr. Hey mentioned special features of both Powers and Hollerith machines, pointing out the special advantages that each had for particular types of work. He then discussed various accounting and scientific applications of the machines, showing the kinds of work (in the statistical field, summing of products for regression analysis, or analysis or variance) which could conveniently be carried out with the aid of punched cards. He said that punched card methods were in general economical and worth while only if two conditions were satisfied: firstly, each hand-punched card should be required for sorting, and tabulation, etc., more than once, and secondly, there must be a considerable volume of work capable of being performed in large batches. The second condition was rarely satisfied in banking, where transactions were posted almost immediately, and accordingly, punched card machines were not popular in business of that kind. If, however, both conditions were satisfied, punched card methods were in general extremely quick, convenient, and gave entirely satisfactory results.

"*Calculating Machines*[2] *Their Nature and Applications*," *by Dr. E. T. Goodwin.*

The lecture began with a description of the general nature of hand- and electrically-operated desk calculators, showing how all machines had certain fundamental features in common, and how individual machines had special features which made them particularly suitable for certain jobs. Dr. Goodwin went on to describe various examples of computation, and emphasized that arithmetic, to be of any use, should not only be right, but should be proved to be right by the application of independent checks. He drew attention to the sufficiency of the totals check in solving simultaneous equations by elimination, since it not only established that in operating on an equation with a multiplier or divisor the arithmetic had been carried out correctly, but also that the correct multiplier or divisor had been used. In conclusion, he described how differencing a mathematical table could not only detect an error, but locate it and supply an estimate of its magnitude; this was illustrated with examples of differencing, some of which had been carried out mechanically on an accounting machine at the rate of 300 lines an hour.

Dr. B. P. Dudding, Vice-President of the *Association of Incorporated Statisticians Limited* and Chairman of its Council writes:

In recent years suggestions have been made in various quarters that statisticians and their less-qualified assistants might be trained, examined and organized along lines similar to those adopted in other professions. Much work on the educational side has been done by the Royal Statistical Society and its Committees, but the Society admits only its own Fellows to its examination, and this will no doubt restrict the numbers who will be able to qualify as statisticians through the Society.

This is particularly true of the more junior staff in statistical offices, who could not, if they would, pass the Royal Statistical Society's certificate examination. Yet it is being increasingly recognized that the junior staff must be properly trained if statistical offices are to function efficiently.

In an attempt to deal with these various problems a group of interested persons has recently obtained powers, under the Companies Act, 1948, to form a non-profit-making "Association of Incorporated Statisticians Limited" with objects stated as follows:

1. To encourage the provision of facilities for training persons in the principles of statistics and their application to the problems of administration and research which arise in industry and commerce, Government departments and in all fields of applied science.

2. To provide an agreed syllabus of study in statistics by which students may become proficient in the subject, and to hold examinations by means of which the attainment of an adequate standard can be tested.

3. To lay down the principles upon which assurance can be given that students who have attained the requisite standard of proficiency in theory and its application have also had adequate experience in one or more fields of practical work.

4. To promote among statisticians the study of the proper use of statistical techniques in the solution of practical problems as they arise.

5. To further and protect the interests of statisticians who are duly registered as having attained the necessary degree of proficiency.

The Council of the Association, under its Chairman, Dr. B. P. Dudding, has already done much preliminary work, and has approved regulations for the registration and examination of students.

The Association recognizes two classes of members, Fellows and Associates, as well as a special class of Registered Statistical Assistants. Qualification as a member requires evidence of adequate training to fulfil properly the duties of a statistician, and in addition, for Fellowship, evidence of ability to fill responsible positions. Qualification as a Registered Statistical Assistant is open to anyone over 18 years of age employed in the compilation of statistics who shows at examination the prescribed standard of proficiency. The sponsors of the new organization place considerable emphasis on the importance of this group, as the efficiency of a statistical office and of its fully-trained staff can be considerably enhanced by assistants properly trained who may otherwise lack the characteristics of a responsible statistician.

Fellows of the Royal Statistical Society will be interested to know that Lord Beveridge, a past President of the Royal Statistical Society, has honoured the new Association by becoming its first President, while members of its Council include Professor R. G. D. Allen, Mr. D. J. Desmond, Dr. C. O. George, Professor Lancelot Hogben and other Fellows of the Society. The Honorary Secretary is Mr. S. Harris, and its offices are at 54, New Broad Street, London, E.C.2.

Dr. H. C. Hamaker, of the Philips Research Laboratories at Eindhoven, has described in the *Proceedings of the Academy of Sciences* of the Netherlands (52, No. 2, 1949) a simple and useful technique for producing random sampling numbers. He uses a ten-sided die made of a light metal. Rolling it down an inclined plane gave biased results, but spinning it in the air from the flat hand and catching it in its downward fall proved quite satisfactory. On catching the die one of the ten side-faces is selected by the thumb and the corresponding digit read off. This procedure excludes any personal bias, such as might be introduced if the top face with the digits on it were visible when the choice of number was made. With practice 1,000 throws can be made and recorded by one person in 40 minutes. A full analysis of 40,000 such throws shows the technique to be reliable. It is clear that such a die provides a convenient and rapid way of generating a random sequence. It could be of value not only in the field but in the instruction of students.

The National Institute of Economic and Social Research announces that the Sir Richard Garton Memorial Prize, of the value of one hundred guineas, is offered for competition for the second time. The closing date for entries is December 31st, 1949.

The purpose of the prize is to stimulate quantitative and empirical studies in applied economics. It is offered for the best original investigation into one of the following topics: (*a*) The factors determining productivity in a particular industry; (*b*) a contribution to the study of the incomes or expenditure of the middle income groups; (*c*) a quantitative study of habits and channels of saving; (*d*) a realistic study of price formation in a particular field; (*e*) regional variations in wages: their causes and their influences on industrial location.

The competition is open to all persons resident or normally resident in the United Kingdom.

Further particulars and the form of entry which must accompany each essay sent in may be obtained from The Secretary, National Institute of Economic and Social Research, 2, Dean Trench Street, London, S.W.1.

The following Fellows of the Society were mentioned in the Birthday Honours List of June, 1949:

C.B.—Mr. H. Campion, C.B.E.; Mr. H. Symon.

C.B.E.—Mr. R. B. Ainsworth, Mr. F. A. A. Menzler, Mr. E. G. D. Northcroft, Mr. R. Shone.

O.B.E.—Mr. H. P. Haddow.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Agricultural Economics Society, Journal of Proceedings—

April 1949—The future of agricultural marketing: *E. H. Whetham.* The economics of grassland management: *J. R. Currie.* The economics of grass conservation: *R. N. Dixey.* The international background: *Sir Henry Clay.*

The Banker—

February 1949—Monetary systems of the Colonies: VIII—Trends and problems: *A Special Correspondent.* Eire's balance of payments problem: *G. A. Duncan.*
March 1949—The future of the Stock Exchanges.
April 1949—Argentina's post-war experiments, I—Control of foreign trade: *N. A. D. Macrae.* Balance of payments of the Colonies: *A. R. Conan.*
May 1949—Building societies and State-financed housing: *H. Wincott.* The future of Lancashire cotton: its role in the national economy: *W. Gwyn Pilkington.* What kind of industry?: *J. Midgley.* Machines and man-power in spinning: *C. Henniker-Heaton.* Bulk buying or Liverpool market?: *T. W. Kent.* Argentina's post-war experiments, II—Control of banking and finance: *N. A. D. Macrae.*

Economica—

February 1949—Prospects of labour: *E. H. Phelps Brown.* The valuation of the social income: *I. M. D. Little.* Economics of the Wheat Agreement: *H. Tyszynski.* The propagation of Ricardian economics in England: *S. G. Checkland.* The distribution of earned and investment incomes in the United Kingdom: *E. C. Rhodes.*

Economic Journal—

March 1949—Professor Meade on planning: *R. F. Kahn.* National income, national expenditure and the balance of payments: *J. E. Meade.* The proportional personal-income tax as an instrument of income creation: *J. C. Hubbard.*

Eugenics Review—

April 1949—The mixing of races and social decay: *The Rt. Rev. E. W. Barnes (Bishop of Birmingham).* The Galton Laboratory: its work and aims: *L. S. Penrose.*

Geographical Journal—

April 1949—The present climatic fluctuation: *H. W. Ahlmann.*

Heredity—

April 1949—The manifold effect of selection: *K. Mather* and *B. J. Harrison.* The distribution of polygenic activity on the X-chromosome of *Drosophila melanogaster: L. G. Wigan.* The quantitative study of populations in the Lepidoptera, II. *Maniola jurtina L.*: *W. H. Dowdeswell, R. A. Fisher* and *E. B. Ford.* Genetical studies in pears. V. Vegetative and fruit characters: *M. B. Crane* and *D. Lewis.*

Journal of Documentation—

December 1948—Bradford's Law of Scattering: *B. C. Vickery*

Manchester School of Economic and Social Studies—

January 1949—Economic change in the north-west: *F. V. Meyer.* The restriction of foreign trade: *A. Henderson.* Some results of the Distribution of Industry Act, 1945: *J. Sykes.* Retail sales per employee: *J. A. Hough.* Internal migration in Victorian England: *A. K. Cairncross.* A shipyard from within: *R. S. Stokes.* Statistics relating to the north-west.

Oxford University Institute of Statistics, Bulletin—

February and March 1949—Juvenile labour supply: *R. Godson.* Britain, O.E.E.C. and the restoration of a world economy: *T. Balogh.* Sweden's economy, 1946–1949: *I. Ohlsson* and *G. Cedarwall.* Notes on estimating national income components: *P. D. Henderson, D. Seers* and *P. F. D. Wallis.*
April 1949—A note on State regulation of farm incomes in New Zealand: *W. Rosenberg.* British industrial recovery: *D. Seers.* The distribution of industry: *T. Balogh.*

Population Studies—

March 1949—European migration potential and prospects: *J. Isaac*. Age at marriage and marital fertility: *C. Clark*. The Canadian sample for labour force and other population data: *N. Keyfitz* and *H. L. Robinson*. Using the *Pao* as the primary sampling unit. Some notes and reflections on the possibilities of a census of China by sampling: *Chih-Ang Chiang*. Summary indices of the age distribution of a population: *H. Hyrenius*. A note on standardized rates: *B. D. Karpinos* and *J. B. Chassan*.

Royal Agricultural Society of England, Journal—

Vol. 109—Crops and plant breeding: *H. W. Howard*. Diseases of animals: *T. Dalling*. Farm economics: *C. S. Orwin*. Dairy farming and dairy work: *J. Mackintosh*. The feeding of livestock: *W. Godden*. Farm implements and machinery: *S. J. Wright*. Soils and fertilizers: *E. M. Crowther*.

INDIA—

Calcutta Statistical Association, Bulletin—

February 1949—Recent trend in prices and wages in India: *S. Bhattacharyya*. Topographic variation in statistical fields: *B. Ghosh*. Estimation of consumption requirements: *K. C. Cheriyan*. On variance factors in weighing designs: *K. S. Banerjee*.

UNITED STATES—

American Academy of Political and Social Science, Annals—

March 1949—Re-appraising our immigration policy (Whole number).

American Statistical Association, Journal—

March 1949—On a unique feature of statistics: *G. W. Snedecor*. An attempt to get the "not at homes" into the sample without callbacks: *A. Politz* and *W. Simmons*. Application of least squares regression to relationships containing auto-correlated error terms: *D. Cochrane* and *G. H. Orcutt*. AOQL single sampling plans from a single chart and table: *D. J. Greb* and *J. N. Berrettoni*. On measuring languages: *S. C. Dodd*. Confidence limits in the non-parametric case: *G. E. Noether*. On a method of estimating birth and death rates and the extent of registration: *C. C. Sekar* and *W. Edwards Deming*. Evaluation of parameters in the Gompertz and Makeham equations: *J. F. Brennan*. On the "information" lost by using a t-test when the population variance is known: *J. E. Walsh*. Wesley Clair Mitchell, 1874–1948, an appreciation: *S. Kuznets*.

Annals of Mathematical Statistics—

March 1949—On the variation of estimates: *G. R. Seth*. On the theory of some non-parametric hypotheses: *E. L. Lehmann* and *C. Stein*. Estimation of the parameters of a single equation in a complete system of stochastic equations: *T. W. Anderson* and *H. Rubin*. Some significance tests for the median which are valid under very general conditions: *J. E. Walsh*. A direct method for producing random digits in any number system: *H. Burke Horton* and *R. Tynes Smith*. On a matching problem arising in genetics: *H. Levene*. A multiple decision procedure for certain problems in the analysis of variance: *E. Paulson*. A modified extreme value problem: *B. Epstein*. On distinct hypotheses: *A. Berger* and *A. Wald*. On approximation to the sampling variance of an estimated maximum value of given frequency based on fit of doubly exponential distribution of maximum values: *B. F. Kimball*. Tests of independence in contingency tables as unconditional tests: *A. M. Mood*. The 5 per cent. significance levels for sums of squares of rank differences and a correction: *E. G. Olds*. Independence of non-negative quadratic forms in normally correlated variables: *B. Matern*. A formula for the partial sums of some hypergeometric series: *H. von Schelling*. The variance of the proportions of samples falling within a fixed interval for a normal population: *G. A. Baker*. The point beserial coefficient of correlation: *J. Lev*. A note on Kac's derivation of the distribution of the mean deviation: *H. J. Godwin*. Correction to "Asymptotic formulas for significance levels of certain distributions": *A. M. Peiser*.

Estadística—

September 1948—La enseñanza de la estadística en Canadá y países Latinoamericanos (Extractos): *R. Rodrigues*. Some problems of the 1950 Census of the Americas: *C. L. Dedrick*. Planificación para la tabulación del Censo de Población de 1950: *L. E. Truesdell*. Sexta revision decenal de las listas internacionales de enfermedades y causas de muerte: *D. Curiel*. Co-ordinación de la demandas nacionales e internacionales de estadísticas en los Estadados Unidos: *D. C. Riley*. Canada's national vital statistics indexes: *J. T. Marshall*. Primera Sesión de la Junta de COTA: Resumen y apéndice.

Harvard Business Review—

March 1949—ERP in operation: *L. Gordon*. The question of steel capacity: *M. Barloon*.

May 1949—Reading habits of business executives: *E. C. Bursk* and *D. T. Clark*.

Journal of Experimental Education—

December 1948—An experimental study to measure the contribution of motion pictures and slide-films to learning certain units in the course introduction to nursing arts: *L. E. Heidgerken*. A summary of mental health survey of Spartanburg County, Spartanburg, South Carolina: *E. C. Hunter*. The stability of mental test performance between two and eighteen years: *M. P. Honzik*, *J. W. Macfarlane* and *L. Allen*.

Journal of Political Economy—

February 1949—Tariffs, the terms of trade, and the distribution of national income: *L. A. Metzler*. Economics of scheduling for industrial mobilization: *S. M. Robbins* and *T. E. Murphy*. The welfare aspects of excise taxes: *E. R. Rolph* and *G. F. Break*. Rostow's proposals for petroleum policy: *J. S. Bain*. The cash ratio in English banks before 1800: *J. K. Horsefield*. Significance of Bank capital ratios: *H. J. Mellon Cooke*.

April 1949—A survey of contemporary economics: *G. J. Stigler*. Have we underestimated increases in rents and shelter expenditures? *S. J. Maisel*. The future locational pattern of iron and steel production in the United States: *W. Isard* and *W. M. Capron*.

Milbank Memorial Fund Quarterly—

April 1949—Old-age problems in the family: *J. H. Sheldon*. The importance of the family in the prevention of mental illness: *K. A. Zimmerman*. Development of international statistics: *W. F. Willcox*. A case study of the international collection of demographic statistics: *F. E. Linder*. World health statistics: *K. Stowman*. Social and psychological factors affecting fertility, IX. Fertility planning and fertility rates by socio-economic status: *C. V. Kiser* and *P. K. Whelpton*.

Quarterly Journal of Economics—

February 1949—Britain's economic problem: *T. Balogh*. The secular trend in monetary velocity: *C. Warburton*. The internal organization of the firm and price formation: an illustrative case: *R. M. Alt*. The mathematical foundations of economic theory: *R. G. D. Allen*.

The Record (American Institute of Actuaries)—

October 1948—Rate functions and their role in actuarial mathematics: *C. J. Nesbitt* and *M. L. van Eenam*. Punched-card "Successive multiplication" as applied to calculation of reserves and other operations: *J. S. Hill*. Mortality tables analyzed by cause of death: *T. N. E. Greville*.

Review of Economics and Statistics—

February 1949—Comments on the G.M.-U.A.W. wage control of 1948: *A. M. Ross*. How to manage the National Debt: Articles by *S. E. Harris*, *L. H. Selzer*, *C. C. Abbott*, *R. A. Musgrave* and *A. H. Hansen*. Some experiments in demand analysis: *A. R. Prest*. Devaluation and the trade balance: a note: *A. O. Hirschman*. The planning of investments in the Soviet Union: *H. Hunter*. Personal income tax reduction in a hypothetical contraction: *M. I. White*. The long-wave depression, 1873–97: *R. Fels*.

BELGIUM—

Bulletin de l'Institut de Recherches Économiques et Sociales—

March 1949—La conjoncture économique de la Belgique: *L. H. Dupriez.*
April 1949—Sociologie et morale: *J. Leclercq.* Les rapports entre la sociologie et la science économique: *J-E. Mertens.* Pratique religieuse et milieux: *C. Leplae.* Un sondage sur la culture des étudiants: *L. de Groote.*

CZECHOSLOVAKIA—

Statistický Obzor—

Vol. XXIX, Part 1—Oblastní plánování a technická práce: *J. Okrouhlý.* Sezónní druhy zboží v indexu životnich nakladu: *J. Mach.* (Czech text, English summaries.)

DENMARK—

Nationaløkonomisk Tidsskrift—

Vol. 86, Parts 5 6—Nágra synspunkter pa inflationsproblemet: *E. Lindahl.* Marshallplanen og dens betydning for Dansk økonomi: *S. Nielsen.* Matematik og økonomi: *B. Fog.* Nationalindkomst, nationalregnskab og nationalbudget: *K. Bjerke.*

FRANCE—

Journal de la Société de Statistique de Paris—

March-April 1949—Le statistique des images: *L. Amy.* Essai sur les bases de la périodisation de l'histoire: *E. Cavaignac.* Normes de la statistique, du calcul des probabilités et des erreurs de mesures: *A. Pallez.* Interprétation statistique des épreuves de prélèvement effectuées dans l'industrie: *M. Dumas.*

Population—

January-March 1949—Faits et problèmes du jour: *R. Peltier.* Des lois dans leurs rapports avec la population: *J. Doublet.* Progrès technique et répartition professionelle de la population: *A. Sauvy.* La prévision des naissances d'après les déclarations de grossesse: *P. Vincent.* Enquête démographique en Océanie française: *C. Valenziani.* La situation démographique de la Grande-Bretagne. Comparaison avec celle de la France (2e partie): *J. Bourgeois-Pichat.*

Revue d'Économie Politique—

January–February 1949—A propos de la co-ordination des transports. Répartition d'un objectif-production entre deux activités: *A. Sauvy.* Structure de la fortune privée en France: *F. Trevoux.* L'or paramonétaire: *F. Herbette.*

HOLLAND—

Statistica—

Vol. 3, No. 1—Statistische ervaringen in 1948: *Ir J. van Ettinger.* Statistische research in de verzekeringswetenschappen: *J. J. M. van Tulder.* De ruis in radiobuizen als statistisch verschijnsel: *H. C. Hamaker.* Ein statistische Verfahren zur qualitativen Bestimmung der Immunität mittels Hautreaktionen: *C. A. G. Nass.* (English summaries).

ITALY—

Giornale degli Economisti e Annali di Economia—

January–February 1949—(Nell'anniversario della nascità di Vilfredo Pareto)—Gli studi di Vilfredo Pareto sulle funzione di domanda e offerta: *G. Sensini.* Nuovi problemi dalla polemica Scorza-Pareto: *E. Zaccagnini.* Una generalizzazione delle equazioni di Pareto-Slutsky (I): *A. Bordin.* L'analisi Pareto-Slutsky della domanda e la teoria delle imposte sui consumi: *E. D'Albergo.* Ricerche sulla curva dei redditi: *R. D'Addario.* Sulla curva dei redditi: *F. Giaccardi.* Alla scuola di Vilfredo Pareto e Maffeo Pantaleoni: *B. Griziotti.*

Statistica—

October–December 1948—Il problema delle file: *L. Galvani.* Caratteristiche sociali della studentesca padovana: *A. de Polzer.* Nuove ricerche sugli indici di cograduazione fra serie con termini uguali: *B. Grazia-Resi.* Prezzi e dazi doganali in Francia nel primo decennio dopo la prima guerra mondiale: *A. Giannone.* La popolazione studentesca universitaria di Palermo dalle origini ai nostri giorni: *A. di Pasquale.* L'organizzazione statistica di una cassa mutua per l'assistenza di malattia: *D. Miani-Calabrese.* Su alcune proprietà dei momenti impiegati nello studio della variabilità, asimmetria e curtosi: *L. Faleschini.*

SWEDEN—

Acta Agriculturae Suecana—

Vol. II, Part 2—Studies in the variations of the calory content of milk: *G. Bonnier, A. Hansson* and *F. Jarl.* Studies on monozygous cattle twins, VII. On the genetical determination of the interdependency between the percentages of fat, protein and lactose in the milk: *G. Bonnier* and *A. Hansson.*

Vol. II, Part 3—Causes of variation in the size and weight of litters from sows: *N. Korkman.* Studies on monozygous cattle twins, VIII: Amount and composition of the milk as affected by frequency of milkings: *A. Hansson* and *G. Bonnier.*

Vol. III, Part 1—The interplay of heredity and environment on growth and yield: *G. Bonnier, A. Hansson* and *H. Skjervold.* The effect of feeding excessive amounts of calcium and phosphorus to milking cows: *A. Hansson.* The effect of premature parturition on udder development and milk secretion: *A. Hansson.*

Ekonomisk Tidskrift—

March 1949—Strukturgränser i svensk industrialism: *J. Akerman.* Nya vindar i den sovjetryska statistiken: *A. P. Aizsilnieks.*

SWITZERLAND—

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker—

April 1949—Die elemente der kollektiven Risikotheorie von festen und zufallsartig schwankenden Grundwahrscheinlichkeiten: *H. Ammeter.*

Schweitzerische Zeitschrift für Volkswirtschaft und Statistik—

February 1949—Der freie Finanzdollarmarkt—eine währungspolitische Notwendigkeit: *A. Bosshardt.* Some aspects of the dollar problem in Switzerland: *P. Jacobsson.* Der Stand der Dollarfrage: *H. Sieber.* Bemerkungen zur Dollarfrage: *V. F. Wagner.* Theorie der Glücksspiele und ökonomisches Verhalten: *O. Anderson.*

INTERNATIONAL—

International Labour Review—

February 1949—Housing and the economic crisis in Great Britain: *M. Bowley.* Job classification in the Netherlands: *A. N. van Mill.*

Revue de l'Institut International de Statistique—

Vol. 16, Nos. 1/4—Les valeurs typiques d'ordre nul ou infini d'un nombre aléatoire: *M. Fréchet.* The analysis of market demand; an outline of methods and results: *R. Stone.* Sur la mesure du côut de la vie et du pouvoir d'achat: *A. Sauvy.*

REVENUE OF THE UNITED KINGDOM

Net Produce in Quarters of 1948, *and in Financial Years ended March* 31, 1948–49, 1947–48, 1946–47, 1945–46

(000's omitted)

QUARTERS ended	March 31, 1948	June 30, 1948	Sept. 30, 1948	Dec. 31, 1948	Total for Calendar Year 1948
	£	£	£	£	£
Customs	202,441	213,742	193,555	217,558	827,296
Excise	186,300	183,000	185,000	183,800	738,100
Stamps and Estate etc. Duties . ..	65,300	61,435	55,978	56,800	239,513
Other Inland Revenue Duties	575	75	25	30	705
Post Office (net revenue)	—	—	—	—	—
Wireless Licences	3,750	2,220	1,955	3,580	11,505
Motor Vehicle Duties	35,779	5,403	4,051	3,272	48,505
Profits Tax	10,850	22,160	44,621	73,159	150,790
Excess Profits Tax	61,183	38,300	19,400	11,805	130,688
Special Contribution	—	4,600	9,900	17,850	32,350
	566,178	530,935	514,485	567,854	2,179,152
Income Tax and Surtax	731,858	227,152	199,880	166,906	1,326,096
	1,298,036	758,387	714,365	734,760	3,505,548
Sale of Surplus War Stores	64,258	18,708	45,700	26,879	155,545
Surplus Receipts from certain Trading Services	25,086	826	..	4,000	29,912
Crown Lands	260	180	220	260	920
Interest on Sundry Loans	3,526	2,012	7,078	2,254	14,870
Miscellaneous Receipts	45,511	14,690	64,280	34,527	159,008
	1,436,677	794,803	831,643	802,680	3,865,803

YEARS ended March 31	1948–49	1947–48	1948–49 (compared with 1947–48) Increase	Decrease	Corresponding Years 1946–47	1945–46
	£	£	£	£	£	£
Customs	823,258	791,101	32,157	—	620,741	569,842
Excise	733,500	629,700	103,800	—	563,500	540,800
Stamps and Estate etc. Duties	233,574	228,309	5,265	—	186,382	145,400
Other Inland Revenue Duties	700	715	—	15	724	712
Post Office (net revenue)	—	—	—		—	—
Wireless Licences	11,700	11,200	500		9,940	5,180
Motor Vehicle Duties	52,716	49,108	3,608		49,456	13,172
National Defence Contributions	—	—	—	—	—	35,485
Profits Tax	199,090	36,120	162,970	—	32,107	—
Excess Profits Tax	79,805	252,568	—	172,763	325,391	432,130
Special Contribution	79,450	—	79,450	—	—	—
	2,213,793	1,998,821	387,750	172,778	1,788,241	1,772,721
Income Tax and Surtax	1,465,470	1,280,948	184,522	—	1,382,497	1,430,415
	3,679,263	3,279,769	572,272	172,778	3,170,738	3,203,136
Sale of Surplus War Stores	99,597	197,231	—	97,634	155,996	—
Surplus Receipts from Certain Trading Services	28,564	101,261	—	72,697	59,000	—
Crown Lands	860	950	—	90	920	960
Interest on Sundry Loans	17,683	23,044		5,361	26,779	11,046
Miscellaneous Receipts	180,623	242,603		61,980	78,312	70,561
	4,006,590	3,844,858	572,272	410,540	3,491,745	3,285,703

NET INCR. £161,732

BANK OF ENGLAND

Pursuant to the Act 7th and 8th Victoria, cap. 32 (1844)

(000's omitted)

1	2	3	4	5	6	7	8
Issue Department						Collateral Columns	
Liabilities		Assets					
Notes Issued	Dates (Wednesdays)	Govt. Debt (£11,015) and Govt. Securities	Other Securities	Gold Coin and Bullion*	Silver Coin†	Notes in Hands of Public	Minimum Discount Rate
£		£	£	£	£	£	
1,400,248	Jan. 7	1,388,187	789	248	8	1,331,257	2%
1,400,248	,, 14	1,388,278	700	248	7	1,306,428	
1,400,248	,, 21	1,388,227	745	248	13	1,285,030	
1,400,248	,, 28	1,388,189	789	248	7	1,268,966	(26.10.39)
1,350,248	Feb. 4	1,338,250	725	248	10	1,257,632	
1,350,248	,, 11	1,338,172	807	248	6	1,247,019	
1,350,248	,, 18	1,338,176	803	248	6	1,235,241	
1,350,248	,, 25	1,338,202	773	248	10	1,231,611	
1,300,248	Mar. 3	1,288,329	649	248	7	1,236,528	
1,300,248	,, 10	1,288,345	629	248	11	1,236,896	
1,300,248	,, 17	1,288,245	730	248	10	1,233,599	
1,300,248	,, 24	1,288,352	625	248	8	1,241,035	
1,300,248	,, 31	1,288,265	707	248	13	1,245,863	
1,300,248	April 7	1,288,267	707	248	11	1,246,617	
1,300,248	,, 14	1,288,170	807	248	8	1,241,765	
1,300,248	,, 21	1,288,174	802	248	9	1,237,525	
1,300,248	,, 28	1,288,178	798	248	9	1,237,845	
1,300,248	May 5	1,288,209	766	248	10	1,242,938	
1,300,248	,, 12	1,288,215	759	248	11	1,247,261	
1,300,248	,, 19	1,288,242	738	248	5	1,249,100	
1,300,248	,, 26	1,288,375	603	248	7	1,244,201	
1,300,248	June 2	1,288,268	705	248	12	1,244,981	
1,300,248	,, 9	1,288,194	786	248	5	1,249,389	
1,300,248	,, 16	1,288,267	706	248	12	1,250,020	
1,300,248	,, 23	1,288,214	763	248	8	1,249,810	
1,300,248	,, 30	1,288,217	755	248	13	1,252,218	
1,300,248	July 7	1,288,164	808	248	13	1,261,411	
1,300,248	,, 14	1,288,214	756	248	15	1,271,971	
1,300,248	,, 21	1,288,257	715	248	13	1,276,469	
1,300,248	,, 28	1,288,169	807	248	9	1,285,041	
1,300,248	Aug. 4	1,288,268	707	248	10	1,288,349	
1,300,248	,, 11	1,288,270	707	248	8	1,277,837	
1,300,248	,, 18	1,288,167	808	248	9	1,262,965	
1,300,248	,, 25	1,288,176	800	248	9	1,253,324	
1,300,248	Sept. 1	1,288,167	812	248	6	1,248,998	
1,300,248	,, 8	1,288,219	761	248	5	1,248,206	
1,300,248	,, 15	1,288,181	796	248	8	1,243,624	
1,300,248	,, 22	1,288,167	808	248	11	1,237,821	
1,300,248	,, 29	1,288,226	749	248	10	1,236,391	
1,300,248	Oct. 6	1,288,168	807	248	10	1,237,851	
1,300,248	,, 13	1,288,169	807	248	9	1,236,053	
1,300,248	,, 20	1,288,122	852	248	11	1,231,613	
1,300,248	,, 27	1,288,165	808	248	12	1,230,772	
1,300,248	Nov. 3	1,288,157	816	248	12	1,233,935	
1,300,248	,, 10	1,288,164	809	248	12	1,235,600	
1,300,248	,, 17	1,288,166	809	248	10	1,233,950	
1,300,248	,, 24	1,288,169	806	248	10	1,233,078	
1,300,248	Dec. 1	1,288,128	849	248	8	1,241,993	
1,300,248	,, 8	1,288,263	713	248	9	1,260,213	
1,300,248	,, 15	1,288,274	704	248	7	1,286,940	
1,300,248	,, 22	1,313,244	735	248	6	1,296,442	
1,300,248	,, 29	1,313,271	706	248	8	1,298,089	

* At 172*s*. 3*d*. per fine oz.

† Coin other than Gold Coin.

WEEKLY RETURN

for Wednesday in each Week, during the year 1948

(000's omitted)

9	10	11	12	13	14	15	16	17	18
BANKING DEPARTMENT									
Liabilities					Assets				
Capital (£14,553) and Rest	Public Deposits	Bankers' Deposits	Other Deposits	DATES (Wednesdays)	Govt. Securities	Discounts and Advances	Other Securities	Reserve (Notes and Coin)	Totals of Liabilities and Assets
£	£	£	£		£	£	£	£	£
3.617	13,783	304,659	101,273	Jan. 7	336,464	13.261	18,840	69,320	437,885
3,657	21,667	295,551	96,341	" 14	308,321	9,930	19,382	94,136	431,769
3,693	22,002	293,516	94,835	" 21	280,834	12,019	20,230	115,517	428,600
3,732	16,322	290,842	93,046	" 28	241,575	12,674	32,702	131,544	418,495
3,766	23,408	283,712	94,300	Feb. 4	286,508	11,944	28,415	92,872	419,739
3,809	17,295	285,801	92,345	" 11	275.999	10,586	23,768	103,450	413,803
3,852	11,654	296,270	93,588	" 18	270,624	11,890	22,138	115,265	419,917
3,888	12,064	290,609	93,324	" 25	263,324	11,277	20,977	118,860	414,439
3,911	18,495	293,542	92,256	Mar. 3	327,644	10,491	20.694	63,927	422,756
3,923	12,624	299,103	93,449	" 10	329.824	4,499	20,660	63,668	423,652
3,948	11.974	305,129	92.799	" 17	326,679	13,443	21,178	67.104	428,404
3,962	11,906	309,311	91,681	" 24	333,599	16,894	21,207	59,712	431,411
3,997	9,008	314,319	94,302	" 31	341,059	14,377	25.895	54,848	436,179
3,157	11,776	308,432	91,366	April 7	346,914	8,977	19,302	54,091	429,284
3,191	14.475	302,059	94,029	" 14	338,529	10,569	20,132	59,077	428,307
3,200	10,291	308,062	93,265	" 21	330,519	11,534	21,037	63,285	429,375
3,266	12,634	307,402	90,333	" 28	320,004	14,490	30.582	63,111	428,187
3,285	20,924	299,553	92,050	May 5	321,229	14,807	36,354	58,074	430,365
3,341	8,642	311,526	92,431	" 12	344,334	9,207	23,321	53,631	430,493
3,374	8,484	317,282	91,923	" 19	353,019	9,967	20,968	51,662	435,617
3,412	10,678	311.754	92,965	" 26	344,319	9.784	22.562	56.696	433,361
3,451	18,869	298,911	92,912	June 2	338,079	14,366	20,263	55,988	428,697
3,488	12,213	302,123	90,908	" 9	336,494	15,786	19,436	51,567	423.281
3,532	14,690	304,482	93,540	" 16	335,819	23,902	20,192	50,884	430,797
3,572	13,577	312,842	89,760	" 23	338,849	23,340	20,923	51,191	434,304
3,610	14,509	325,414	92,181	" 30	351,699	17,786	32,060	48,724	450,268
3,661	9,597	309,285	92,362	July 7	347,665	22,012	20,198	39,383	429,158
3,678	14,069	303,309	88,901	" 14	364,144	10,210	21,101	29,052	424,510
3,722	14,710	307,048	89,125	" 21	366,869	15,616	22,133	24,540	429,158
3,764	11,247	311,026	89,351	" 28	366,344	13,398	34,149	16.050	429,941
3,823	42,035	305,423	90,141	Aug. 4	405,199	10,556	27,439	12,781	455,975
3,843	53,370	299,750	87,958	" 11	402,154	8,324	25,592	23,404	459,474
3,864	46,018	316,676	90,126	" 18	404,124	6,150	22.631	38,332	471,237
3,882	50,352	300,269	90,115	" 25	283,744	5,400	22,021	48,006	459,171
3,906	58,660	307,030	94,798	Sept. 1	400,969	3,901	21,675	52,402	478,947
3,929	60,280	308,129	93,482	" 8	397,479	8,522	21,332	53,050	480,373
3,954	62,075	307,767	94,577	" 15	395,824	7,919	21,425	57,758	482,926
3,977	69,943	301,024	92,230	" 22	389,554	6,588	21,846	63,739	481,727
3,993	75,935	299,970	93,324	" 29	361,849	25,037	35,419	65,440	487,775
3,145	17,748	296,980	96,032	Oct. 6	315,919	27,130	21,176	64.233	428,458
3,179	25,219	303,091	94,023	" 13	325,529	26,396	22,620	66,420	440,965
3,194	28,760	301,493	93,930	" 20	322,144	25,323	22,366	71,097	441,930
3,253	32,707	307,460	93,043	" 27	322,289	19,303	37,352	72,072	451,016
3,280	34,994	304,125	94,421	Nov. 3	325,194	23,728	33,369	69,072	451,373
3,327	21,320	312,086	93,073	" 10	327,364	22,378	27,039	67,578	444,359
3,360	28,089	313,436	92,435	" 17	351,039	26,862	24.636	69,336	451,873
3,385	26,565	302,729	99,206	" 24	324,054	28,853	23.308	70.223	446,438
3,430	25,168	313,410	91,909	Dec. 1	332,719	22,927	31,534	61,290	448,470
3,467	24,063	308,689	89,841	" 8	355,819	17,665	23,912	43,217	440,613
3,508	22,303	301,087	91,052	" 15	370,644	21,150	23,921	16,788	432,503
3,544	29,013	310,026	92,116	" 22	379,149	14,355	23,126	32,622	449,252
3,578	29,118	314,487	92,063	" 29	380,754	29,118	20,301	30,622	453,799

VOL. CXII.] [PART II.

Journal of the Royal Statistical Society

SERIES A (GENERAL)

PART II, 1949.

International Statistics

By H. Campion, C.B.E.

[Read before the Royal Statistical Society, January 19th, 1949, the President, Dr. David Heron, in the Chair]

Members of this Society have always taken great interest in the development of international statistics. It was in London at the Great Exhibition of 1851 that the first proposals and the first arrangements were made for regular meetings of statisticians from all over the world. In 1885 the International Statistical Institute was formed at the Jubilee Meeting of the Royal Statistical Society. It was in London in 1919 that the first discussions took place about statistical work to be undertaken by the League of Nations and its relation to the International Statistical Institute. It was again in London that the Statistical Commission of the United Nations was set up in 1946.

I am presenting to you to-day a paper on a subject which is therefore of traditional concern to this Society. My paper describes the steps which have been taken to re-establish and develop international collaboration on statistical questions, more particularly since the war, and outlines the scope of the work which the international organizations hope to do.

I had myself little experience at first hand of the work of the International Statistical Institute or of the Committee of Statistical Experts of the League of Nations before the war, although I had read and used their reports. When, therefore, in 1946 I became one of a group of statisticians from different countries asked to consider the organization of the statistical work to be done by the United Nations and its associated bodies, I sought the advice informally of Fellows of this Society.

I would have liked to have been able to talk again with Flux and Stamp. I remember that one of my last conversations with Lord Stamp was about international statistics. It was one evening in 1940, soon after the Germans had overrun Norway and Denmark and the Low Countries, and I had gone to see him in his room on the first floor of the Cabinet Office in Richmond Terrace. He was then Chairman of the Cabinet Committee on Economic and Financial Plans, and I had to discuss with him a paper I had prepared for him which he wanted to use for a Committee the following day. It was getting late, and our talk was brief and to the point. Then, when he found he had plenty of time before he had to leave for his dinner appointment, his conversation turned to other topics. Picking up the evening paper from a side table, he looked at the headlines of what was happening in Belgium and Holland. He stopped, and then said quietly, "I hope Flux will be all right in Denmark, and that nothing has happened to Methorst in The Hague or Julin in Brussels." M. Julin was the President and M. Methorst the Secretary-General of the International Statistical Institute, of which he himself was Treasurer. He then talked about the work of the Institute and of the Committee of Statistical Experts of the League of Nations, of which Flux had been Chairman, and he emphasized how important it was that the work of these two bodies should be revived when the war was over. I listened to him expound his ideas of what these and other international organizations should try to achieve.

Stamp was killed in an air raid in April 1941, and Flux died in occupied Denmark during the war. From them and from other Fellows of the Society I learnt what I understood should be the principal aims of international collaboration on statistical matters. Briefly they were:

(1) Statistical journals and publications should be exchanged freely between different countries, so that statisticians all over the world should be kept informed of the work of each other and draw inspiration from them.

(2) There should be means by which statisticians from different countries should be able to meet regularly. They should be free to discuss their ideas, their work, to give their reasons why they used certain methods in compiling their statistics.

(3) There should be a strong international organization or groups of organizations working together, which should assemble figures received from all countries of the world and publish them. The figures should be built up into world totals where such totals would have meaning. The statistics should be published for each country in as comparable a form as possible and with the necessary technical explanations.

(4) Statisticians from different countries should try to agree to publish their statistics in comparable form. "Eccentricity, so interesting in social life, loses its charm when applied to the compilation of statistics."* There are often compelling reasons why an individual country cannot use an international classification for its own domestic purposes, but this should be no excuse for not collecting statistics in sufficient detail to permit of re-arrangement for international comparison. And

(5) to beware of the danger of levelling down the standards of national statistics for the sake of international comparability. What was needed was to get the views of the experts in a particular field of statistics to see what they would recommend as the best methods to employ. The need was not only for standards which could be adopted immediately by most countries, but also for standards which, although they might be adopted only by a few countries immediately, would set goals to be attained by others in the future.

1. *The Development of International Statistics up to the Second World War*

The landmarks in the development of international statistics stand out clearly; the first proposals for international statistical congresses made at the time of the Great Exhibition in London in 1851; the foundation in 1885 of a continuing organization, the International Statistical Institute; the establishment of the Economic and Financial Section of the League of Nations in 1919; and the Committee of Experts of the League in 1928. It is convenient to write the history in periods of years around these dates, but it would be wrong if the impression was thereby given that the development was other than a gradual evolutionary process over the century. It started with the evangelical enthusiasm of a few individuals intensely interested in statistics, who thought it would be useful to meet and discuss each other's work. Out of their meetings grew the world-wide and somewhat complex system of international organizations in which governments, business men, learned societies and individuals now all have their place, and there are international commissions and international conventions.

It began quite simply. During the early years of the nineteenth century public interest was aroused by the attempts made for the first time to get facts about how people lived and worked. Many of these enquiries were undertaken by private individuals, but official statistical offices were later set up in Belgium, Holland, France, Prussia, Saxony and the Scandinavian countries. In England, G. R. Porter was appointed head of a new statistical department in the Board of Trade established in 1832, and Farr became "Compiler of Abstracts" at the General Register Office in 1839.

The statistics compiled and published by these private and official enquiries were the subject of discussion and criticism, and statistical societies were formed in many countries, particularly in the 1830's, where papers could be presented and discussion follow. The Statistical Society of Saxony was started in 1831, and in England the Manchester Statistical Society started in 1833, and the London (later the Royal) Statistical Society in 1834. The American Statistical Association was founded in Boston in 1839.

At first the main preoccupation of members of the societies was to get facts to stir the public conscience so that the social reformers could persuade the governments of the day to carry out the measures they wanted to relieve poverty, to improve health, and generally to raise the standard of living of the people.

The years 1830–50 were a period of enthusiasm for statistics in England and throughout the continent of Europe. Inevitably comparisons began to be made between the statistics collected for the first time by the different countries, and between the kind of enquiries undertaken and the

*Hailey, Lord, *An African Survey, p. 1353.*

methods used. The questions were asked: why should not each country undertake the kind of statistical enquiries which had proved so valuable in England, France, Belgium and other countries?; when these enquiries were done, why should they not be undertaken in the same way so that the results were comparable?; why should there not be official organizations in each country to undertake these enquiries and to make sure the results were comparable?; and why should not the organizations in each country work together in a joint European organization which would lay down the principles by which each country should collect its statistics? It was suggested that the statisticians of the different countries should meet to discuss these proposals.

These ideas no doubt occurred to the statisticians of more than one country, but the initiative in calling the first statistical congress was taken by Quételet, the distinguished Belgian statistician. He had been a teacher in mathematics to Prince Albert, the Consort of Queen Victoria, and had friends in England. His name will always be associated with the foundation of the Royal Statistical Society. Babbage recalled in a letter to Farr, written on December 14th, 1860, the part that Quételet played:

> "At length, the conviction of the importance of the value of Statistical Science becoming widely extended in other countries, M. Quételet saw that a fit time had arrived for summoning an European Congress. The results of such meetings are invaluable to all sciences, but more particularly to statistics, in which names have to be defined, signs to be invented, methods of observations to be compared and rendered uniform; thus enhancing the value of all future observations by making them more comparable as well as more expeditiously collected."

The occasion when Quételet made his proposals was the time of the Great Exhibition in London in 1851, when there were many delegates from foreign countries. Quételet was asked to arrange for such an international congress, and it was agreed that it should take place in Brussels in September 1853. 150 persons from 26 countries attended the meeting, including three from the United States.

The items on the agenda for this first congress in Brussels were—

1. Statistical organization.
   (Proposal for a central statistical commission in each country, and for these commissions in turn to be attached to an international congress charged with establishing comparability between statistics published by different countries.)
2. Censuses of population and vital statistics.
3. Definition and measurement of territories.
4. Statistics of migration.
5. Censuses of agriculture.
6. Censuses of industrial production.
7. Statistics of external trade and navigation.
8. Family budget enquiries.
9. Statistics of unemployment and poverty.
10. Statistics of education.
11. Statistics of crime.

The meeting in Brussels was a success, and was followed by a series of congresses in different capitals of Europe at intervals of two or three years.

		Numbers attending
1853	Brussels	153
1855	Paris	311
1857	Vienna	542
1860	London	586
1863	Berlin	477
1867	Florence	751
1869	The Hague	488
1872	St. Petersburg	488
1876	Budapest	442

After the first few congresses it became clear that something more was needed than merely a meeting every two or three years. There should be some link between the various congresses; otherwise the same subjects were discussed over and over again. It was proposed that there should be a small Permanent Commission which would meet between sessions to study the various recommendations made at the congresses, to bring them to the notice of the Governments of the participating countries and to prepare papers for discussion at the forthcoming congresses.

The Permanent Commission was first proposed by Engel, Director of the Statistical Office of Prussia, at the meeting in Berlin in 1863, but it was not established until 1872 at the time of the congress in St. Petersburg. The Commission held four meetings—in Vienna in 1873, in Stockholm in 1874, in Budapest in 1876 and in Paris in 1878. Much time, however, seems to have been spent at these meetings in discussing the role of the Commission. Unfortunately, not only did it try to assume responsibility for acting as an organization for continuing the work of the congresses between sessions, but it also sought to establish itself as a body for imposing the recommendations of the congresses on the various member states and to make the recommendations binding on governments. This immediately led to objections from participating countries, and the work of the whole organization of the international congresses and of the Permanent Commission itself came to a stop. The meetings fixed for 1879 in Rome and 1880 were not held.

Looking back now at this period from 1850-1880, all the difficulties of securing effective international collaboration in statistics are evident. There is no doubt that the congresses created a very powerful stimulus to the development of statistics in Europe. They had a vitalizing effect, and resulted in great progress in the development of official statistics in the European countries. Professional interest in statistics was aroused. Even to-day the speeches of Quételet, Farr, Newmarch, Engel, Keleti and others at these meetings are well worth study. At the meeting in London in 1860 there was an outstanding opening address by the Prince Consort, and Florence Nightingale (who became a member of our Society in 1858) presented a notable paper on hospital statistics.

But there were difficulties.

(i) In those days there were none of the mechanical aids which now exist to facilitate discussion at international meetings—such as simultaneous interpretation, quick reproduction of documents and telephone and telegraph communications. Those attending the congresses had to rely on being able to express themselves in French, or in the language of the country where the congress was being held. At many meetings the discussions could not be followed by many of those attending the congress.

(ii) The congresses were arranged for too short a period and too many subjects were covered. The result was that the discussions were often diffuse, and too many recommendations were adopted hastily and without adequate consideration.

(iii) Too many persons attended the congresses, and many of them were not professionally interested in statistical matters. The status of the persons to be invited to the congresses was never satisfactorily settled. Some of them were officials sent by their governments, and it was not always clear whether they were speaking on behalf of their governments or in a personal capacity. In addition there were representatives of learned societies. The status of those attending the Congresses became important when the Permanent Commission sought later to enforce on the participating countries the recommendations passed at the congresses.

(iv) The early congresses suffered from not having a continuing organization to prevent duplication of discussion between congresses. The fact that the Permanent Commission when elected went too far in assuming powers for itself to enforce its decisions did not prove that a continuing organization was not needed.

Foundation of the International Statistical Institute

During the years which followed the congresses, the statisticians of the different countries had time to reconsider what kind of international organization there ought to be in the future. It was inconceivable that after the success of the early congresses there should be no international organization at all during a period in which statistics in so many countries were developing rapidly. An opportunity to think again was provided by the invitations extended by the United Kingdom

Government to foreign countries to send statisticians to attend the Jubilee Meeting of the Royal Statistical Society held in London in 1885. A special session of the Jubilee Meeting was arranged to review the successes and failures of the congresses, and to see what proposals would gain general support for a future international association.

A paper was read by Neumann-Spallart of Austria at the Jubilee Meeting, and in the discussion which followed various suggestions were made. Out of these were drafted the statutes of the International Institute of Statistics. These were agreed in principle among those present in London, and discussed further at the 25th anniversary of the foundation of the Statistical Society of Paris, which was held in Paris immediately afterwards.

Some changes were made in the draft statutes first proposed, but Article I of the statutes of the International Institute of Statistics as finally adopted was as follows:

"L'Institut International de Statistique est une association internationale autonome qui a pour but de favoriser le progrès de la statistique administrative et scientifique:

1. En recherchant et en recommandant les méthodes propres à obtenir, autant que possible, l'uniformité dans les cadres et dans le dépouillement des relevés de la statistique, afin de rendre comparables les résultats obtenus dans les différents pays;
2. En appelant, par des voeux, l'attention des gouvernements sur les questions à résoudre par l'observation statistique;
3. En faisant des publications internationales destinées à élucider les questions de statistique et à établir des rapports permanents entre les statisticiens de tous les pays;
4. En concourant, s'il y a lieu, par d'autres publications, par l'enseignement et par divers moyens, à propager les notions de statistique et à interesser les hommes d'Etat et les savants à l'étude numérique des faits sociaux."

There was still some doubt among some European statisticians as to how far the proposed International Institute of Statistics was merely re-establishing the international statistical congresses and how far the Institute differed from the Permanent Commission. These doubts were expressed by Becker and Blenck of Germany in a letter to Sir Rawson W. Rawson, President of the Royal Statistical Society at the Jubilee Meeting, who became the first President of the International Statistical Institute. The reply which the President sent to them on October 5, 1885, explained the difference between the new International Statistical Institute and the previous bodies:

"The Association is purely a private and scientific body. It is altogether different from the International Statistical Congresses or their successor, the Permanent International Commission. It has no official character. It seeks to exercise no official authority or influence, though it is not without hope that its labours will be useful in furnishing information and suggestions to the Governments, as well as to the statisticians, of the world. It seeks the co-operation of the chief statistical authorities in the several countries, because they are the most competent to aid it with their experience and advice, the most likely to take an interest in its proceedings, and the most certain to ensure respect for the results of its deliberations.

"It is not without hope that these results may, by their merits, obtain the attention of Governments, and may often lead to the adoption of methods, or enquiries, which will promote the advancement of statistical knowledge, with, or without, an increase in the uniformity of national statistics. But it is not its purpose to make this uniformity the special object of its labours, nor has it any intention of attempting to impose its views either upon Governments or upon its own members.

"It may, like the Royal Society of England, find itself in a position to offer recommendations or suggestions to the Governments, but it would probably exercise this privilege through the voluntary action of its own members.

"To sum up, I may say that while the direct object of the Congresses and Permanent Commission was to influence Governments, that of the International Statistical Institute is to acquire and perfect statistical knowledge, and to furnish information which may be useful to those Governments who may pay attention to its proceedings."*

The International Statistical Institute was to be an international scientific organization to consist of members in their personal capacity. Originally the number of ordinary members was limited to 150, but was later raised to 225. The number of honorary members was not to exceed 25.

* *Bulletin de l'Institut International de Statistique*, 1886, Tome I, p. 33.

From 1887 up to the beginning of the first World War the Institute met once every two years.

Although the Institute had assumed the role of a private scientific organization, it still had something of the standing of an inter-governmental organization. Sessions of the Institute were held in different countries at the invitation of the national Governments. Governments were free to include persons who were not members of the Institute in their national delegations to the sessions of the Institute. Those participating at sessions therefore included members of the Institute who might or might not be official delegates of their governments or other national organizations, and non-members who might be official delegates from other countries.

The papers read at the meetings of the Institute were of a scientific and technical character, and the quality of the papers reflected the high standing of the membership of the Institute. No attempt was, however, made to pass resolutions which would be binding on governments, but in some cases the Institute set up committees to study special problems, and on the basis of their reports passed resolutions which they hoped Governments would accept. Out of the work of one such committee, of which Bertillon was chairman, was developed the International List of Causes of Death which the Institute at its meeting in Christiania in 1899 recommended for adoption by the statistical offices of Europe.

Establishment of the Permanent Office of the International Institute of Statistics

Owing no doubt to the unhappy experience of the Permanent Commission, no attempt was made to create a continuing organization for the Institute, but the need for a permanent secretariat became evident. The creation of a Permanent Office of the Institute was canvassed in 1908 by van der Borght, the Director of the German Office of Statistics, and at the meeting of the Institute in Paris the establishment of the Permanent Office was proposed. Plans for the Permanent Office were discussed further at the meeting at The Hague in 1911, and the Office was finally established in 1913. The Office was to be in The Hague and H. W. Methorst was appointed as its Director, a post which he has filled with so great distinction up to 1947.

The Permanent Office was given the following terms of reference:

"1. The permanent office of the International Institute of Statistics shall gather, study and preserve in its library and archives the statistical documents of the various States and international offices. All information, which is useful for international comparison, will be selected; in the first place such information as bears on *Demography*.

"2. It shall, through permanent activity, facilitate the unification of methods, means of elaboration, ways of publishing, so that the results may be as much as possible comparable.

"3. As soon as possible it shall publish an international Statistical Year Book, a periodical, and if there be occasion, other works as well, in which side by side with statistical tables a bibliography will be found, and notices bearing on the progress and the new proceedings which take place in certain countries, and which it would be expedient to make universally known.

"4. It shall help the 'Bureau de Présidence' to prepare the programme of each session."

One of the most important tasks of the new office was to begin to collect and publish international statistics. The proposal for an international statistical yearbook had been made more than once during the previous half-century. It was not, however, until 1913 that the Office of the International Institute was given this specific task, and even so it was instructed to start by collecting demographic data.

So far there had been no serious challenge to the position of the International Statistical Institute as the central organization for consultation internationally on statistical matters. Even by 1913 there were signs that other international organizations were beginning to demand responsibility for statistics in their own fields. International organizations on railways and postal and telegraphic services were compiling their own statistics. An international organization on labour questions had been established in Basle in 1901, but it had not as yet begun any serious statistical work. The International Institute of Agriculture had been set up in 1905 in Rome, and had published an international statistical year book on agriculture. An International Bureau of Hygiene had been established. The pattern of "Specialized Agencies" dealing with different branches of statistics had begun to appear, and the international Institute of Statistics had to some extent already lost ground as the central body for the collection of international statistics even by the time the Permanent Office was established in 1913.

The Permanent Office of the International Statistical Institute was set up in 1913. Before it could get to work effectively the first world war had begun, and the contacts which the International Institute had with statisticians in member countries could not be maintained during the war. Since, however, the seat of the International Statistical Institute was in The Hague, and Holland was a neutral country during the first world war, it was possible for Dr. Methorst and his staff to start the work originally proposed for the Office. During the war years of 1914–18 the Office published important reports, giving demographic data for the European countries.

League of Nations

A completely changed situation, however, arose in 1919, when the League of Nations was created and the International Labour Organization established. Both these organizations needed to collect statistics from member countries, and to compile them so far as possible in a comparable form. It was not clear how far the new organization of the League of Nations should assume the functions for collecting statistics entrusted to the Permanent Office of the International Statistical Institute, and how far the League of Nations should take over from the International Institute its role of making general recommendations on statistical questions to governments.

A conference to explore the possibilities of international co-operation in statistics was therefore called by the League of Nations in London on August 14th and 15th, 1919. Mr. Salter (now Sir Arthur Salter), then Director of the Economic and Finance Section of the League of Nations, presided. Representatives attended from the International Statistical Institute, the International Institute of Agriculture, and the proposed International Labour Organization. At this meeting the difficulties of collaboration became evident, and the problem arose how far the work of the collection of statistics should be shared between the various international organizations. It was proposed that the International Institute of Agriculture should continue to collect agricultural statistics, the International Labour Organization labour statistics, and for the time being the Permanent Office of the International Institute should collect demographic statistics. It was proposed also that there should be a Central Advisory Council on Statistics to meet normally at the seat of the League.

There was still a long way to go before these ideas were accepted. There were spirited defenders of the authority of the International Statistical Institute who were not willing to surrender their rights readily to the League of Nations. At a further meeting in Paris called by the League of Nations in October, 1920, Bodio, the Italian President of the International Statistical Institute, proposed and got the majority of those present to support his view that the functions of the existing organizations as regards the collection and publication of statistics should be retained, and the International Statistical Institute be given the role of becoming the independent technical and scientific advisory body to the League of Nations on statistical questions. This proposal was rejected by the General Assembly of the League of Nations when it met in September, 1921.

Mixed Committee of the League of Nations and the International Institute of Statistics

A way was found, however, to harmonize the views of the International Statistical Institute and the League of Nations. A Mixed Committee, consisting of representatives of the International Statistical Institute, of the League of Nations and of the International Labour Organization was set up. This committee, under the skilful guidance of M. Methorst and Mr. Loveday, achieved great success.

The reports and suggestions prepared by the Mixed Committee were submitted in the first instance to the biennial conferences of the International Statistical Institute, and then subsequently to the Economic Committee of the League of Nations.

Thus, detailed principles concerning the methods to be employed in the compilation of statistics of international trade and fisheries were prepared by the Mixed Committee for the meeting of the Institute at Brussels in 1923 and approved by the Institute. Similarly, at Rome in 1925 draft resolutions relating to censuses of industrial production, and at Cairo in 1927 draft resolutions relating to indices of industrial activity were examined and adopted.

The resolutions finally approved by the Institute for consideration by the Economic Committee of the League of Nations were circulated by the League to Governments for their consideration. From the replies received a general desire was expressed on the part of many governments that

definite steps should be taken to secure the adoption by the various governments of certain of the methods proposed. Accordingly in 1927 the Council of the League of Nations approved a proposal that—

"A conference to which all governments should be invited to send official statisticians should be convened during the course of the year 1928 with this end in view."

International Convention Relating to Economic Statistics (1928)

The conference on economic statistics met in Geneva in November and December 1928, and out of their deliberations a draft convention relating to economic statistics was prepared. Article 2 of this Convention (which is reproduced in full in Appendix I to this paper) asked the contracting parties to compile and publish the following classes of economic statistics, subject to certain reservations:

I. *External Trade and Shipping.*

(*a*) Annual and monthly returns of the quantity and value of imports and exports.
(*b*) Annual, and, if possible, quarterly or preferably monthly, returns showing the net tonnage of mercantile vessels engaged in international trade.

II. *Occupations.*

Statistics showing the occupations of the population.

III. *Agriculture and Fisheries.*

(*a*) General censuses of agriculture.
(*b*) Annual statistics showing both the distribution of area under the main crops and quantities harvested.
(*c*) Periodic returns of the number of live-stock.
(*d*) Periodic surveys of forest resources, to be taken by the chief timber-producing countries.
(*e*) Returns by countries to whose economy fishing is important, showing the products of the main sea fisheries landed.

IV. *Mining.*

Annual returns of the output of minerals and metals.

V. *Industry.*

(*a*) General censuses of industrial and, if possible, also of commercial establishments.
(*b*) Figures, as adequate as each country finds possible, of industrial production.
(*c*) Statistical series at regular intervals indicating the variations of the activity of the most representative branches of production.

VI. *Index Numbers of Prices.*

Index numbers expressing the general movement of wholesale prices and cost of living.

Article 2 of the Convention thus covered all the economic fields of major importance except labour (already covered by a separate convention, under which the International Labour Office was established); transport and communications (covered by a branch of the League of Nations); and finance and related matters (covered by the Economic Committee of the League of Nations).

The Annexes to the Convention set out the various methods to be recommended for adoption in collecting the various kinds of economic statistics, and were based on resolutions adopted by the International Institute of Statistics. Thus, after all, the International Institute found itself in the position of having made proposals for comparability of international statistics which Governments were to put in effect.

This Convention on Economic Statistics represented a great achievement; it became the foundation on which the League of Nations from 1928 onwards built most of its statistical work. The Convention itself was signed by 26 states, including the United Kingdom, but not by the United States of America, as it was not a member of the League of Nations.

League of Nations Committee of Statistical Experts

Article 8 of the Convention provided that a Committee of Technical Experts should be appointed, and the Committee authorized to—

"Make any suggestions which may appear to it useful, for the purpose of improving or amplifying the principles and arrangements laid down in the Convention concerning the classes of statistics dealt with therein. It may also make suggestions in regard to other classes of statistics of a similar character in respect of which it appears desirable and practicable to secure international uniformity. It shall examine all suggestions to the same ends which may be submitted to it by the Governments of any of the High Contracting Parties."

Certain fields of economic statistics were, however, excluded from consideration by the Committee under the Convention:

"The Committee shall not make any suggestions in respect of statistics relating to public or private finance (public debt, revenue and expenditure, banking, the money market, the stock exchange, etc.), or without the previous agreement of the appropriate international institutions or organizations in respect to statistics relating to agriculture, labour or transport."

The reason for these exclusions was that international statistical organizations already existed in these fields. Later, however, in 1936 the Committee of Experts was empowered to prepare reports concerning financial statistics.

This Committee of Statistical Experts met once a year between 1931 and 1939. The members of the Committee were not official delegates of their countries, but acted in their personal capacity. The Convention contained no limitatión as to the number of members of the Committee, but the number during most of the period of the existence of the Committee was ten. There was no provision in the Convention to prevent the appointment of more than one member of the Committee from the same country, but in practice all the members were from different countries. In addition to the members chosen from individual countries, the Committee included one member from the International Labour Office and one from the International Institute of Agriculture. There were no official representatives from the International Statistical Institute, though most members of the Committee were also members of the Institute.

Of the methodological studies made by the Committee, six were published in the League of Nations series, "Studies and Reports on Statistical Methods."

1. Statistics of the gainfully-occupied population.
2. Minimum list of commodities for international trade statistics.
3. Timber statistics.
4. Statistics relating to capital formation.
5. Housing statistics.
6. Indices of industrial production.

Other topics on which the Committee has made definite recommendations in its reports were:

"Minimum List of Countries (Statistical territories) to be shown in the Statistics of International Trade by Countries."

"Experiment in the Recording of Commodities by Countries of Provenance and Destination."

"Recommendations on Mining and Metallurgical Statistics."

"Indices showing Variations in Quantum and Prices of International Trade."

"International Tourist Statistics."

The activities of the Committee were interrupted by the outbreak of the war. At that time they had begun the study of—

Banking Statistics.
Balance of Payments Statistics.
Building Statistics.
Price Indices and Related Price Statistics.

The work on banking statistics had reached a formative stage by 1939, and some work had also been started on statistics of balance of payments and on building statistics. The study on price indices and related price statistics had just been resumed after being shelved for some years. The

Committee had also decided in accordance with its general mandate under the Convention to take up the problem of industrial censuses and current production statistics and "to include in its programme for the next few years the statistical measurement of national income."

International Conferences of Labour Statisticians

By agreement under the Convention of 1928 the Committee of Experts of the League of Nations did not concern themselves primarily with labour statistics. The International Labour Organization itself set up a body of experts on labour statistics, and the first Conference of Labour Statisticians was held in Geneva in October, 1923. In all, five conferences were held between 1923 and 1939.

Topics discussed:

1923 Classification of industries and occupations.
Statistics of wages and hours of work.
Statistics of industrial accidents.
1925 Cost of living index numbers.
Unemployment statistics.
International comparisons of real wages.
Classification of industries.
1926 Methods of family budget enquiries.
Statistics of collective agreements.
Statistics of industrial disputes.
Classification of industries.
1931 International comparisons of real wages.
1937 Convention concerning statistics of wages and hours of work.

Regional Consultations on Statistical Questions

There were important developments during the inter-war years to secure closer co-operation on statistical matters between the countries of the British Commonwealth and Empire and also between the Scandinavian countries.

The experience of the first world war had shown the need for closer collaboration between the statisticians of member countries of the British Commonwealth and Empire. The first "Conference of Government Officers engaged in dealing with statistics in the British Empire" took place in London in January and February, 1920. This conference made recommendations about the kinds of statistics to be collected and the statistical methods to be used. It was suggested that there should be a British Empire Statistical Bureau responsible for collecting statistics for the British Empire. This bureau was not established, and the Board of Trade continued to prepare their Statistical Yearbook for the British Overseas Dominions and Protectorates.

At the Imperial Conference, 1930, the report of a statistical sub-committee of the General Economic Committee was adopted, recommending that a second conference of statistical officers of the British Commonwealth should be held in September 1932, in Ottawa. This second conference did not, however, meet until 1935 in Ottawa.

Ever since 1888 there had been arrangements for conferences between the statisticians of the Scandinavian countries. The first meeting was arranged in 1888 by Kiaer, Director of the Norwegian Central Bureau of Statistics, with his colleagues from Denmark and Sweden. The purpose of these conferences was to establish so far as possible a greater degree of comparability between the statistics of the Scandinavian countries. The conferences were extended later to include representatives from Finland in 1924 and from Iceland in 1927. These meetings were arranged regularly, though not at stated intervals of time. The results of these conferences were to achieve greater uniformity between the statistics of the Scandinavian countries, and gave support to the representatives of the Scandinavian countries who sat on the committees of the international organizations.

The position in 1939, at the outbreak of the second world war, was very different from what it had been in 1914.

(1) There were now established international organizations, the League of Nations, the International Labour Office and the International Institute of Agriculture, publishing statistics regularly relating to many countries of the world and presenting them so far as possible in a comparable form.

(2) There were two standing committees of experts—one under the authority of the League of Nations and the other under the International Labour Organization—whose task it was to study the statistics collected and to make recommendations for their improvement.

(3) The International Statistical Institute after an inspired period in the 1920's, when it gave great support to the League of Nations in formulating the International Convention on Economic Statistics, had resumed its more traditional role of an international scientific organization, although it still collected and published demographic data.

(4) The arrangements between the League of Nations and the other specialized organizations, such as the International Labour Organization and the International Institute of Agriculture, were working smoothly. These specialized agencies were few, and they did not themselves overlap seriously in any way in the work they had to undertake. There were arrangements for regular consultation between the statisticians of the League of Nations and these organizations to prevent duplication of work.

(5) The main support for the statistical work of the international organizations still came mainly from the European countries, but increasing help was coming from the United States, even though it was not a member of the League of Nations. A representative from the United States sat on the Committee of Experts of the League of Nations. Compared with the International Congresses of the 1860's, the new organizations can more rightly claim to be "international." There were still, however, large areas of the world for which no adequate statistics were collected, particularly in Central and South America, Asia and Africa.

The second world war brought a more serious break in the arrangements for international collaboration in statistics than the first world war had done. Although the seat of the League of Nations was in Switzerland, contacts between Switzerland and the rest of the world proved difficult. The Secretariat of the League of Nations itself were divided between Geneva and Princeton in the United States. The statistical staff of the International Labour Office moved to Montreal in Canada. The International Statistical Institute, which had been able to continue its activities in a neutral country during the first world war, found itself in an occupied country in 1940, and the work of the Institute almost ceased. Members of the Permanent Office of the International Institute of Statistics were approached by the Germans in 1941 for assistance in creating an Inter-European Statistical Institute. No members of the Office gave their support either officially or in their private capacities.

There were, however, two important developments during the war:

(i) Collaboration on statistical matters between the United Kingdom and other members of the Commonwealth of necessity became very close—even more than they had during the first world war. Further, close relations were established between the statisticians of the United Kingdom and the United States following the introduction of the Lend-Lease Act, and then later American participation in the war. During the war a large number of British statisticians visited the United States on official business to work alongside their American colleagues.

(ii) In addition, the need for collaboration on statistical matters between the United States itself and other States of the Americas became more urgent, and in order to stimulate this development the Inter-American Statistical Institute was set up in 1940. The basis for this organization was laid by the American members of the International Statistical Institute present at the Eighth American Scientific Congress held in Washington in May 1940. After a preliminary period of organization, the first meeting of the executive committee of this new body was held in August 1942. The objective of the Inter-American Institute has been to encourage the development of statistical science and administration throughout the Western Hemisphere, and in particular to improve the methods used in the collection, tabulation, analysis and publication of both official and unofficial statistics and to obtain a greater degree of international comparability in these

statistics. The Inter-American Statistical Institute grew in strength during the war, and by the end of the war was a vigorous and thriving organization.

II. *Development of International Statistics since the end of the Second World War*

Before the end of the war plans were already being discussed for the organization of the United Nations to replace the League of Nations. A Preparatory Commission was set up to recommend the form of the organization of the new body. The first session of this Preparatory Commission was held in San Francisco in June 1945, immediately after the Charter of the United Nations was signed, and the second session began in London in November, 1945. On the basis of the recommendations of this Preparatory Commission it was decided to establish a permanent Statistical Commission to report to the new Economic and Social Council. A temporary Statistical Commission was created in the meantime to make recommendations to the Economic and Social Council on the role of the Statistical Commission, its terms of reference, and more generally on the statistical work to be undertaken by the United Nations.

The principal tasks which this temporary Statistical Commission had to consider at its session in New York in May 1946, were:

(i) The composition and terms of reference of the Permanent Statistical Commission of the United Nations, including the creation of sub-commissions.

(ii) The statistical organization and functions of the United Nations secretariat.

(iii) The disposition of statistical activities conducted by the League of Nations.

(iv) The general character of statistical relationships between the United Nations and Specialized Agencies.

(v) The general character of the statistical relationships between the United Nations and other organizations of a quasi-governmental or non-governmental character, including those organs on a regional basis.

All these topics gave rise to considerable discussion, but the Commission itself was able to present to the Economic and Social Council a unanimous report which, except for one or two points, was accepted by the Council.

Terms of Reference of the Statistical Commission

The terms of reference of the new permanent Statistical Commission were agreed:

"The Statistical Commission be charged with the task of—

(*a*) promoting the development of national statistics and improving their comparability.

(*b*) the co-ordination of statistical work of Specialized Agencies.

(*c*) the development of the central statistical services of the Secretariat.

(*d*) advising the organs of the United Nations on general questions relating to the collection, interpretation and dissemination of statistical information.

(*e*) promoting the improvement of statistics and statistical methods generally."

It is important to note that in these terms of reference nothing was said about the International Convention on Economic Statistics, and the powers and functions of the Statistical Commission were henceforward to be derived directly from the authority of the United Nations itself, and not from the International Convention as the Committee of Experts of the League of Nations had done.

Since the Statistical Commission was to be a technical body, it was suggested by the temporary Commission that the size should be restricted to twelve in number, and that the members should serve in their personal capacity and not as official government representatives. The Economic and Social Council agreed to this limitation of the size of the Commission to twelve members (a smaller number than for most other Commissions), but did not accept the proposal that the members of the Commission should act in their personal capacities and decided instead that they should act as official government representatives.

An arrangement was made, again based on the experience of the League of Nations, that the Council might appoint, "in their individual capacities, not more than twelve corresponding members from countries not represented on the Commission." This arrangement was designed to help to secure continuity in the work of the Commission over the years, as existing members had to retire from the Commission.

The experience of the League of Nations had shown how important it was that there should be a strong statistical organization within the United Nations. It was agreed that a central statistical unit should be organized within the Secretariat. Although for administrative reasons the Statistical Office of the United Nations was to be placed within the Department of Economic Affairs, the Statistical Office of the United Nations was established to serve all Departments and organs of the United Nations.

The first and most important step to be taken by the United Nations was to make arrangements to carry on without interruption the valuable statistical work of the Economic, Financial and Transit Department of the League of Nations and other statistical activities carried out under the sponsorship of the League. Arrangements were made to transfer the statistical records of the League of Nations from Princeton and Geneva to the United Nations in Lake Success, so that as from the beginning of 1947 the *Monthly Bulletin of Statistics* and the various other publications of the League could be continued and adapted to meet post-war needs.

Some difficulty arose, however, on the question of taking over the legal powers under the International Convention on Economic Statistics and the functions of the Committee of Experts established under the Convention. The Statistical Commission was of the opinion that the functions of this Committee should be taken over as rapidly as possible by the United Nations. On the other hand, the Statistical Commission was anxious to place on record "its understanding that transfer of responsibilities under the International Convention of the League of Nations to the Statistical Commission should in no way be construed as limiting functions and responsibility given to the Commission by the Economic and Social Council, or as modifying or impairing the respective obligations and duties of the United Nations or the specialized agencies on the agreements existing between them."

The Statistical Commission was equally anxious that the clauses which appeared to limit the range of subjects which could be discussed by the Committee of Statistical Experts should not be used to restrict the future work of the Statistical Commission. All that was intended was that the responsibilities of the contracting parties to the League of Nations under the International Convention should be transferred to the United Nations as part of the legal procedure of transferring responsibilities between the League of Nations and the United Nations. It would retain in being the International Convention for which the League of Nations and the International Statistical Institute had striven so hard in the 1920's. This transfer of activities, after much legal and political argument, was achieved at the General Assembly in Paris in November, 1948, and the International Convention still exists.

Relationship between the United Nations, the Specialized Agencies and Other Organizations

Before the war there were only two principal specialized organizations working on statistics in association with the League of Nations—the International Labour Organization and the International Institute of Agriculture. The League of Nations and the International Labour Organization were in Geneva, and the International Institute of Agriculture in Rome, so that the working relations between them were relatively easy. It was now proposed that there should be at least ten Specialized Agencies working in association with the United Nations:

International Labour Organization.
Food and Agriculture Organization.
International Civil Aviation Organization.
United Nations Educational, Scientific and Cultural Organization.
International Bank for Reconstruction and Development.
International Monetary Fund.
World Health Organization.
International Refugee Organization.
Universal Postal Union.
International Telecommunications Union.

And there were plans also for the—

International Trade Organization, and
United Nations Maritime Organization.

Not only was the co-ordination of statistical work between the United Nations and each of these bodies necessary, but arrangements were needed to prevent duplication of statistical activities between the Specialized Agencies themselves. To take one example, not only was the United Nations interested in international trade statistics, but the International Monetary Fund was concerned so far as such figures were needed for estimates of balance of payments; the Food and Agriculture Organization wanted figures of imports and exports of agricultural products; and if the International Trade Organization were set up later, trade figures would be wanted for the day-to-day work of that organization. The problem of securing effective working relations between these organizations was likely to be the more difficult because some of the organizations were to be in North America and others in Europe—thousands of miles apart.

The co-ordination of statistical work between the Specialized Agencies and the United Nations is a continuing function, and cannot be achieved only by occasional meetings of the Statistical Commission, which is essentially an advisory body meeting at intervals of a year. It was agreed that effective working relations could be secured only by—

(i) A review at each annual meeting of the Statistical Commission of the work being done by each of the Specialized Agencies and of their programmes for future work.

(ii) A Consultative Committee on Statistical Matters, on which the heads of the statistical branches of the Specialized Agencies and the United Nations would meet regularly to discuss co-ordination of their statistical activities.

The Statistical Commission gave general guidance to the Economic and Social Council on the kind of relationships which should exist in the formal agreements to be established between the United Nations and the Specialized Agencies. The agreement concluded between the International Labour Organization and the United Nations is typical of similar clauses introduced into agreements with the other Specialized Agencies:

"1. The United Nations and the International Labour Organization agree to strive for maximum co-operation, the elimination of all undesirable duplication between them, and the most efficient use of their technical personnel in their respective collection, analysis, publication and dissemination of statistical information. They agree to combine their efforts to secure the greatest possible usefulness and utilization of statistical information and to minimize the burdens placed upon national governments and other organizations from which such information may be collected.

"2. The International Labour Organization recognizes the United Nations as the central agency for the collection, analysis, publication, standardization and improvement of statistics serving the general purposes of international organizations.

"3. The United Nations recognizes the International Labour Organization as the appropriate agency for the collection, analysis, publication, standardization and improvement of statistics within its special sphere, without prejudice to the right of the United Nations to concern itself with such statistics so far as they may be essential for its own purposes or for the improvement of statistics throughout the world.

"4. The United Nations shall develop administrative instruments and procedures through which effective statistical co-operation may be secured between the United Nations and the agencies brought into relationship with it.

"5. It is recognized as desirable that the collection of statistical information should not be duplicated by the United Nations or any of the specialized agencies whenever it is practicable for any of them to utilize information or materials which another may have available.

"6. In order to build up a central collection of statistical information for general use, it is agreed that data supplied to the International Labour Organization for incorporation in its basic statistical series or special reports should, so far as practicable, be made available to the United Nations."

The Statistical Commission had also to consider the relationship between itself and other commissions of the Economic and Social Council and of the regional organizations which might be established under the authority of the United Nations in Europe, Asia and the Far East, the Middle East and in Central and Latin America. The Statistical Commission realized that, while the work must be shared among different Commissions, it must be itself the central body on

statistical matters. The Statistical Commission had the general responsibility to survey and make proposals on the whole field of international statistics.

An important development within the organization of the United Nations itself was the establishment in 1946 of the Population Commission. This new Commission had been suggested in the report of the Preparatory Commission which considered the organization of the United Nations in 1945 and 1946, but the Population Commission was set up after the temporary Statistical Commission had met and made its report to the Economic and Social Council. Since much of the work of the Population Commission was likely to be of a statistical nature, arrangements were made for a representative of the Statistical Commission to sit on the Population Commission. There was also some overlap of membership between the two commissions.

In its report to the Economic and Social Council the temporary Statistical Commission hoped that appropriate means could be devised to bring the International Statistical Institute into harmonious and mutually advantageous relationship with the United Nations. There was no suggestion however that the International Statistical Institute should be represented officially on the Statistical Commission and, in fact, it could not be, once the Council decided that the membership of the Commission should consist of representatives of governments. Any decision of what the relationship with the International Statistical Institute should be was deferred until the first session of the Institute, which had been arranged for September 1947.

The Permanent Statistical Commission of the United Nations

The first meeting of the Permanent Statistical Commission took place in January, 1947. The membership of the Commission included representatives of the following countries:

Canada, China, France, India, Mexico, Norway, Netherlands, Turkey, Ukraine, United Kingdom, U.S.S.R., United States.

The second session was in August, 1947, and a third session in April, 1948. Henceforward the sessions of the Statistical Commission will be once a year—in the same way as the Committee of Statistical Experts of the League of Nations.

The first two sessions of the Statistical Commission in 1947 were perforce largely devoted—as the session of the Temporary Commission had been—to questions of organization. An increasing proportion of the work of the Statistical Commission was spent in the second and third sessions on the study of the collection and compilation of different classes of statistics.

The Statistical Commission realized that because of the break in the collection of statistical data during the war years, many countries could now consider making changes in the compilation of their statistics, since it would no longer involve a serious loss of continuity. There was, therefore, a unique occasion to persuade as many countries as possible to introduce more up-to-date methods and, as far as possible, to compile them in such a way as to make international comparison easier.

1. *Collection and Publication of International Statistics*

The Statistical Office of the United Nations has taken over the publication of the *Monthly Bulletin of Statistics*, previously issued by the League of Nations. New series have been introduced and new information collected from countries. During the war it was not possible for the League of Nations to maintain close relation with member countries, and the statistical data for past years have been re-submitted to countries for checking and revision. In addition, a Supplement has been issued to the *Monthly Bulletin*, giving the definitions and descriptions of the series submitted by member countries. Arrangements are now working satisfactorily by which more than 55 countries send each month to Lake Success by cable or air mail letter the latest figures for their countries on economic and social conditions.

Arrangements have been made for the publication of a Statistical Year Book, and the series for the different countries are now being checked and revised.

2. *Industrial Classification*

In 1938 the Committee of Statistical Experts of the League of Nations recommended an industrial classification for international use. The experience of the war years showed that some changes were necessary in this classification. Since many countries were proposing to take

censuses of production in 1948 and following years, and also censuses of population in the years about 1950, intensive work was done to bring up-to-date the industrial classification previously recommended by the League of Nations.

A committee of Experts was appointed to study various proposals. A draft was sent to Governments for comment, and on the basis of the replies received a new classification was proposed. This new Industrial Classification for international use was adopted by the Statistical Commission in May, 1948, and by the Economic and Social Council two months later.

3. *Minimum List of Commodities for International Trade Statistics*

Before the war the Committee of Experts of the League of Nations prepared a Minimum List of Commodities entering into international trade. This list has been re-examined by the Statistical Commission, and a revised draft, based on suggestions made by different countries, is at present being prepared.

4. *Statistics of Occupations and of Personal Status*

In view of the forthcoming censuses of population in many countries, a Committee of Experts of the International Labour Office and the Statistical Commission of the United Nations have been reconsidering the question of whether it is possible to devise a classification of occupations for international use. Certain proposals were made by a special conference of labour statisticians called by the International Labour Organization in April 1948, and their proposals will be discussed again by the Statistical Commission in 1949.

5. *World Census of Population*

It has always been a hope that all countries would take a census of population at about the same time, so that there would be in effect a world census of population. The Population Commission and the Statistical Commission have been working together to try to achieve this for the years 1950/51. A special committee was set up by the Inter-American Statistical Institute to make arrangements for a census of all the Americas in 1950 and, in addition, many countries both in Europe, Asia and Africa have agreed to take censuses about that year. Already thirty countries have agreed to take censuses in 1950–51. The Population and Statistical Commissions have discussed what might be the minimum list of questions to be asked in such censuses, and have recommended this list for consideration by countries taking censuses. They have also suggested definitions to be used in asking these questions.

6. *World Census of Agriculture*

As well as the world census of population it is hoped to persuade as many countries as possible to take censuses of agriculture in 1950. The Food and Agriculture Organization is making every effort to secure this. Special regional centres have been set up to provide training facilities so that censuses of agriculture may be undertaken on comparable lines in as many countries as possible in 1950.

7. *National Income*

In view of the great developments in the preparation of national income estimates since before the war, one of the first tasks of the Statistical Commission was to recommend that the Statistical Office of the United Nations should collect and publish the figures of national incomes of as many countries as possible, and indicate how far the figures differ because of methods of compilation. This important study has now been completed.

8. *Demographic Yearbook*

With the creation of the Population Commission, considerable attention has been devoted to the collection of demographic data for different countries. Before the war the work on the collection of demographic data was shared between the International Statistical Institute and the League of Nations. The draft tables of the first Demographic Yearbook have now been approved, and returns are being made by different countries to the secretariat of the United Nations. It is hoped that the first Demographic Yearbook will be published in 1949.

Apart from this work, which is now nearing completion, preparatory work is being undertaken under the sponsorship of the Statistical Commission of a number of other subjects:

(i) *Index numbers of prices.*—The Committee of Statistical Experts of the League of Nations undertook work on studies of index numbers of prices and of their relationships. This work is now to be taken up again in more detail.

(ii) *Index numbers of production.*—Many countries have now had experience in trying to use the methods originally proposed by the Committee of Experts of the League in compiling index numbers of production. Some difficulties have been found in carrying out the recommendations and it is proposed to make a report on the experience of different countries and to issue a report.

(iii) *Capital formation.*—Many countries during recent years have attempted to compile figures of gross and net investment, and have had experience in collecting the basic material for use in compiling such estimates. It is felt that the time has come to pool the experience of different countries on this subject.

Reference has already been made to the collaboration between the Specialized Agencies and the Statistical Commission of the United Nations on statistical questions during the last two years. The International Monetary Fund has made a study of the methods for preparing estimates of balance of payments. Following a meeting of experts in September 1947, it has issued a manual for the collection of balance of payments estimates. The World Health Organization, working with the other interested international organizations, has carried through a revision of the International List of Causes of Death and, further, has incorporated in it a classification to cover all diseases and injuries, fatal and non-fatal.

Sub-Commission on Statistical Sampling

At the meeting of the temporary Statistical Commission in May 1946, it was realized that the needs for statistics by national governments and by the international organizations were likely to be much greater than before the war. It was suggested that Governments should be advised to consider how far sampling methods could be used in the collection of basic data. Great progress had been made during the last 20 years in the development of sampling methods.

For this purpose it was decided to set up a Sub-Commission on Statistical Sampling. The members serve in their personal capacities. The present members of the Sub-Commission are:

P. C. Mahalanobis, India; G. Darmois, France; W. E. Deming, United States; F. Yates, United Kingdom; and Professor R. A. Fisher, Consultant to the Sub-Commission.

The terms of reference of this Sub-Commission are:

"(*a*) Give special consideration to the use of statistical sampling methods in meeting the actual needs of the United Nations, the specialized agencies and non-governmental organizations brought into consultative status with the United Nations with a view to filling gaps in the information needed by them.

"(*b*) Make recommendations regarding the use of sampling methods in conjunction with an attempted complete enumeration in order to assess the reliability of that enumeration, or to obtain early returns and supplementary information.

"(*c*) Give special consideration to those circumstances which make sampling methods preferable to complete enumeration in respect to reliability, speed, cost, continuity and other factors.

"(*d*) Examine the use which is being made of statistical sampling in different countries and in different fields of subject-matter.

"(*e*) Promote the improvement of methodology in statistical sampling and its applications."

The Sub-Commission has so far held two meetings, and has considered sampling methods to be used in the 1950 world census of agriculture, and the use of sampling in censuses of population, and in family budget enquiries. Mr. Deming, on behalf of the Sub-Commission, has prepared a memorandum giving "Recommendations concerning the preparation of reports of sampling surveys." The Sub-Commission is acting as a professional technical advisory body on sampling methods not only to the United Nations, but also to the Specialized Agencies.

International Statistical Conferences in September 1947

In view of the great changes which have been made in the organization of collection of official statistics and the repercussions which these developments had on the relations with the

International Statistical Institute and other scientific organizations, it was felt that an opportunity should be given to the statisticians of different countries, between whom contact had broken down during the war, to meet together again and hear at first hand of the plans and programmes of the various organizations.

The International Statistical Institute had originally intended to hold a meeting in Washington, D.C., in 1939, but this session was postponed because of the war. The session was then arranged for September, 1947, and the opportunity was taken to hold at the same time in Washington a series of meetings of other international statistical organizations. The meetings arranged in Washington in September 1947, included:

1. The United Nations World Statistical Congress, sponsored by the Economic and Social Council of the United Nations, in which the work of the United Nations, the Statistical Commission and the Specialized Agencies was described and discussed;

2. The first post-war session of the International Statistical Institute to discuss its future role of work;

and meetings of

3. The Inter-American Statistical Institute;

4. The Econometric Society;

5. The International Union for the Scientific Investigation of Population Problems;

6. The International Association for Research into Income and Wealth. (This new association was formed following discussions at the Conference on Research and Wealth in New York in December, 1946.)

In view of the experience of the international statistical congresses which were held from 1853 to 1876, the World Statistical Congress in Washington was organized only for the purpose of general information and exchange of views between statisticians of different countries. No attempt was made to pass resolutions or recommendations on statistical matters. The meetings were attended by more than 600 persons, representative of many countries. The success of the Conferences owed a great deal to the United States Arrangements Committee, of which S. A. Rice was Chairman. In Appendix II to this paper a list of the papers read at the Conference is given. The Arrangements Committee hope to publish in 1949 a complete set of these papers.

Revised Statutes of the International Statistical Institute

Perhaps the most pressing reason for the meetings in Washington was to review the role of the future work of the International Statistical Institute.

For various reasons it was only possible for 45 members—less than one-third of the surviving members of the International Statistical Institute to attend. The membership of the Institute had fallen in recent years, because it had not been possible to elect any new members to the Institute since 1938, and a number of members had died.

Unfortunately, M. Julin, the Belgian President of the International Statistical Institute, was not able to attend the meeting in Washington, but the report he submitted covering the years 1938 to 1947 stated clearly the decisions which the Institute had to face:

> "From time to time every organization should review the bases on which it rests and the means by which it may fulfil its purposes. An institution such as ours should have a dynamic quality able to adapt itself constantly to the exigencies of the times while maintaining its fundamental character of an autonomous international academy.
>
> "One cannot deny that after the troubled period through which we have passed, the conditions under which international scientific collaboration operates have been profoundly modified. The creation of the United Nations, whose sphere of activity also includes statistics, has forced us to consider the position of the Institute in relation to that of the United Nations in the same field.
>
> "A similar situation, although in a different form, arose after the first world war, with the creation of the League of Nations. Events later proved that the problem of mutual relations such as the avoiding of overlapping functions had not been satisfactorily resolved at that time.

". . . I will mention the creation of international organizations, created either before the war or projected for the future, which are concerned with fields in which statistical analysis is an essential technique: demography, econometrics, mathematical statistics, national income, public opinion measurement.

"The formation of such organizations, regional on the one hand and specialized on the other, expects a tendency to decentralization, 'horizontal' as well as 'vertical,' in the international organization of statistics."

At the meeting of the Institute a report was submitted by an Advisory Committee on revision of statutes. It was proposed that the International Statistical Institute should not attempt to retain its position "as a semi-governmental instrument for the collection of international statistics but should become a voluntary and scientific society"—to quote M. Julin's words, "an international statistical academy." "It would be close to national governments and inter-governmental bodies but beyond their domination. It would serve and abet their efforts towards international statistical progress but it will not exercise their official functions." These words are not very different from those written by Sir Rawson W. Rawson to the German Statisticians in 1885 when the Institute was established.

It was proposed at the meeting in Washington that the new terms of reference of the International Statistical Institute should be:

"The International Statistical Institute is an autonomous society devoted to the development and improvement of statistical methods and their application throughout the world, in particular—

"(*a*) By encouraging the international association of statisticians, the exchange among them of professional knowledge, and the growth among them of a collective interest in the advancement of such knowledge.

"(*b*) By aiding in the establishment of such relations among statistical societies and other official and unofficial organizations having statistical interests as will further the international integration of statistics.

"(*c*) By establishing and maintaining professorships, lectureships, and fellowships for advanced studies in statistics.

"(*d*) By promoting the training of competent statisticians.

"(*e*) By studying statistical theories, appraising statistical methods and practices, encouraging statistical research, and furthering the use of statistical methods in diverse subject-matter fields wherever useful.

"(*f*) By promoting the use in all countries of the most appropriate statistical methods.

"(*g*) By furthering international comparability of statistical data.

"(*h*) By fostering public appreciation of sound statistical practice and the usefulness of statistical methods."

After the Washington session, the new Statutes of the Institute were submitted to a vote of the whole membership by mail. They became effective in July 1948.

Apart from the conferences arranged by the International organizations themselves, a number of meetings have taken place since the war between representatives of different countries to discuss statistical questions.

The regular meetings between the statisticians of the Scandinavian countries have been resumed. The first meeting after the war was held in Sweden in 1946, and a further meeting was held in Iceland in 1948, when there was a discussion on plans for the population and agricultural censuses in 1950, the form of "national accounts," and the contents of the international tables which appear regularly in the statistical yearbooks of the Scandinavian countries.

Partly as a result of the Customs Agreement between Belgium, Holland and Luxembourg, which was drawn up in London in September 1944, steps have been taken to achieve closer co-ordination of statistics of the three countries. In 1946 and 1947 a number of meetings were held, and the work by the co-ordinating committee is of great interest in showing the problems arising in combining the statistical procedures of the three countries. Among those topics discussed have been whether gross or net weights of imports and exports should be used, the determination of import values, what constitutes transit trade, whether imports and exports of gas, electric power and drinking water should be included in foreign trade statistics. Considerable attention has also

been paid to achieving comparability in the methods of construction of index numbers of wholesale and retail prices. A test has been made to compare the absolute price levels in the three countries by having the same goods priced in the three countries on the same date. Samples of Dutch articles, mainly textiles and small household articles, were circulated in the three countries in order to ensure comparability. Discussion has also taken place on the methods of compiling figures showing changes in wage rates and earnings.

Under the sponsorship of the Special Commissioner for South-East Asia, a conference of statisticians of the countries in South-East Asia was held in Singapore in January, 1948. The purpose of the meeting was to try to improve the methods of collecting statistics of rice and other food crops, and also to discuss the difficulties experienced in compiling figures of changes in retail prices.

III

I started this paper by setting out what I understood to be the main reasons for international collaboration on statistical matters. I have tried to describe the kind of organization which has been built up to achieve this—an organization which has been fashioned by the successes and failures of the last 100 years. There are now to be two focal bodies; the Statistical Commission of the United Nations, responsible for co-ordinating the statistical work on the official level between governments, and between the United Nations and its associated organs; and the International Statistical Institute, "an autonomous international statistical academy" with affiliations with other scientific organizations and learned societies responsible for the development and the study of statistical methods and for furthering the interests of statisticians throughout the world. The division of functions between the Statistical Commission of the United Nations and the International Statistical Institute is much clearer than it was between the League of Nations and the International Statistical Institute; but it can never be precise, since it is impossible to draw a clear line between responsibility for the discussion of statistical methods as such and their application in practice by governments.

Work of the Statistical Commission of the United Nations

I hope, and believe, that the present organization as now devised by the United Nations and its associated organizations will succeed. Paradoxically, it suffers from being too burdened by formal agreements between the United Nations and the Specialized Agencies and between the Specialized Agencies themselves, while at the same time it is too loose in so far as it is not proof against possible "empire building" in which any one of the organizations may engage. It should be realized that since most countries, in contrast to the United Kingdom and the United States, have one central bureau responsible for the collection and publication of most of their statistics, it is impossible to hide from the statistical offices of national governments any possible duplication in the work being done by the United Nations and the Specialized Agencies.

At present there is a spirit of co-operation between the members of the secretariat of the different international organizations which augurs well for the future. During the last two years it has been possible to secure satisfactory working arrangements between statisticians of different countries and between the statisticians of the international organizations. In spite of the fears which some may have had, the meetings of the Statistical Commission have been very successful indeed and, except on two matters which had political implications—the transfer of powers under the International Convention relating to Economic Statistics from the League of Nations to the United Nations, and the recommendation by the United Nations for the adoption of an an International Standard Industrial Classification—all their recommendations have been unanimous.

The Statistical Commission of the United Nations has resumed the work of the Committee of Experts of the League of Nations in the development of standards for various kinds of economic statistics, but it has been asked also to see what can be done to help in the improvement of statistics in the countries where statistics are at present relatively undeveloped. For example, a survey recently undertaken by the Regional Commission of the United Nations for Asia and the Far East showed that in many Asiatic countries many basic statistics about the economies of these

countries were lacking. The Secretary-General of the United Nations stressed the need to help these countries when he opened the World Statistical Congress in Washington in September 1947:

> "It is shocking to realize that no statistics worthy of the name exist for probably half of the world's peoples. No one knows exactly how many people there are, how they make their living, or the characteristics of their social, economic and political institutions.
>
> "There is ignorance about the economic and social resources available for the development of adequate and dignified standards of living, ignorance about the most elemental facts that would highlight the condition of the society and point to possibilities of improvement.
>
> "I do not mean to imply that those concerned are unaware of the usefulness of statistics. I only want to emphasize the tremendous size of the task which must be undertaken in your field if the world is to reap the advantages of concerted action upon important common problems.
>
> "It is obvious, of course, that an adequate statistical system is to a high degree dependent on the political and economic organization within a given country. Nevertheless, the development of statistics in the areas where they are lacking is one of the urgent tasks to which we all must give our best attention."

The provision of expert assistance to these countries is a matter which will be discussed at the next meeting of the Statistical Commission. It is not merely a question of introducing those statistical methods which have been found suitable for European or American countries, but of considering the best methods of collecting basic statistics relating to population, agricultural and industrial output, prices, and labour conditions in the countries of Asia, Africa, Central and South America, whose economies are very different from those of Europe and North America.

International Statistical Institute

It is too early to say what kind of work the International Statistical Institute itself will undertake in the future, since only one meeting has been held since the war, but the papers read at the meeting held in Washington show the wide range of subjects in which it is interested.

One of the most important topics to be discussed at its next meeting in Berne in September 1949, will be what can be done internationally to provide more facilities for education and training in statistics.

Future Work

In this paper I have deliberately restricted myself to describing the organizations which have been built up to achieve international collaboration on statistical matters. The test whether these organizations will succeed is not merely whether they will work smoothly as administrative bodies, but whether they tackle seriously the many questions which statisticians throughout the world are debating amongst themselves and on which they would like to come to a common view. The chances of getting agreement internationally on, for example, what is "a family" or even "a house" may be remote, but there are many questions on which agreement could be achieved.

Labour Force

There are still differences in the definitions adopted by different countries for the gainfully occupied in the censuses of population and in estimates of changes in the labour force. Most countries agree to include the armed forces, the unemployed and those working in institutions (such as prisons, hospitals and labour camps), though there are minor differences even here between the practice of different countries. On the other hand, a great deal has still to be done before agreement can be reached on how best to treat unpaid family workers, particularly in agriculture and also domestic workers. These are problems which need to be discussed if comparability is to be achieved between the censuses of population of different countries in 1950/51.

Occupations

The International Labour Office is taking steps to devise an occupational classification which could possibly be adopted for international use. In view of the great differences in the economies of countries, it is very doubtful how far this can be achieved. The Committee of Experts of the

League of Nations was reluctant to tackle this particular question, and expressed the view that this was a question of national, rather than of international, concern. In practice, classification "by occupation" tends to get confused with classification by "personal status," and even the usual broad groups of classification by personal status, namely, "employers," "workers on own account" and "persons in receipt of wages and salaries," are becoming difficult to follow in many countries.

Migration

Figures of migration are still in a very relatively primitive state, although there has been much discussion on how these should be compiled. Inevitably, the figures of immigration do not agree with those of emigration, and the figures of movements of population between countries are hard to reconcile. In addition, at the moment there are particular difficulties relating to "stateless" persons. These raise difficulties in taking censuses of population when questions relating to nationality, legal status and birth places are asked. There are proposals for a standard statistical form for international use for recording particulars of migrants.

Vital Statistics

The new international list of causes of death and morbidity has to be tested nationally on a wide scale. Many countries still experience great difficulty in the collection of vital statistics, and registrations of births, marriages and deaths are known to be incomplete. There is need for a study of the registration systems in different countries. Some countries are experimenting with methods of linking up records of births, marriages and deaths where they are kept separately.

National Income

During the last ten years a great amount of work has been done in compiling estimates of national income for different countries, and there is now almost international agreement on the definitions to be used in estimates of net national income at factor cost and gross national product. There are still important problems to be solved in finding the right methods of converting these estimates into a common currency, and there are still considerable differences in the methods used in estimating real income over time and in the methods used in preparing social accounts for different sectors of the economies.

Index Numbers of Production

Most countries are now compiling index numbers of production regularly, and the main differences between them appear to be in how far they cover all branches of manufacturing, building and service trades. There is no agreement as to how far the formulae used in compiling index numbers of production should be different from those used to measure changes in real national income over time, or what is the most satisfactory classification to be used in subdividing between capital and consumer goods, or any other alternative classification.

International Trade

Most countries have resumed their collection of figures of external trade, and the main outstanding question is how far agreement can be reached on a Minimum List of Commodities entering into international trade. Special problems of recording imports and exports are being created by the development of air traffic, and also by the growth of trade by parcel post, which is generally not analysed in full detail in the export statistics of the different countries.

Capital Formation

Many countries are experimenting for the first time in collecting figures of capital formation with varying success. Two methods are being employed; the first uses returns made by producers of the output of capital goods; the second uses returns of expenditure made by those acquiring capital goods. The need for statistics of capital formation is becoming important as more attention is given in different countries to the development of long-term capital investment programmes.

So far the results obtained by the two methods, even for the same country, are often hard to reconcile, and there is need for exchanging ideas on the best methods of collecting figures of capital formation. No satisfactory attempt has yet been made to match the estimates of capital formation obtained with estimates of national capital.

Balance of Payments

The International Monetary Fund has been devoting considerable attention to devising a model schedule for balance of payments. In practice it is very difficult often to distinguish between movements of funds on "capital account" and on "current account." More might be done to exchange information between countries on "invisible items" such as tourist expenditure.

Use of Sampling Methods

There is urgent need for an international study of the application of sampling methods in different subject fields made by different countries, and it is hoped that the Sampling Sub-Commission of the United Nations will assemble and analyse the experiences of different countries. In many countries the only way to obtain reliable basic statistics may be by sample surveys.

I could add more to this list of questions which need to be discussed internationally between statisticians of different countries, and other Fellows of this Society could add more. If progress is to be made in tackling these questions, all the help which statisticians in countries all over the world can give will be needed. International statistics cannot be improved by the efforts of a few individuals. The work done by statisticians, whether at meetings of the Statistical Commission, the International Statistical Institute or other international organizations cannot fail to be of benefit to all for the reasons which the Prince Consort so well expressed at the opening of the International Statistical Congress in London in 1860:

> "It is the social condition of mankind, as exhibited by those facts, which forms the chief object of the study and investigation undertaken by this Congress; and it hopes that the results of its labours will afford to the statesman and legislator a sure guide in his endeavours to promote social development and happiness. The importance of these international Congresses in this respect cannot be overrated. They not only awaken public attention to the value of these pursuits, bring together men of all countries who devote their lives to them, and who are thus enabled to exchange their thoughts and varied experiences; but they pave the way to an agreement among different Governments and nations to follow up these common inquiries, in a common spirit, by a common method, and for a common end."

APPENDIX I

International Convention Relating to Economic Statistics, 1928

Article 2 of the Convention asks the High Contracting Parties to compile and publish the following classes of economic statistics at the intervals stated subject to certain reservations:

"I. *External Trade*

"(*a*) Annual and monthly returns of the quantity and value of imports and exports.

"(*b*) Annual, and if possible quarterly or preferably monthly, returns showing the net tonnage of vessels engaged in external trade entered at and cleared from the ports of the country concerned, according to their nationalities.

"II. *Occupations*

"Returns of the population according to occupations to be compiled and published at least once in each decade, and to relate to the closing year of the decade, or to a year as near as possible to such closing year.

"III. *Agriculture, Live-Stock, Forestry and Fisheries*

"(A) General censuses of agriculture, to be taken if possible once in each decade, on the lines proposed, and, if possible, for the year proposed by the International Institute of Agriculture.

"(B) Annual returns showing:

"(1) The distribution of the cultivated area amongst the principal crops, stating if possible, in cases where such information is of importance, both the areas sown or planted and the areas harvested; and

"(2) The quantities of such crops harvested.

"(C) Periodical (if possible annual) returns of the numbers of the chief species of live-stock, showing sex and age where possible.

"(D) In the case of countries to whose economy timber production is important, periodical returns of forest resources, showing forest area and, whenever possible, timber content, annual growth and annual cut; distinction to be made as far as possible according to the species of the timber.

"(E) In the case of countries in which fisheries are an important and organized branch of economic activity, annual returns showing: (1) the products of the main sea fisheries landed and, if possible, of inland-water fisheries; (2) the nationalities of the boats by which such products are landed; (3) the numbers and classes of national boats engaged in fisheries; and (4) the number of persons employed on such boats.

"Whenever it is not possible to render complete returns, an approximate indication of the extent to which such returns are defective shall be given.

"IV. *Mining and Metallurgy*

"Returns (at least annual) of the quantities produced by any of the minerals and metals mentioned below, the production of which in the country concerned is of national importance:

"(1) Non-Metallic Minerals:

Coal (bituminous or anthracite), lignite and coke.	Phosphates.
	Potash minerals.
Petroleum and natural gas.	Sulphur.
Nitrates.	

"(2) Metallic Minerals and Metals:

"(*a*) Ores of the following:

Iron.	Aluminium.	Tin.	Manganese.
Copper.	Lead.	Zinc.	Nickel.

"(*b*) Smelter production (actual or estimated) of the following:

Iron and steel.	Tin.	Antimony.	Silver.
Copper.	Zinc.	Tungsten.	Gold.
Aluminium.	Manganese.	Molybdenum.	Platinum.
Lead.	Nickel.	Bismuth.	

"V. *Industry*

"(A) Statistical surveys at regular intervals, and if possible at least once in every ten years, of:

(*a*) Industrial establishments, including at least all such establishments of any considerable importance; and

(*b*) If possible, commercial establishments.

"Such surveys may be carried out in connection with a census of population or with a census of industrial production or independently, and show show, inter alia:

"(1) With regard to such establishments, the number of persons of each sex employed therein; and, so far as possible, such persons according to categories of their employment and distinguishing adults from young persons, the age at which this distinction is made being stated

"An estimate shall also be made, if possible, of the numbers of persons employed in establishments which are not included in the surveys.

"(2) In the case of industrial establishments, the nominal capacity of the prime movers (if any) installed, distinguishing, if possible, between (i) steam engines, (ii) internal-combustion engines, and (iii) hydraulic engines; and the nominal capacity of the electric motors installed (if any), indicating whether the electric energy is generated in the establishment or elsewhere. In each of the above-mentioned classes, prime movers and electric motors normally in use should, if possible, be shown separately from those idle or in reserve.

"(B) Returns of industrial production, as comprehensive as it may be possible in the case of each country to furnish with a sufficient degree of accuracy.

"(C) Statistical series in the form either of absolute figures or relative figures referring to a period taken as a basis of comparison, at regular intervals, if possible quarterly or preferably monthly, showing the variations of the industrial activity of the most representative branches of production.

"VI. *Index Numbers of Prices*

"Index numbers:

"(*a*) Showing the general movement of wholesale prices, to be compiled and published monthly, and

(*b*) Showing the general movement of the cost of living, to be compiled and published at least quarterly.

The cost-of-living indices may be based on data relating to a single town or to several towns selected as being representative and taken either separately or collectively.

"Each statement of index numbers mentioned above shall contain a reference to a short official publication showing the items the prices of which have been used, and the methods employed in the calculation of the indices.

"In addition to the indices mentioned above, the wholesale prices, in absolute or relative form, of the principal individual commodities shall, so far as practicable, be published for the same periods."

APPENDIX II

LIST OF PAPERS PRESENTED AT THE INTERNATIONAL CONFERENCES HELD IN WASHINGTON, D.C., IN SEPTEMBER, 1947

(Papers presented jointly to the International Statistical Institute and the Econometric Society and the Inter-American Statistical Institute are shown below under the programme of the International Statistical Institute. The titles of the papers may be altered before they are published.)

WORLD STATISTICAL CONGRESS

Statistical Activities of the United Nations

The Work and Programme of the Statistical Office of the United Nations, by A. D. K. Owen.
The Role of the United Nations Statistical Commission, by Herbert Marshall.

Statistical Activities of the Specialized Agencies

International Labour Organization, by Robert M. Woodbury.
United Nations Educational, Scientific and Cultural Organization, by Theodore Besterman.
Food and Agriculture Organization, by Valentino Doré.
International Civil Aviation Organization, by A. M. Lester.
World Health Organization, by Knud Stowman.
International Bank for Reconstruction and Development, by J. S. Gould.
International Monetary Fund, by J. J. Polak.

Recent Developments in Statistical Activities of National Governments

Recent Developments in British Official Statistics, by J. Stafford.
State Statistical Services in Poland, by Stefan A. Szulc.
The Independently Controlled Statistical System of the Chinese Government, by Jennings P. Chu.
Co-ordination of National, State and Municipal Statistics through the Brazilian Institute of Geography and Statistics, by Arranio de Carvalho.
Recent Developments in the Statistical Work of Turkey, by Sefik Inan.
Recent Developments in the Statistics of Czechoslovakia, by F. Fajfr.
Recent Developments in Canadian Statistics, by Herbert Marshall.
Planning Economic Statistics, with Special Reference to a National System of Accounts, by Arne Skaug.
Industrial Statistics of Czechoslovakia: Scope and Method, by Karel Malik.
Progress in the Training of Statisticians in France: The Work of the School of Applied Studies of the National Institute of Statistics and Economic Studies, by F. Louis Closon.
Measurement of Agricultural Output in Denmark, by Einar Cohn.
Statistics as a Means of Improving the Educational System, by Philip J. Idenburg.
A Census of Nine Hsien in Szechwan, China, by Kuo-Pao King.
The Economic Statistics of the Lebanon, by Mustafa Nsouli.

Activities of Secretariat of United Nations

The Development of International Comparability of Statistics, by William R. Leonard.
The Development of International Demographic Statistics, by Forrest E. Linder.
Analysis of International Population Problems, by Frank Notestein.
The Comparability of National Income Statistics, by J. B. D. Derksen.

INTERNATIONAL STATISTICAL INSTITUTE

General

Report of the President of the Institute, 1936–38, submitted on behalf of A. Julin.
Report of the President of the Institute, 1938–47, read on behalf of A. Julin.
Report of Advisory Committee on Revision of Statutes, submitted by Stuart A. Rice.
Other Questions Referred to the General Assembly by the Advisory Committee, by Gunnar Jahn.
A Future Role for the International Statistical Institute in International Statistics, by Stuart A. Rice.

Statistical Methodology—General

Analysis of Probability Relationships, by G. Darmois.
A New Method of Factor Analysis, by Pierre J. Delaporte.
The Means of Samples, by Corrado Gini.
On Statistical Relations and Their Inversion, by Corrado Gini.
On Random Variations in Statistical Figures, by Wilhelm Winkler.

Engineering Research and Quality Control

Experimental Design Principles in Industrial Research, by George W. Brown.
The Application of Statistical Theory to the Preparation of Industrial Specifications, by John H. Curtiss.
The Role of a Statistical Consultant in a Research Organization, by Churchill Eisenhart.
The Dependence of Earnings on Choice of Specifications, by Merrill M. Flood.
Statistical Quality Control in American Industry, by W. R. Pabst.
Standard Sampling-Inspection Procedures, by W. Allen Wallis.

Social Statistics

The Standard of Living of Native Workers on the Estates in Java, by H. M. J. Hart.
Report on the Activities of the Joint Committee on Criminal Statistics, by Alessandro Molinari.
Methods of the Statistics of Social Insurance, by Demetrious Calitsounakis.

Methodological Problems of the Statistics of Literary Production, by Désiré Elekes.
A Middle Class Family from One World War to the Other (1914–1947), by Ugo Giusti.
Statistical Data of Social Mobility in Italy, by Livio Livi.
Cost of Living, Salaries, Consumption, and the Situation of the Italian Middle Classes, by Guglielmo Tagliacarne.

Demographic Statistics

Future Development of the Greek Population, by Demetrious Calitsounakis.
The Mortality Experience of the Metropolitan Life Insurance Company, 1911–1946, by Louis I. Dublin.
The Birth Rate in Italy from 1931 to 1946, by Livio Livi.
Measurement of Demographic and Economic Phenomena in Relation to Their Definitions, by Alfred Sauvy.
The First Census of Population Taken in Iceland in 1703, by Thorsteinn Thorsteinsson.
Analysis of Recent Demographic Trends, with Particular Reference to the Causes and Significance of Changes in the Birth Rate, by Alfred Sauvy.
The Fertility of Successive Cohorts of Women in the United States, by Pascal K. Whelpton.
Principles Underlying International Statistical Classification of Diseases, Injuries and Causes of Death, by Dario Curiel.
Age Distribution and Its Interrelation with the Elements of Natural Increase, by Wilhelm Winkler.
On the Calculation of a Monthly Infant Mortality Rate, by Henri Bunle.
Current Problems in Morbidity Research, by Haven Emerson.
Evaluation of the Methods of Measuring Net Fertility, by Alfred J. Lotka.
Factors of Urban Growth in China, by Ta Chen.
Outlines of the Demography of the Jewish Population of Palestine, by R. Bachi.

Social Security and Health Statistics

Problems in the Development of Comparable Statistics in the Social Security Programmes of Different Countries, by Wilbur Cohen.
Problems in the Development of Health Indexes, by Manuel de Viado.
Recent Developments in Applications of Statistics to Tuberculosis Control Programs, by Jacob Yerushalmy.

Public Opinion Research

Recent European Developments in Public Opinion Research, by Jean Stoetzel.
Public Opinion Research in Latin America, by Alberto Jose Castelli.
U.S. Methodological Research in Public Opinion Measurement, by Samuel A. Stouffer.

Experimental Design

The Theory of Experimental Design, by Ronald A. Fisher.
Application of the Theory of Experimental Design in Biology, by George W. Snedecor.
Limitations and Potentialities of the Application of the Theory of Experimental Design in Sociology, by Margaret J. Hagood.

Economic Statistics

Recent Developments in Marketing Statistics, by Ross Cunningham.
Recent Developments in Federal Government Statistics of the United States:

a. The Census Bureau Statistical Program, by Philip M. Hauser.
b. The Bureau of Labor Statistics Program, by Ewan Clague.
c. The Agricultural Statistics Program, by Charles F. Sarle.
d. Social Security Statistics, by I. S. Falk.

Proposal for the Better Co-ordination and Faster Compilation of International Barometric Indexes, by Livio Livi.
Statistical Determination of Irreducible Consumption, by Yves Mainguy.
International Statistics on Motive Power, by Alessandro Molinari.

Other Contributed Papers

An Experiment in Collection of Data on Population Movement in Turkey, by Celal Aybar.
The Particulars of Turkish Population Censuses, by Celal Aybar.
A Note on Intellectual Statistics, by Vincenzo Castrilli.
Regularities in the Proportion of the Sexes, by Aloys Dolanyi.
Methods of Evaluation of the Statistical Characteristics which Constitute Guides to Economic, Social and Fiscal Policy, by Georges-Antoine Hostelet.
Infant Mortality of the First-Born and Later-Born, by Franco Savorgnan.
Methods of Apportioning Seats in the House of Representatives Among the States, by Walter F. Willcox.
Variate Difference Method, by Rascho Zaycoff.
Some Remarks on the Representative Method, by Rascho Zaycoff.
Principles and Objectives of an International Classification for Statistical Materials, by Robert Guye.
Origin, Functions, and Future of the British Central Statistical Office, by E. H. Godfrey.
Need for an International Subject Classification for Statistical Materials, by Stanley G. Lyon.

Probability Theory in Economic and Social Sciences

Possibilities and Limitations of the Application of Mathematics, and Probability Theory in particular, in the field of Social and Economic Phenomena, by Maurice Fréchet.
Statistical Inference from Non-Experimental Observations, by Jacob Marschak.

Statistical Inference

Statistical Estimation of Parameters, by Maurice Fréchet.
Inductive Reasoning or Inductive Behavior—Modern Conceptions Relative to Statistical Inference, by Jerzy Neyman.

Sampling Theory

The Influence of Agricultural Research Statistics on the Development of Sampling Theory, by Frank Yates.
Recent Developments in Sampling Theory in the United States, by W. G. Cochran.
Sequential Analysis, by Abraham Wald.
History of the Uses of Modern Sampling Procedures, by Frederick F. Stephan.
Sampling of Human Populations, by Morris H. Hansen.
Commercial Uses of Sampling, by J. Stevens Steck.

National Income and National Capital

Recent Experiments in Social Accounting: Flexible and Dynamic Budgets, by J. Tinbergen and J. B. D. Derksen.
National Income and Industrial Structure, by Simon Kuznets.
The Effect of the Role of Government on International Comparisons of National Income, by Arthur Smithies.
The Measurement of National Wealth, by G. Findlay Shirras.

World Census of Agriculture and Population

World Census of Agriculture, by Conrad Taeuber.
The 1950 Census of the Americas, by Calvert L. Dedrick.
Methodology in Agricultural Statistics of the Americas, by Hernán Montoya.

THE ECONOMETRIC SOCIETY

General

A Future Rôle for the Econometric Society in International Statistics, by Charles Roos.

Statistical Analysis of Economic Relationships

Estimation of Economic Relationships, by H. O. Wold.
Errors and Shocks in Economic Relationships, by L. Hurwicz.
Prediction from Autoregressive Schemes and Linear Stochastic Difference Systems, by J. R. N. Stone.
Further Contributions to the Scatter Analysis, by N. Georgescu-Roegen.
The Estimation of Parameters in Autoregressive Time Series, by M. G. Kendall.

Econometrics and Private Business

Laws of Production Cost, by I. Jantzen.
Statistical Analysis and Charting for Business Control, by Charles Roos.

Econometrics of International Economic Relations

Some Remarks on the Problem of Dollar Scarcity, by Jan Tinbergen.
A Geometric Presentation of the Double Equilibrium of Exchange and International Trade, by F. Divisia.
Exchange Rate Stability Considered, by L. Metzler.

Economic Growth and Fluctuations

Theory of Economic Growth, by Colin Clark.
The Law of Divergence, by B. Chait.

Theory of Choice and Utilization of Resources

Social Yield and Social Productivity, by M. Allais.
Optimum Utilization of the Transportation System, by Tjalling C. Koopmans.
Choice by the State-Producer: Application of the Theory of the Producer and Consumer Surplus, by François Perroux.
The Practice of Economic Planning and the Optimum Allocation of Resources, by Oscar Lange.
Optimum Utilization of National Resources, by J. Stafford.
From the Theory of Choices to Family Budgets, by René Roy.
Economic Choice in a World Subject to Uncertainty and the Notion of Marginal Expectations, by M. Massé.

Inflation and Unemployment

The Concept of Inflation in the Light of a Theory of Variations in the General Price Level, by J. Rueff.
Capital Accumulation and the End of Prosperity, by E. Domar.
Prices and Money, by L. Amoroso.
Monetary and Fiscal Framework for Economic Stability, by M. Friedman.

Other Papers

Sampling Aspects of the Problem of Relationship from the Error-in-Variable Approach, by R. C. Geary.
Structural Matrices of National Economics, by W. Leontief.
Operational Concept in Economy, by J. Dumontier.
Limitations of the Use of Mathematics in Economics, by G. H. Bousquet.
The Problem of Fair Division, by Hugo Steinhaus.
Metastatics, by G. Lutfalla.
Effort Supply and Demand of the Working Man, by Robert Hénon.

Inter-American Statistical Institute

General

A Future Role for the Inter-American Statistical Institute in International Statistics, by Halbert L. Dunn.

Statistical Training Methods

Committee on Statistical Education—Programme and Organization, by Carlos E. Dieulefait.

Minimum Plans of Study for the Training of Statistical Technicians: Preliminary Report, by Carlos E. Dieulefait.

Statistical Teaching in the Western Hemisphere, by Milton da Silva Rodrigues.

Foreign Trade Statistics

Current Status of Foreign Trade Statistical Classification Activities in the American Nations.

Notes and Experiences on International Standardization of Statistics of International Trade, by Carlos Procaccia.

Methodology of Foreign Trade Statistical Practices in the Americas, by Santiago Woscoboinik.

Problems Involved in Supplying Statistics to International Organizations

National Viewpoints on Problems of Supplying Statistics to International Organizations.

Concept of a National Focal Point for International Statistics, by Halbert L. Dunn.

Co-ordination of National and International Statistical Requirements in the United States, by Donald C. Riley.

Industrial and Mining Statistics

Memorandum on Statistics of the Manufacturing Industry in the American Nations, by Santiago Woscoboinik.

Methodology in Statistics of Mineral Industries in the American Nations, by Bjorn Koch.

Other Papers

Preliminary Notes on Statistics of Education and Culture in the American Nations, by Germano Jardim.

Discussion on Mr. Campion's Paper

Mr. Alexander Loveday: It is my privilege to move a very cordial vote of thanks to Mr. Campion for his paper, and I do so with the greater pleasure because I have read it with the most lively interest. It so happens that I have lived through a considerable portion of the history he records, but I have never had my own experience put in a historical setting in the way that Mr. Campion has done in his paper. So far as I can judge, he is an extremely accurate historian. He was quite correct in suggesting that there was a certain amount of friction in 1919 between the League of Nations and the International Institute, but such friction was in the end lived down. It was the greatest mistake to force a newborn lamb like the League at that time into the lion's den of the old international statistical bodies. We got around that difficulty by means of the Mixed Committee.

I find it difficult to know where to start in commenting on this paper, but I will make one or two remarks with a view to supplementing what the author has said. I do not think he put enough emphasis on the background work done by the Secretariat. In my experience, by far the most competent person to put forward constructive proposals for improving international comparability is the man who, in the course of preparing an international statistical publication, is forced to study the statistics of each country day by day and find out what discrepancies there may be, what the respective definitions are, and what are the different phenomena which the statistics measure. It was really this background work in the League, especially in the field of trade statistics, that made it possible for the League to have a Conference in 1928, and conclude a convention involving a series of more or less binding premises with regard to the methods to be adopted

I emphasize this because it is, I think, important that the Statistical Committee of United Nations should work on the assumption that the general background work of the Committee must be done by the headquarters' staff.

My second point is in connection with the reason why that particular convention of 1928 covered economic statistics, and not financial statistics. It has been stated that the reason was that there were already some bodies interested in financial statistics. That was not the real reason. The

real reasons were two; in the first place we had not got nearly as far in analysing financial statistics as in analysing economic statistics. The second reason was that I did not believe that one Committee of experts could be really competent in both economic and financial statistics. We had it in mind to supplement this Economic Statistical Conference by a Financial Conference later. Then came the depression of the 'thirties; Conferences became difficult to organize; the League Council accordingly requested the Committee of Experts to appoint a sub-committee to deal with the financial field. I mention this because it seems to me that there might be a case to-day for supplementing the 1928 Convention by a parallel one for financial statistics.

Another method we adopted in order to try to improve comparability was, with the assistance of the Rockefeller Foundation, to get an official in a statistical office in some less advanced country seconded for some months to another country, in order to study the methods adopted there. The underlying idea was that recommendations concerning end-results were not enough. Statisticians would require to know also the whole apparatus by which these results could be achieved—what questionnaires should be sent out, what records kept—and we endeavoured to enable a certain number of people to gain that knowledge by studying the methods adopted in the more advanced countries. On completing their tours the statisticians passed through Geneva, and informed us of their experience, which enabled us to judge both of the benefit they had derived and of the difficulties with which they were faced at home.

To revert to the question of the International Institute, the change made in the Statute of the Institute is, as I see it, a change in the right direction. The Institute must be an Academy, and to some extent it must be always ahead of what Governments can really do. But what are really important in the long run, from the point of view, at any rate, of an international official concerned with promoting comparability, are the meetings held under its auspices. I myself found the International Institute absolutely invaluable. By its meetings one got to know statisticians in different parts of the world; and if, later, one wanted to persuade some Government to change its methods, it was obviously much easier to do so if one could begin one's letter by referring to some amusing incident at a recent Conference instead of plunging directly into some controversial issue of statistical methodology. It is through its meetings, and the contacts especially, that the Institute performs, and must perform, its most valuable service in the cause of statistical comparability.

Perhaps I may conclude by emphasizing again what I said at the very outset. It does seem to me of enormous importance that the United Nations should look to their Secretariat to do the basic work for their Statistical Committee, and I hope that it will not be afraid of a certain amount of overlapping. The essential is to have a sufficiently strong and competent Central Office. What I should like to see is the Editor of the United Nations Statistical *Year Book* feeling himself directly responsible for every table published in it, not treating the book as a sort of compendium of tables furnished by the various agencies, but himself criticizing and thinking about each individual table, and the comparability of the national figures composing it, and considering at the same time how tables from one Specialized Agency correlate with those from others. In that way he would be able to give a conspectus of the whole field, which I do not believe anybody else can do. It seems to me that the function of the Committee of Experts is to organize the work in such a way that the Editor and editorial staff of the *Year Book* or, as regards demography, the Editor of the demographic Year Book, would be looked to for furnishing the major part of the essential information which is required for formulating recommendations.

Professor A. L. Bowley, in seconding the vote of thanks: The first part of this paper is a matter of not very well known history. The beginning of international statistical discussions dates back a hundred years, and put into an easily assimilable and coherent form, will be of permanent value as a record. I accept Mr. Campion as a statistical historian so far as the matters about which he writes have come within my knowledge. Much of what I would have said has already been said by Mr. Loveday, and I shall deal not quite so closely with the paper as he has done but make some rather more general comments.

The main problem of international statistics obviously lies in the difficulty of comparison between two different countries. There is a great deal about this difficulty of definition in the paper, and there might be a great deal more. I have had rather a long experience with varied classes of statistics, and nowhere have I found their co-ordination easy, even within one country, amongst the things I could see and understand myself; and I have very seldom ventured to make comparisons between one country and another, because I felt sure that it was not possible for an outsider to understand many of the statistics of the country in which he had not actually lived or worked.

Various international statistical bodies, such as those of the League of Nations and the International Labour Office, have always given a great deal of attention to the question of definition, and I hope they will continue to do so. But I hope also that it will be remembered that a series

of figures can very often be safely given when the definition of the items is still vague and discrepant. That makes a good deal of difficulty in our present era of statistics. Since everything is to be planned, it is not so much a matter of careful study of present-day statistics as of the study of the statistics of to-day as compared with the expected statistics of 1952 or 1958, or to whatever date the planning is to reach.

In this great effort at being up to date there is bound to be a loss of accuracy. Careful statisticians, when they give an estimate, after taking a measurement as far as they can, allow some kind of margin. That is not commonly given in the publication, if I may say so, which appears under the aegis of Mr. Campion's office. It is not commonly given in the statistics which I come across, but it should be given wherever possible. I read recently a Paper in the *Journal of the Statistical Society of Paris* in July of last year in which some doubt was expressed as to the validity of French agricultural statistics. It was said—I am quoting from memory, and this is in no sense an accusation on my part—that everybody concerned took the most favourable view of the returns they made. It was even stated that when a Government Inspector called at a farm and asked for statistics of produce, his conscience was a little quietened by a pound of butter, and if he was a more important inspector, by as much as a kilogram or 2 lb. of butter; and if a more important official came, the tocsin sounded and the inhabitants came out with cudgels. An aerial survey of cattle was attempted, but since it was a hot sunny day, most of the beasts had taken shelter and evaded the camera. In this way one could not claim accurate statistics for agriculture. That may be a travesty of what actually occurs, but it contains a great element of truth in respect of the difficulty of obtaining, in agricultural as well as in other countries, statistics of income and production. I do not know how the International Statistical Office can count the produce of farms.

I suppose it would be proper for me to say something about the International Institute, of which I have been a member for very many years, and which has a place in Mr. Campion's paper. I would myself compare the International Institute with this Royal Statistical Society. It is not bound to accept statistics from anywhere. This Society and the International Institute can discount any political or international bias in statistics. Another thing that can be done and is done by these bodies is to break new ground in statistics. One of the important methods undertaken by the International Institute was the appointment of committees, or commissions, to study a particular question from that point of view.

I should like to emphasize Mr. Loveday's statement that the importance of the International Statistical Institute lies largely in the meetings it arranges of statisticians from many countries. They can exchange their ideas freely, just as in this Society we have Civil Servants who have often come and told us what they have learnt in their offices, but have told it in a freer way than would be thought fit, or possible, for public issue. One thoroughly agrees that when people from different countries have been in actual contact in this way it is easier for them to establish good permanent relations.

The vote of thanks was put to the meeting and carried unanimously.

Mr. H. M. Allix said that, although an official of the Ministry of Food, he spoke in a purely personal capacity. He had followed Mr. Campion's survey of the progress so far achieved in the field of international statistics with great interest. Mr. Campion had clearly illustrated how statistics had come to occupy an important place in the administrative machinery of governments and international bodies. Collaboration in its broadest sense amongst statisticians of the world was, however, still in its infancy; food statisticians felt that much more could be done in this field. The aims of free interchange of statistical data and of meetings between statisticians of different countries should be much more energetically pursued, and there was much to be said for bringing statisticians together on a technical level to clear up misunderstandings and survey the ground before international conferences took place. There was also a strong case for regular meetings of food statisticians. One example was the conference of European Food Statisticians called in London by the Emergency Economic Committee for Europe in September, 1945. This conference discussed the wartime advances in the technique of analysing food statistics, and attempted to reach some agreement on the methods by which comparable statistics could be prepared. This pioneer work was the foundation on which the F.A.O. of the United Nations based their studies of *per capita* consumption of the main foodstuffs in some 70 countries of the world, the results being published by them in 1946, under the heading *World Food Survey*.

Another example of successful collaboration in this field was the work of American, Canadian and British food statisticians on international consumption levels during the war, the official publication of which, in all three countries, created widespread interest.

International statistics relating to the field of food and agriculture fall broadly into two classes: those associated with production, prices, trade and consumption of food and feeding-stuffs, which

might be called performance statistics, and those relating to the scientific study of nutrition, which might be called requirement statistics. The latter afforded the standards for computing the requirements of the human race in its different environments and according to its varying needs and preferences. It was necessary not only to report facts of international significance relating to production, trade, prices and consumption, but to analyse them as a guide to international action. Much pioneer work in this field had been done by the International Institute of Agriculture in Rome, and had found expression in what are now familiarly known as the Rome Year Books, and in other occasional publications. To-day the Food and Agricultural Organization of the United Nations had, as we had heard, taken over this work with hopes of enlarging upon it. But there would always remain some additional statistical research to be done by individual nations to satisfy their own special requirements. In this respect the United States Department of Agriculture had set an example for others to emulate, and through its ability to render great assistance to those organizations entrusted with the task of tackling post-war food and agricultural problems (for instance, the International Emergency Food Committee) had demonstrated the need for all nations to be statistically well equipped in order to bring their own specialized judgment to bear in international discourse. A successful beginning had been made in this country, but he (the speaker) would not rate it higher than that. At the Ministry of Food they had attempted to assemble statistics relating to production, prices and availabilities abroad. In addition they were continuously investigating other nations' requirements, in order to judge how far they competed with our own demands on a limited effective world supply of food and feeding-stuffs. This had all become necessary in consequence of the development of rationing, Government bulk buying abroad, long-term food contracts, and all the other paraphernalia of a Government system of controlled food distribution over the last ten years. Whatever might be said of this development on other grounds, it had afforded the opportunity for the creation by the Government of a medium for international food intelligence and statistical work in this country. The results of this work formed part of the basis for the general system of Government planning in this country. It had, unfortunately, not yet been possible to introduce a wider public to it, except in one or two instances such as the published study on international consumption levels mentioned before. The speaker looked forward, however, to the much wider publication of the fruits of specialized studies by official and private bodies in this country on international problems of food and agriculture, as a contribution to informed discussion here as well as in the world at large.

Great difficulties, however, lay in the path of all nations eager to improve their food and agricultural statistics. Nations had developed their statistics according to their own technical resources and for purposes which might not agree with the aims of the international statistician. Until a far greater standardization of international terminology was achieved, little progress could be made towards automatic comparability of statistical data. In the case of cereals, for instance, the great variation in harvest times created one important element of incomparability. Areas referred to were sometimes those sown, and at other times those harvested. Data on foreign trade in wheat, for example, might or might not include trade in flour. Varying extraction rates made it difficult to bring flour or milled rice statistics back to thin grain equivalent. In Indonesia the term paddy included rice in the husk plus part of the stalk; close definition of available statistics was therefore required to ensure comparability with other countries. These few examples of difficulties in crop statistics could be multiplied, but they were few in comparison with those encountered in the field of livestock and livestock product statistics. For instance, many countries did not make annual estimates of livestock numbers. Where statistics existed the dates to which they related varied from country to country. There were wide seasonal fluctuations in numbers of many species of livestock to be taken into consideration. Statistics of milk utilization were often non-comparable because of the different treatment of the incidence of skim milk. In addition, different countries reported whole milk fed to calves in different ways. France, for instance, included all the milk fed to calves, while Australia and New Zealand disregarded it altogether. The comparative recent development of international trade in dried milk and dried eggs had added difficulties of conversion and comparison of a type familiar to the food statistician, from his previous experience with such items as canned and boned meat. The problem of calculating "oil content" of oilseeds and nuts, and the almost complete absence of reliable statistics in the field of actual production and production capacity with regard to certain types of vegetable oils, by-product animal fat, and even such an important food as fish, which was, after all, the dietary basis of many economically less advanced peoples, might serve as further examples of some of the more obvious problems with which the worker in the field of international food statistics had to contend.

Until the secretiveness and the non-collaboration of many Governments, especially those of the great food-producing areas of the Soviet Union and Eastern Europe, was overcome, and until the statistical backwardness of other areas, such as the Far East, Latin America and certain colonial

territories, had been mitigated, any international picture relating to food and agriculture must contain considerable elements of doubt. Our ability to tackle international food problems would depend both on the progress achieved in closing these great gaps, and also on the degree of refinement that could be brought to existing information. It was, for example, possible before the war to review the relatively slow change in the pattern of international trade in, and production of, foodstuffs against the background of the statistics provided by the Rome Institute. There was little alteration in this pattern within each of the three main periods of prosperity, depression and recovery between 1922 and 1939. It was, however, quite impossible to ascertain, still less analyse, the causes and effects of the major changes between one period and another. The inadequacy of statistics relating to prices and to the sharing of rewards between producers and distributors made it difficult to analyse statistically some of the possible causes of these changes, while on the other hand their effects could not have been fully appreciated because of the prevailing lack of international data on levels of nutrition and household budgets. Such elaboration became all the more desirable to-day in a world of state-regulated production and distribution, and of development in the science of nutrition and the increasing interest in its application.

The interdependence of nations in feeding their peoples adequately would, to some extent, foster the desire for collaboration in this field of international statistics. Such collaboration, he felt, was bound to lead not only to an ultimate raising of food standards over much wider areas of the world, but would bring this about much more quickly.

Dr. Percy Stocks said that he was rather embarrassed by the enormous increase of statistics other than vital in recent years, and he had to confess that the present paper had left him with a rather remote feeling. Perhaps he might be allowed to expand some of the author's references to the recent work on mortality and morbidity statistics. He thought it could be said that a greater degree of international agreement had now been reached about statistics of disease and death than perhaps about any other kind of statistics. As Mr. Campion had reminded them, there had been an international list of causes of death for over fifty years now, and that had been revised by international conferences of statisticians and medical experts in Paris every ten years. The constructive effort towards producing that list really began in 1853 at the Brussels Conference. At that Conference Farr and d'Espine were asked jointly to prepare a nomenclature suitable for all nations.

It was later seen that there were two urgent requirements in connection with the international list, the first, the classification of diseases and injuries, apart from mortality, and the second, an agreement about the method of dealing with death certificates on which more than one cause of death was stated. Owing to the increasing verbosity of doctors more than half the death certificates now stated multiple causes of death. Because of this, in 1940 a diabetes death rate of 134 per million in England meant the same thing as a rate of about 250 per million in America and in Canada. Two people, at least, came away from the Conference in 1938 determined to get these things put right. Dr. Dunn, the Chief Officer for Vital Statistics in the United States, met him after the Conference and they resolved that neither of them would rest until this had been done. Then, almost immediately, the war came, and the United States, Canada and England found themselves caught unawares without having any morbidity classification, which was urgently needed, and each country had to set about making such a classification for itself.

The Committee of the Medical Research Council in this country, working on a draft prepared by Dr. Robert Smith and himself, produced a classification of diseases and injuries which had been quite widely used in this country. At the same time—1944—the United States produced a somewhat similar classification. Very fortunately, both of these were based on the framework of the international list of causes of death, and when the three lists were studied Dunn and he thought that it ought to be possible to get an agreed classification which would serve the purposes both of mortality and of morbidity statistics. That would get rid of the ridiculous situation whereby a soldier admitted to a military hospital and then transferred to a civil hospital and afterwards pensioned off and dying, would have half-a-dozen code numbers successively for the same illness, or, if in America, would have half-a-dozen different numbers, and in Canada yet others.

At the end of 1945 he went to Washington, and the first difficulties of settling the general principles with a large committee of American and Canadian experts were overcome. A small working party, including a representative from the League of Nations, sat down for two months and hammered out an agreed draft, with the help of numerous specialists. They succeeded in producing a list which was subjected to intensive study by a medical advisory committee and other consultants, and then, early in 1947, they all met again in Ottawa and produced a revised list, which was handed to an Expert Committee set up by the Interim Commission of the World Health Organization. The approved list was circulated to 70 nations, and their criticisms and comments were invited. Replies were received from over 40, and another meeting took place in Geneva to

incorporate as many of the proposed amendments as seemed advisable. Then an International Conference of Statisticians was held in Paris last year when 28 nations sent representatives.

In the meantime the United States and Canada had decided to adopt the English system of dealing with the problem of multiple causes, if the Paris Conference would agree to accept it internationally. That Conference did accept it, and also the new classification. Only one more hurdle remained, and that was to get all this agreed to by the World Health Organization. The First Assembly, in June of last year, adopted the recommendations and decided to print the new manual in English, French and Spanish. The first set of international regulations was also produced, making the use of the classification obligatory from 1950 onwards unless a nation decided within a year to contract out. The regulations also required the use of the English system of stating the underlying cause of death. This manual with the new classification was now on its way from Geneva, and it should be available for use in Britain within a few weeks. The result would be that if next year he happened to be riding a bicycle and was knocked down by a taxi, and got a septic compound fracture of the tibia, no matter whether this happened in Amsterdam, Copenhagen, Baltimore, or Oxford, his number would be E8134–N8233; and whether his "number was up" or not he would appear on a machine card with the same eight little holes!

Mr. Goudswaard (Director of the Permanent Office of the International Statistical Institute at The Hague) said that it had been a great pleasure to him to attend a meeting of the Royal Statistical Society, which was the mother of the International Statistical Institute. He had not had an opportunity of reading Mr. Campion's paper before the meeting, but from what he had heard of it he could say that Mr. Campion had given a clear and illuminating summary of the position. He himself would add some remarks on the present status and future developments of the International Statistical Institute.

Mr. Campion had already mentioned that the position of the International Statistical Institute in the period between the two world wars was rather confused. The chief difficulty was to distinguish between the terms of reference of the League of Nations and the International Statistical Institute, which functioned more or less officially. After the war the field governed by the United Nations and the field governed by the International Statistical Institute were much clearer. It had often been said that there was a reluctance in the International Statistical Institute to adopt the new role, which was more of an international academy than of a semi-official international organization. Perhaps there had been some reluctance, especially after the first world war, when the situation was somewhat analogous, but the main reason was a purely practical one—the possibilities of financing. In the national field, it is much easier to finance an official agency than a professional society, and it is the same in the international field.

However, the acceptance of a more professional role for the future did not resolve all the difficulties. Not only had the position of the Institute in relation to the United Nations and its specialized agencies to be reconsidered. A growing tendency to decentralization in the professional field had also to be faced.

Before the recent war some international societies interested in a field of statistical specialization were already in existence—the International Union for the Scientific Study of Population Problems and the Econometric Society. In 1947 the Biometric Society, the International Association for Research in Income and Wealth and the International Association for Public Opinion Research had been created. This "horizontal" decentralization was paralleled by a "vertical" regional decentralization: the formation of the Inter-American Statistical Institute in 1940.

Such a development had its advantages, but on the other hand it might be a serious handicap to the efficient functioning of the international statistical system in the professional field. The new Statutes of the International Statistical Institute, adopted in 1948, provided for the possibility of affiliation of these organizations with the International Statistical Institute. It was hoped that in this way it would be possible to avoid overlapping, and to arrive at a coalescence of international statistical interests. The attitude, with regard to this policy, of the "specialized" international organizations concerned had proved to be very favourable.

A logical outgrowth of the new policy just described was to try to get into closer contact with the national statistical societies in various countries, and to provide an opportunity for these societies to affiliate themselves with the Institute. He was glad that the Council of the Royal Statistical Society was amongst the first to propose affiliation with the International Institute.

Mr. Campion had mentioned that the Institute was setting out to do what was possible on an international basis to provide facilities for education and training in statistics. This was one of the main new projects, and was indeed of primary importance. Before the reorganization of the Institute one of the main objectives was to promote the development and international comparability of statistics in a direct way; now, they were trying to aim at this from an indirect point of view. Statistics were lacking in many countries. One of the main reasons for this was lack of

statisticians, and one of the main reasons for the lack of statisticians was the absence of facilities for education and training. If, therefore, the International Statistical Institute were in a position to provide improved facilities for education it would result in an increase and improvement of statistical information. The difficulty, of course, was the financing of such a project. However, it was possible to start working on it with the help of a grant received from Unesco for the purpose.

Statistical education would be under discussion at the next session of the Institute, to be held in Berne in September of this year. Another important topic for this session, which had also resulted from the adoption of the new policy of the Institute, the conception of inter-organizational co-operation, was the following:

In view of the growing specialization in the field of statistics, it was difficult for people working in one field to keep track of current developments in other fields. It was planned, therefore, that each of the "specialized" international organizations, affiliates or prospective affiliates of the Institute, should submit at the session a report on recent progress in their field, especially for the information of statisticians working in other fields. This was one of the ways in which the Institute could serve as a clearing-house between the various fields of specialization.

The international statistical system, governmental as well as non-governmental, was becoming more and more complicated. In the non-governmental field this was illustrated by the "horizontal" and "vertical" tendencies to decentralization, mentioned earlier. On the governmental side a distinction might be made between collecting statistical agencies, such as the Statistical Office of the United Nations, and Advisory Bodies, such as the United Nations Statistical Commission. As the number of both was increasing, the problem of co-ordination was serious for the collection of statistics and for the activities of the U.N. Statistical Commission on the one hand, and the Statistical Advisory Committees of the Specialized Agencies on the other. Effective working relations were not only necessary in order to prevent overlapping, but also in order to establish an over-all priority scheme for projects to be undertaken.

What were the chances of a particular topic being included in the work programme of one of the U.N. statistical organs? It depended on its importance, but also on whether the U.N. Statistical Commission was responsible for it, or the Statistical Advisory Committee of one of the Specialized Agencies, because the former had to cover a much wider field than the latter. Therefore, in order to spend as efficiently as possible the total funds available for statistical purposes, an over-all priority scheme would be needed.

Finally, it might be mentioned that very useful work was being done by a number of non-governmental organizations in the field of the collection of international statistics, e.g. the World Power Conference, the International Tin Study Group and the International Rubber Study Group.

Professor J. H. Richardson said that during the greater part of the 1920's he took part, as a member of the staff of the International Labour Office, in the compilation of labour statistics, which involved co-ordination with the statistical work of the League of Nations in association with Mr. Loveday. He also attended the Rome and Cairo meetings of the International Statistical Institute.

He wished to refer to two aspects of international statistics, namely, the organizational side and the improvement and development of the statistics themselves. Once the international governmental organizations were set up after the first world war their resources of finance and personnel gave them facilities with which other bodies could not compete, and this was increasingly recognized by the International Statistical Institute. The Governments sent representatives to conferences convened by the League of Nations and the International Labour Office, and agreements were reached defining the statistics which Governments should compile; at the same time these international organizations were obtaining statistics from the various countries, and, as far as the crude data permitted, were making international comparisons. Although some problems of duplication arose between the agencies of the League of Nations, the difficulties were slight and easily overcome, and he believed they would be similarly overcome under the United Nations. For example, the International Labour Office was primarily interested in wages, cost of living, hours of work and employment, but these were connected with movements of the trade cycle, statistics of which were compiled by the Economic Section of the League. The two organizations thus had to work closely together.

He desired to lay stress upon the fact that the agencies of the League of Nations and of the United Nations were governmental agencies, and although they might bring in experts for advisory purposes from outside the ranks of Government departments, they were really closed corporations, the experts being selected by governmental bodies. One of the advantages of the International Statistical Institute was that it had greater freedom in its membership, as it could include statisticians who were outside the government services; also it was less bound by practical considerations, and could study theoretical aspects which were of interest for the long-term development of statistics.

Turning to the problem of improving standards, it had been indicated by an earlier speaker that the statistics compiled in some countries were very meagre, and much progress was needed in these backward countries. This led to a danger emphasized in Mr. Campion's paper, namely, that of levelling down the standards of national statistics for the sake of international comparability. The standards of some countries were so low that it would be impracticable to bring their statistics quickly into line with those of the advanced countries. A method by which some international conferences had attempted to deal with this problem was to establish two standards—a minimum standard which backward countries might reach fairly quickly, and a higher standard which other countries could adopt, so as to obtain greater comparability among themselves.

He thought the problems were so complex that progress must be very slow. One difficulty was that each country was reluctant to change its methods because this would reduce comparability of time series within the country itself. In this connection he agreed with Professor Bowley that time comparisons, showing trends separately for each country, were usually more reliable than place comparisons at a given date. That view was supported by the experience of the International Labour Office.

Mr. J. L. Gibson said that the United Nations sub-committee on Contributions on which Mr. Campion had sat was instructed by the General Assembly to have regard not simply to relative national incomes, but to income *per head* of population of the countries concerned. He felt quite sure that in view of pending developments and from a practical point of view this problem was the most immediate and urgent which they had to face to-day. He need only mention one field in which America and Great Britain had collaborated in this matter. In 1947 the two countries shared *equally* in the financing of German civilian supplies, with the result that the British taxpayer was taxed at 55*s*. per head as opposed to 15*s*. per head paid by the American taxpayer. This anomaly was pointed out by General Clay's financial adviser, and Mr. Secretary Marshall was so much impressed by this disparity that in September, 1947, he called a meeting of the Appropriations Committee of the House and Senate, and persuaded them to reassess the contributions due from America and Great Britain. The upshot was that in 1948 the relative contributions of Great Britain and America for Germany were reassessed more in relation to the national incomes of the two countries. The new figures, resulting from the Anglo-American negotiations, showed that in 1948 Great Britain was called upon to pay only £20m., mostly in sterling, as opposed to £80m. paid in 1947, mostly in dollars. Yet in 1948 the *combined* cost was much higher than in 1947.

He found after consulting Mr. Jackson of the Central Statistical Office, Professor R. G. D. Allen, and Mr. Campion himself, that there were no statistics relating to what might be termed the marginal national income. This matter was of vital importance. In his view, the British contribution to any agreed international purpose should be, as compared with the United States' contribution, more in the ratio of 1 to 10 than of 1 to 6. The national income of the United States was six times that of Great Britain, but the average income per head of the 150 million inhabitants of the United States was double the income per head of the 50 million inhabitants of Great Britain. But the citizens with double a given income do not pay merely a double tax. Where there was combination for an agreed political or economic end, like U.N.R.R.A. or the Atlantic Pact, the computation should be not on the straight national income basis, but on what might be viewed as the *taxable margin* of income per head. In other words, for such functional ends common citizenship came into being, and all such citizens should be called upon to subscribe on a common scale according to their assessed taxable margin of income.

He hoped Mr. Campion would support him in that proposition.

The following written contribution was received after the meeting:

Mr. J. W. Nixon: Perhaps I may be allowed to supplement and expand the references made by Mr. Campion, in his excellent and comprehensive paper, to the work of the International Labour Organization, in the field of international statistics, as it is only briefly touched on,and has developed on somewhat different lines from those adopted by the League of Nations and the United Nations. The paragraph on page 114 headed "International Conferences of Labour Statisticians" mentions together the "body of experts on labour statistics, and the Conference of Labour Statisticians" without indicating their independence and their different functions. The "Conferences," of which there have been six (the sixth, held in August, 1947, is not mentioned in the paper) were confined to government representatives, and were called primarily to consider the possibility of securing greater uniformity in the compilation of statistics on certain topics of interest to the International Labour Office. Their recommendations had no binding power, and were circulated to Governments in the hope that they would take them into consideration when considering changes in their statistics. Their functions were thus akin to those of the sessions of the International Statistical

Institute, as described by Mr. Campion; but the recommendations carried, I think, more authority, because they were drawn up by official statisticians and were based on comprehensive international surveys. This function remained true of all the conferences called up to the present, except the fifth, which was called for a special purpose.

The "Committee of Statistical Experts," on the other hand, was a small body of statisticians not representatives of their governments, set up primarily to advise the Office on various statistical inquiries which it had undertaken or was proposing to undertake, and its recommendations were addressed to the Governing Body of that Office. Its origin was due to the investigations which the Office had made into the relative levels of "real wages" in different countries—enquiries originally started by the Ministry of Labour. The results of such studies were to show that real or purchasing power of wages of workers (or of a particular category of worker) in country A were x per cent. higher or lower than those in country B. Although all the data on which the computations were made (wages, prices and consumption data) were published, the methods expounded, and the qualifications to be attached to the results fully set out, the figures were strongly criticised in some countries. The most vocal were the Germans (the results showed that the real wages of German workers were less than those in certain European countries), and they even threatened to invoke Article 11 of the Covenant of the League of Nations, which states that each member has the right "to bring to the attention of the Assembly or the Council any circumstance whatever affecting international relations which threatens to disturb international peace or the good understanding between nations upon which peace depends." Thus international statistical comparisons may lead to international political complications !

This Committee of statistical experts (of which Mr. Ramsbottom was one of the leading and most helpful members) gave valuable advice to the Office on this and other problems submitted to it, but its activities came to an end with the war. Its last meeting was in 1937, when its original functions were broadened to enable it to give advice on the desirability of submitting to the International Labour Conference an international convention on labour statistics and, if desirable, as to which branches were suitable for inclusion in such a convention. In this connection it should be pointed out that the International Labour Organization is unique among the "specialized agencies" of the United Nations, in that its constitution provides for an annual conference, whose primary function shall be the drawing up of "international labour conventions." These conventions differ from those drawn up by other international bodies. When adopted they *must* be submitted to the competent authority of each country, which is free to ratify them or not, but once ratified their provisions have the binding force of an international treaty. The Committee recommended that the statistics of wages and hours of work might form the subject of such a convention, and the subject, in view of its technical nature, was therefore first submitted to a conference of government labour statisticians in 1937. This is briefly referred to on page 114 of the paper where, the subject of the fifth Conference of Labour Statisticians is mentioned. This Conference drafted a proposed convention which was submitted to the International Labour Conference in 1938, which adopted it with certain amendments, and it came into force in June, 1940. Owing to the war ratifications were very few, but now 16 countries have ratified or promised ratification (the latest ratification being that of the United Kingdom). This Convention differs from the Convention on Economic Statistics, which is fully dealt with by Mr. Campion on p. 112. It not only lays down, in some detail, the scope of the statistics, the method of compilation, the frequency of collection, but provides that each ratifying country must submit an annual report on the manner in which it has carried out the provisions of the convention, reports which are examined not only by a special body of experts, but by the annual International Labour Conference, which has the power to make "representations" concerning non-application. Complaints may even be referred to the International Court of Justice. This first experiment in developing international statistics by means of an international and "supervised" convention of this nature is worth calling attention to; had not the war intervened in 1939 more countries would probably have ratified it, and it is possible that the method might have been applied to other branches of labour statistics.

These differences between the methods of improving, in both quantity and quality, international statistics, adopted by the International Labour Organization (through the three means of conferences of official statisticians; advisory Committee of Experts to the Office and its Statistical Section; and international conventions) and those adopted by the League of Nations, the United Nations and other international bodies, are perhaps worth calling attention to.

Two minor points may be referred to. On pp. 120 and 125 Mr. Campion refers to the steps being taken to devise an occupational classification which could possibly be adopted for international use, and considers it "very doubtful how far this can be achieved," in view "of the great differences in the economies of countries." This is perhaps too depressing a view, especially as a standard classification of industries covering about 230 industrial groups has, as Mr. Campion

states, just been adopted by the United Nations in spite of "the great differences in the economies of countries." It is of course impossible to draw up an accepted international classification of occupations in the same detail as, e.g., the United States Classification of Occupations, which classifies about 20,000 occupational titles under a six-number code system, but considerable progress can be made on reaching agreement on broad occupational groups, on certain characteristic occupations common to all countries, and on eliminating the confusion in some countries between occupation and personal status, to which Mr. Campion refers, and also, it may be added, between occupation and industry. A preliminary scheme has already been drawn up by the International Labour Office with the help of experts, and will be submitted to a conference of labour statisticians (the seventh) in October, 1949.

Also on page 125 Mr. Campion refers to the definition of the labour force as one of the questions on which agreement could be achieved, but does not state that a series of recommendations on this subject was adopted by the Sixth International Conference of Labour Statisticians in 1947, and that the Population Division of the United Nations has issued a report on "definitions of the labour force" in view of the censuses of 1950/51. The definitions recommended by the Conference of Labour Statisticians are followed in the United States, Canada and Japan in their current estimates, and the new monthly manpower statistics published by the Ministry of Labour and National Service since February, 1949, conform very closely to them. Many other countries will, it is hoped, adopt them for their next population census.

Mr. Campion thanked the speakers for their kind remarks about his paper. He subsequently wrote as follows:

I agree with Mr. Loveday on the importance of the background work done by the secretariat of the international organizations. If their work is not competently done it is very hard for committees of representatives from different countries meeting infrequently and then only for a short time to make rapid progress in the development of international standards for the collection and publication of statistics.

I share the views of Professor Bowley and other speakers on the dangers of comparing indiscriminately the statistics from different countries, and I agree that the improvement in national statistics cannot be secured merely by inspiration from international organizations, but is something which each country must tackle itself, profiting by the experience of other countries.

I was therefore much interested in the remarks made by Mr. Loveday on the arrangements made by the League of Nations before the war for statisticians from the under-developed countries to visit other countries. Similar arrangements are now being discussed as part of the programme of technical assistance for economic development to be sponsored by the United Nations and the Specialized Agencies. One of the quickest ways to get improvement in the statistics of under-developed countries is either for acknowledged experts in particular subjects to visit these countries and to give advice on the spot or, alternatively, for statisticians from these under-developed countries to have an opportunity of learning at first hand how the collection and analysis of statistics are done in other countries. I am glad to hear from Mr. Goudswaard of the role the International Statistical Institute proposes to play in providing facilities for education in statistics.

The question raised by Mr. Gibson is a very difficult one. Even if agreement could be reached between experts of different countries on the theoretical concepts involved, it would be very difficult to get agreement on actual figures. Mr. Gibson is not quite right in saying that contributions to the United Nations are based only on national incomes or income per head. A number of other factors are taken into account, such as the extent of war damage, amount of foreign exchange available, relative contribution per head of the population: and even apart from these, there are other semi-political considerations which the General Assembly takes into account in making their decisions.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

Raymond Vincent Baron.	Brian Gerald Nepean Fowler.
William Frederick Frere Bentley.	Peter Wilfred Freeman.
Ronald Brech.	Francis Richard Gillett.
Alexander William Boyne.	Victor William John Heard.
Frances Mary Chadwick.	Edwin Hole-Baker.
Edward Corrin.	William Arthur Read.
John Daniel Dale-Green.	Norman Llewellyn Smith.
Mervin George Dixon.	Marcus Emanuel Weinstein.

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Benjamin G. C. Newcombe, *representing* The Bowater Paper Corporation.

LOCAL GOVERNMENT STATISTICS—WITH SPECIAL REFERENCE TO THE HEALTH SERVICE

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Introduction

THE object of the present paper is to discuss the way in which statistical theory and practice might be applied to improve the efficiency of the local government services, with particular attention to public health administration, which is the writer's primary field.

This is not an original theme. Many of the suggestions made in the paper were put forward in earlier years by administrators who have long since left the offices from which they vainly struggled to convert ideals into practice. From its inception as a distinct authority the London County Council instituted a statistical department, with responsibility for recording facts relating to the work carried out by the Council. Amongst other things they published annually a volume entitled *London Statistics*, which was a comprehensive statistical review of the local government services, illustrated by diagrams.

The first volume was somewhat rudimentary, but the second volume, covering the years 1891–92, contained statistics on—

Population.—Preliminary details of 1891 Census.
Taxation.—Incidence of local taxation. Yearly changes in total rateable value. Analysis of Council's expenditure.
Debt.—Loans raised and amounts expended.
Pauperism.—Analysis of types of paupers, able-bodied, lunatics, etc.
Crime.—Total offences dealt with and cost of police.
Education.—Number of schools and pupils. Cost per child.
Health.—Death-rate. Cost of services and division of responsibility. Open spaces. Unsanitary houses dealt with.
Lighting.—Average illuminating power of gas.
Water Supply.—Annual changes in water consumption.
Labour.—Standard rates of wages and relation to costs of production. Report on workmen's trains.
Trade.—Price of corn.

There was also a summary table comparing London, in relation to the above, with the rest of England.

Whilst many of the tables were only broadly descriptive rather than analytical in relation to the factors determining the various services, there was already here a wealth of raw material from which to test the efficiency of municipal administration.

In 1915 the Council published a volume of *Comparable Municipal Statistics*, in which figures similar to those published in *London Statistics* were employed for the following representative towns: London, Birmingham, Liverpool, Manchester, Sheffield, Leeds, Bristol, Newcastle, Cardiff, Swansea, Glasgow, Edinburgh, Dundee, Aberdeen, Belfast, Dublin.

The preface to this volume is of interest:—

"The present volume of comparative municipal statistics is the result of efforts made . . . for several years past. The subject was brought up in connection with an enquiry into the work of the statistical department of the Council in the year 1912, when a strong opinion was expressed

that endeavours should be made to secure the publication of municipal statistics on scientific lines. A return prepared by the Council in that year relative to the comparative cost of certain municipal services in a few large English towns brought out several interesting facts and stimulated further inquiry. The Council subsequently resolved to compile annually a volume of comparative municipal statistics, similar to those issued in France, Germany and Austria, . . .

"It was deemed desirable to enlist the co-operation and obtain the advice of the other great municipalities, and a conference of officials was accordingly convened by the Council. . . . The undermentioned cities, amongst others, were represented: Birmingham, Leeds, Liverpool, Manchester, Sheffield, Aberdeen, Dundee and Glasgow.

". . . It was decided, subject to the acquiescence of the Council, to publish a preliminary survey at an early date, following, as far as possible, the lines of the volume of *London Statistics* already published by the Council.

". . .

"The first object of a volume of this kind is to ascertain whether any economies can be effected by a yearly comparison of the cost of municipal services; in other words whether any standard of expenditure on particular services can be laid down, even approximately. . . .

"A second object is the increase of efficiency. The value of trustworthy official figures in relation to tramways, the supply of water, gas and electricity, is obvious; or, again, take the question of the improvement of health and the elimination of disease. There is no need to lay stress on the importance of careful statistics in the war which local administration has to wage against disease, and the amount to be learned by a careful comparison of the results of different methods in different areas.

"It may be contended that this work falls within the province of the Imperial Government. The reply to this contention is that the third object in view in comparative municipal statistics is that of self-defence. The question at issue is that of the relationship between local and Imperial taxation. . . . It is well known and recognized that the system of assigned revenues has broken down and that the Exchequer contributions are inadequate. Not only has the cost of the national services which are defrayed locally increased enormously, but additional duties are constantly being imposed by Parliament which threaten to bring about local bankruptcy. . . .

"Fourthly, comparative municipal statistics are necessary in the interest of the reform of our local self-government, which is rapidly becoming chaotic. There are innumerable detailed reforms required in the law of rating and assessment, the poor law and other local matters. Apart from the reform of the law, there is the question of its gradual consolidation with a view eventually to a complete municipal code. Under the present system of central government these reforms can only be obtained if the information is prepared by the local authorities. Otherwise, when the central government is at length compelled to listen to the cry for reform, it may be declared that necessary information is lacking.

"Moreover, apart from legislation, various administrative questions which are of great importance to local administrators, but have no interest to politicians, will always be shelved unless local authorities act together. . . ."

Unfortunately this volume was not only the first but also the last to be published; before the war of 1914–18 was over there had been a change of policy on the part of the Council, and this and other statistical projects were abandoned. Since 1915 very little progress indeed has been made. The London County Council and other large municipal authorities have continued to publish comprehensive volumes of facts relating to local services, but the impulse and, indeed, the skill for the critical examination of these facts has, with rare exception, been lacking. The short-sighted view has prevailed that social studies were of little value to ratepayers, who have been so anxious to govern cheaply that they have not risked the small expenditure involved in examining the efficiency of their administration. Such studies have been left to voluntary effort, to free lance social economists and reformers.

During and since the war of 1939–45 sheer necessity once more focused attention upon the importance of adequately planning social endeavour. The War Office may remain the butt of the comedian, but the fact is that during the war the Armed Forces displayed a degree of scientific organization never attained before. The planning of research preceding production and the planning of the actual production of the instruments of war has provided some classic examples of fact-finding and analysis by teams of workers over a wide field. As with destruction, so with

construction; whatever divergent political view may obtain regarding the National Health Service, all will agree that the preparatory statistical work was of high quality. Behind all this lies the realization that "organization is a problem of analysis and its converse, synthesis. It involves first the exact determination of the separate elements to be co-ordinated, whether these be sources of supply, processes of manufacture or divisions of administration. Secondly, these elements must be translated into the appropriate functions which they necessitate; for example, specialized branches of labour or steps in accounting procedure. Finally, the separate functions must be integrated into a coherent pattern, sufficiently stable to be economical of effort while remaining sensitive to changes in any of the basic circumstances with which the plan has to deal."

"Whether it be a question of selling deaf aids or supplying armies or repairing roads, the plan of action must be preceded by fact-finding and the careful arrangement of the facts in relative importance to the problems under review, the assessment of time factors and secular trends, the forecast of emerging quantities, the skilful balancing of compensatory factors. The more complete these processes, the more easily is the plan derived. Subsequently the construction of the plan, the final synthesis of functional components, is a task calling for breadth of vision, a capacity to visualize the pattern of action in its entirety without being distracted by detail, and an appreciation of all that is involved in the relation between the plan and the problem which it is to solve—an ability to keep the formula flexibly linked to the material from which it derives."(1)

The Need for Local Government Statistics

This is especially pertinent to local government, which attempts to bring the "greatest good to the greatest number," and by its very nature embraces a wide variety of types of person and of personal needs. Here is an expansive field of administration and a great diversity of functions. From drains to fire engines, from day nurseries to restaurants, from libraries to public baths; from the cradle to the grave we are constantly in need of local services. Yet if the organization is to succeed it must be understood by all who are affected by it; simplicity must be the objective in order that every member of the community may consciously co-operate in the machinery provided to satisfy his own needs, and thus to contribute mutually to the attainment of the greatest degree of freedom of action for all. There is another important aspect. Government, central or local, is labour-consuming; it is a drain on the production resources of the community, and it is imperative that there should be the maximum achievement from the smallest effort; there is no room for wasted man-power and equipment. Too often, however, there are large numbers of different services which, though they react on each other, are run by watertight sections of a local government organization, resulting in duplication of effort, delay in movement, and the creation in the mind of the man-in-the-street of a feeling of frustration and inhumanity. In most cases councillors are enthusiastic, and officials are industrious, but few of them are sufficiently concerned to see whether the best methods are being used, and if the purpose of the service is being achieved at the lowest cost.

A local authority cannot operate efficiently without making a continuous study both of the communal needs which it attempts to satisfy and of the effectiveness of the machinery by which social services are run. The collection of comparative statistics provides practically the only means by which this study can be made; and no department of a local authority can escape the responsibility for engaging in some statistical research. Yet if there is a crying need for statistical activity in local government, there is equally a need for a greater degree of skilled direction of this activity than has hitherto been exercised.

Most local authorities collect statistics. The trouble is either that they collect too much of the wrong kind, or that they spend too much time in collection and too little time in analysis. A statistician engaged in local government should make paper control his first priority, in order to ensure that (*a*) no fresh piece of paper goes into circulation if the information is already available from existing records; (*b*) that existing records contain only the irreducible minimum of information for the adequate fulfilment of an essential purpose; that they do reach the officials for whom they are intended with the minimum of delay; and that they do not persist in use beyond the minimum period during which it is necessary to operate them. Many authorities allow each and every small section of the administrative machine to collect returns and keep records without any check as to whether the information is already available elsewhere, or whether the records are

designed to produce the essential facts with the least expenditure of labour. There is inadequate inter-sectional liaison. The result is that there is considerable wastage of labour involved, both in the use of individual returns and in their overlapping with other returns. Paperwork multiplies, and the public or the officials on whose co-operation the accuracy and promptitude of the information depend become alienated.

It will be advantageous now to turn to concrete illustrations of some of the aspects of local government administration where statistics may have important applications.

Finance Department

The financial burden of local services is greater in the more necessitous areas, so that the greater local expenditure falls upon those who are less capable of bearing the burden. To meet this difficulty the Central Government either makes direct *ad hoc* grants for special services, e.g. education, or provides a "block grant" to equalize the local expenditure by giving more assistance to the poorer than to the richer areas. These "block grants" are provided from national taxation, and are shared among the local authorities by having regard to their rate income and to the social conditions, such as population (and the number of children in that population), degree of unemployment, overcrowding, etc. In order that an authority may protect its interests when these block grants are periodically revised, it is essential for these factors, upon which the grants are based, to be kept continually under review. On the general question of the distribution of expenditure between central and local funds, there is bound to be some conflict between the central legislature and the local authorities. In this conflict the strength of the local authority will lie in the adequacy of the social information at its disposal. There is scope for statistical research into the whole structure of local government finance, and for the examination of differential levels of valuation. Examples of this type of study which immediately come to mind are the valuable researches of Professor Hicks on the incidence of local expenditure and rate charges(2).

With regard to local expenditure, it will be appreciated that the local authority neither meets its liabilities nor receives its rate income on one particular day in the year. The financial transactions are continuous, and arrangements must be made to regulate income and expenditure, so that there is always money in the till, i.e. a working balance, or a carefully calculated scheme of temporary borrowing by overdraft to meet current commitments. Furthermore, the authority cannot wait until its liabilities have matured before it levies a rate. It has to ask for the money in advance. It is, of course, a legal obligation that a budget must be prepared and an estimate made of future liabilities. The task of preparing these estimates begins with the spending departments, which furnish statements of the services they expect to maintain, and the probable cost. The statements are then examined by the Finance Officer. It does not need a statistician to prepare these estimates, but it may often be possible for the statistician to suggest some approximate rule which will enable these estimates to be checked by the Finance Department without the need for detailed recapitulation of the original calculations. By isolating the more important variables concerned, the statistician may be able to provide, for example, a formula by which the cost of redecorating a school may be related to the whole volume of the building, total window space, or other details which, once scheduled, can be repeatedly used. Although a cash balance is necessarily carried by a local authority, it is not withheld from the rate-payers indefinitely but is, of course, taken into account in fixing the subsequent rate requirements. For financing capital expenditure it may be necessary at certain times for these cash balances to be augmented in order to bring the liquid resources of a council to a level required for the repayment of maturing or for getting new capital commitments without recourse to borrowing. At other times, according to the trend of interest rates, an entirely contrary policy may be necessary. A local authority can never become bankrupt, but the maintenance of the appropriate degree of fluidity in its assets, coupled with stability in the rate levied, is an important undertaking requiring careful and expert adjustment, and involving some degree of statistical skill.

The statistician is not concerned with accounting procedure, but its scope for statistical research in the organization of internal audit and financial control. The complete audit of accounting transactions and calculations is often laborious and costly. It may not always be necessary. It seems possible that sampling investigation of the incidence of different types of error, together with a quantitative assessment of the financial cost of such errors, might provide a valuable guide to the

degree of audit required. In restricting audit a finance officer necessarily takes the risk of undetected errors. He would be more confident of his procedure if he were able to measure this risk.

Costing analyses are an essential part of the machinery for assessing the efficiency of local services. These operations are properly the function of local accountants or other technical officers, but complex problems of allocation frequently arise in which the statistician, or at least statistical knowledge, can be of assistance.

Housing and Valuation

Since the local authority depends upon the rate income based upon property within its area, it is obviously essential that it should possess intimate knowledge of the rateable values involved. Accurate assessment of these values is, of course, the function of the Valuation Officer, and it would be most undesirable for the statistician to trespass in this field; but there are cognate problems of a statistical character in the solution of which statistical advice might be valuable. Changes in the relative values of different types of property are made by quinquennial adjustments in the valuation lists. In so far as these adjustments are determined by changes in the economic structure of the nation as a whole, it should be possible to discern the direction of movement and to make some assessment of differential trends and values. New properties are constantly appearing, and older properties are being rebuilt or demolished. There is a constant movement reflecting the absence of contraction of local industries, movements in the dormitories of the population, new fashions in entertainment, or the broader redistribution involved in long-term planning. The planning of capital expenditure already referred to also entails the analysis of these changes.

The local authority is commonly not only a direct provider of housing accommodation, but also exercises considerable central control over all building development within its area. At the present time the local authorities are still facing a tremendous accumulated demand for dwellings, arising from the cessation of normal building during the war years and acutely aggravated by war damage. The short-term problem is to marshal all available resources by the requisition of unoccupied property and the provision of temporary bungalows, and the allocation of this immediately available accommodation to those in need of it. In measuring priorities, due consideration must be given to grading applicants according to whether their existing living conditions are unhygienic, and to their occupation and the necessity for living close to the place of employment. Account must be taken of size and age and sex distribution of family, illness or incapacity of other members of the family, etc. Doubtless in many cases ingenious systems of numerical rating have been evolved. In any case, the arrangement of facts in such a way that selection of the more appropriate candidates can be efficiently accomplished requires a specialized skill.

Where the municipal body is charged with the formulation of a long-term town-planning scheme, considerable statistical research is involved. In discussing the plan for the redevelopment of London, which was produced for the London County Council by J. H. Forshaw, the architect, and Professor Patrick Abercrombie, the authors say(3): "If a plan is to be authoritative, the planners must first make a profound study of existing conditions by means of a civic survey of the most detailed character; secondly they must familiarize themselves with the wishes of the people, whether as inhabitants of houses, or workers in offices and industries, masters and men; they must consider the views of the public authorities . . . they must discuss their proposals with the great statutory undertakers, railway companies, Port of London Authority, the Metropolitan Water Board, the Post Office, and others; they must analyse and assimilate numberless plans and proposals, official and otherwise, that have been prepared from time to time . . . the civic survey which has been prepared as a basis for a plan has covered a very wide field and has broken much hitherto unchartered ground. . . . Besides recording facts, the numerous surveys have had a secondary, but still important and indeed essential, use, namely of dividing the problem of London into its many component parts and of making it possible to comprehend, both separately and in relation to the whole. Without this approach the problems and complexities of London appear almost overwhelming." The great detail of the County of London Plan will, for those who have examined it, be evidence of the large volume of research involved in long-term planning. Apart from the prodigious survey work of an architectural or engineering nature, a statistical analysis was made of many problems, such as housing density, the industrial and occupational distribution of the population, traffic needs, the need for hospitals, schools, public utility services,

etc., trends in the migration of industry and population, the present use of river frontage, distribution of open spaces, and many other aspects of town planning. All of this was essential.

Education

As a result of the Education Act of 1944 school authorities now have to provide educational services from the nursery to the adult stage. The school-leaving age has been raised, and many different types of secondary schools have to be provided. Apart from actual education, other services are to be provided, e.g. medical inspection and treatment, milk distribution, and school meals centres. The numbers emerging year by year in any particular category will be sensitive to short-term fluctuation in fertility or in employment conditions and, to maintain an economic balance between the different branches of education, forward planning is essential. If the best use is to be made of actual school accommodation, it is essential that the trend of the infant population should be kept under observation. For this purpose the author has already been asked to provide school population estimates for several years ahead. These estimates, which were based upon the trend of fertility and mortality, had also to take into account the intentions of the Council with regard to the "decanting" of the population from the county to out-county areas. More interest is now taken in the comfort of children while at school, and the author has had to meet demands for scales of measurement of children according to age, in order to assist in the standardization of types of furniture and other equipment which needs to be graded in size according to age. Considerable attention is being paid to the provision of adequately nutritious meals at school restaurants, and the author has been called in for statistical advice in connection with the assessment of relative degrees of vitamin destruction in different types of cooking and in the examination of wastage in the restaurants. A random sample of children who have begun to take school dinners after having been fed at home are at present under observation, to ascertain whether their growth curve shows any susceptible change in either direction as a result of this change in feeding habit.

Mention might be made here of the psychometrical methods which have been developed in recent years to furnish criteria for educational grading. This is properly the field of the expert psychologist, but the statistical methods which have evolved in development of factor analysis and of which an excellent account has been given by Thomson(4) have wider applications. They have been applied to the assessment of social attitude by Eysenck(5). They provide a valuable control over multiple regression analysis which cannot be neglected by the social statistician. Further reference will be made to this in a later section.

Supplies and Municipal Trading

Many of the larger authorities engage in commercial activity of some kind, whether it be the bulk buying of such commodities which, being common to the requirements of many branches of its service, may be more cheaply obtained by such practice, the direct conduct of restaurants, libraries, or the sale of food preparations in welfare centres. Some authorities engage in farming activities. Where there is actual production of commodities or where there are central purchasing transactions, the adequate provision for fluctuations in supply and demand may give rise to problems in multiple correlation or other aspects of market research. In matters such as these the local officials, accountants, etc., are expert, but there may be times when it is difficult for them to see the wood for the trees and there is scope for independent consultation. In many instances it is not the problem itself but its recognition that demands skill. This is true of much statistical experience.

Engineering

In normal times a municipal authority must organize the constant repair of the roads for which it is responsible, and occasionally the building of new roads. It would appear that the planning of a programme of road work, which would employ a constant and therefore well controlled fund of labour, rather than one of frequent rushes and sudden reinforcement of labour force with loss of efficiency, must be a problem of careful experiment and observation. Road surfaces, like persons, have lives the length of which depends upon their constitution and wear and tear. Just as the mortality of a group of persons can be observed, and the future wastage by death forecast, so also ought it to be possible to forecast the road surfaces which will fall due for repair in a

particular year. A drainage scheme or the laying of a cable might cause the premature death of a road surface, but a basic programme would be valuable. Baths and washhouses and even the collection of refuse are projects likely to be influenced by a varying demand which is capable of quantitative assessment. It should be possible to observe an increased use of baths long before queues appear; and it seems a pity that the effect of population growth in new urban areas upon refuse collection cannot be anticipated before irate letters appear in the local press, as they so often do.

Staff

The recruitment of local government staff is normally quite haphazard. Vacancies accumulate, and periodically groups of officers are appointed by examination or otherwise. The result is unfortunate. Promotion bottlenecks frequently occur. The officer who is unluckily appointed just after a block of young staff have moved up the scale finds himself waiting for the retirement of men only a few years older than himself. He becomes disgruntled, and either takes his experience elsewhere or loses interest in his work. The public service suffers. Much of this frustration could be avoided if staff recruitment were to be planned and based upon a proper study of the wastage in the various grades. The problem may be expressed in the following simple terms: An authority employs certain numbers of staff in grades A, B, C, D, etc. It recruits at age 17, and promotion between grades ranges over specified periods of age. What should these periods of age for promotion be, and what should be the retirement policy of the authority in order to assure specified promotion prospects to new entrants? The application of the service-table technique has already been set out in simple terms by Jones (6), and more completely by Seal(7).

It might be appropriate to mention here, rather than in the Health Section of this paper, another application of statistics in staff work, namely, the analysis of sickness records. It is important that an employer should keep adequate records of sickness– first, to provide knowledge of the health of the *individual* employee, to ensure that he, or she, is fit for work and is employed in satisfactory conditions, and secondly, to provide information about the health of occupational *groups* of employees, to ensure proper supervision of the conditions of work. By continuous review of the records and comparison with similarly employed groups in other organizations, an assessment can be made both of the ill-effects of particular conditions, and the value of any measures applied to reduce the incidence of sickness. The statistical requirements are comparatively simple, and for the most part are necessarily recorded in connection with the payment of salaries or wages. It is a matter rather of suitable organization than the institution of entirely new records.

Ambulance Service

Before dealing with the work of the Health Department, it is also appropriate to consider separately the statistical organization involved in the continuous test of the efficiency of the Ambulance Service, in which the arrangements to be described are of general application to other types of organization involving the deployment of vehicles or other equipment, which is subject to a demand which is always urgent and varies from hour to hour and from district to district. To meet emergency demands for ambulances, the London (Accident) Ambulance Service maintains some 30 ambulances distributed at stations spread over the County. To ensure that peak demands at various points can be met without undue wastage at off-peak times, and that demands are met with the least possible delay, the operation of the Service is subjected to continuous statistical control. As each call is made a summary of the following details of the call is submitted to the Statistical Section:—

Date, day of week and time of day of call.
Agency through which ambulance was called, e.g. police, hospital, etc.
Nature of the call, e.g. street accident, illness, maternity, etc.
Number of patients involved in the call.
Metropolitan borough in which the call arose.
Ages and sex of the patients.
Name of station answering call.
Mileage run.
Times taken to reach case, to pick up case, to reach hospital, and to return to station.
Name of hospital to which patient was taken.

The information is entered on a card of the standard size for mechanical sorting, and the card is so printed that the information can be given by striking through, in respect of each detail, an item in a list of possibilities, or by entering code numbers. Thus, very little writing is involved. The cards are then analysed mechanically to answer questions of the following nature:

What is the peak time of day or day of week for a particular type of call or for a particular ambulance station ?

In which sub-district of London is the pressure heaviest ?

What is the average mileage run ?

What, for different types of call, is the average time taken to reach the patient ?

How many patients are taken to each of the various hospitals in London ?

In this way a careful check can be kept on the adequate deployment of the available ambulances, and on the general efficiency of the Service.

Public Health

There is no aspect of local government in which the value of statistical analysis is more recognized than in the organization of preventive medicine; and it is no accident that where the professional statistician is found in a local authority it is more often in the public health department than elsewhere. The early epidemiologists left the statistical method as a legacy to the Medical Officers of Health by whom they were succeeded, and there are few who do not find it an essential part of their equipment even if there are many who do not sufficiently exploit it.

Social Medicine has often been described as a battle for health, and in the warfare against disease it is essential that the Medical Officer should know what forces are required, and how they can be deployed to the best advantage. He must know where the enemy is attacking and how the battle is going. There must be an efficient intelligence service. The language is, perhaps, too colourful, but the analogy is sound. It is a question of eternal vigilance. Past dangers may return, and new threats are constantly arising.

In public health organization the statistical field has been extended in recent years and is now wide. As Sir Andrew Davidson (8) remarks: "Gradually it is being realized that all diseases can be studied by epidemiological and statistical methods. . . . More than half a century ago Karl Pearson defined the scientific method as 'the accurate classification of facts and observation of their correlation and sequence.' We are now gradually applying the scientific method to medicine. . . . A new era is dawning: an era in which what Whitehead has termed 'the irreducible and stubborn facts' will be elucidated, after which careful analysis and study should provide more and more information about disease, about the types of disease to which particular categories of people are susceptible, and about the methods of prevention that may be applied."

This more extensive study of the actiology of disease is only one side of the picture. The very organization of preventive medicine itself is being more extensively made the subject of scientific study to match that organization to the changing pattern of social life. Steiglitz (9) speaks of the need to investigate "the weaknesses of past and present medical practice, with the purpose of evolving new values, objectives, and techniques more appropriate to the changing social environment of these arduous times. That great changes will occur can be taken for granted. But we must not assume complacently that these will be wholly desirable changes unless they are predicated upon full appreciation of the requirements of society and the capacities and limitations of medical science. Analysis must inevitably precede synthesis."

The first objective must be a study of the constitution of the population for the care of whose health the service is responsible.

(i) *Race*

Where there is a large proportion of immigrants in the community their susceptibility to disease may be different from that of the native population. This differing susceptibility may be due to early environmental influences in the country of origin, to a subnormal standard of living which is sometimes endured by immigrants, or to differing social habits. Whatever the cause, race is a real factor to be borne in mind and special measures of control may be necessitated, the extent of which will depend upon the strength of the racial element in the population. One

recent example may be quoted. The Prophit Survey (10) has shown that groups from rural Ireland and Wales have a lower incidence of sensitization to tuberculin (than urban groups in those countries, and lower also than rural groups in England), but, more important still, evidence was found suggesting that given equality of exposure and environment, the morbidity of Irish and Welsh groups was much higher than in other groups, and the report goes so far as to suggest a genetic factor, i.e., that there are racial differences in resistance to tuberculous infection even within the United Kingdom.

(ii) *Sex*

Apart from the obvious differences in occupational risks as between the two sexes, giving rise to differing morbidity experience, and to the special risks of pregnancy and childbirth which affect women, there is evidence of constitutional, as distinct from environmental, causes of variation in susceptibility to certain diseases. An arbitrary selection of statistics illustrating these differences is shown in Table I.

TABLE I.—*Sex Factor in Disease: Some Contrasts in London A.C. in* 1938

	Male	*Female*	*Ratio Male to Female*
Infectious disease.			
Diphtheria—Notification-rate per 1,000 living at ages 0–4 years	10·62	8·97	1·18
Malignant disease.			
Cancer death-rate, per 1,000 living, 55 years and over	8·91	6·40	1·39
General disease.			
Diabetes death-rate, per 1,000 living, 55 years and over	0·612	0·838	0·73
Respiratory disease.			
Bronchitis, Pneumonia, Influenza death-rate, per 1,000 living, all ages	1·423	0·888	1·60
Digestive disease.			
Peptic ulcer death-rate, per 1,000 living, 35 years and over	0·536	0·117	4·58
Degenerative disease.			
Nephritis death-rate, per 1,000 living, 35 years and over	0·623	0·493	1·26

This table is based upon 1938, the last year for which the Registrar-General furnished sex age-group estimates of the London (A.C.) population. (Such estimates for subsequent years have been made for departmental use, but it was thought preferable to use authoritative figures for the present purpose.)

The need for the separate observation of maternal morbidity is not, therefore, the only reason for requiring a separate assessment of the numbers of each sex at risk in the population.

(iii) *Age*

The pattern of disease changes from age to age throughout life. The principal causes of death in successive age-groups are shown in Table II. The figure shown in brackets is the percentage of all deaths in the sex age-group assigned to the specific cause or group of causes.

TABLE II.—*Principal Causes of Death in Age-Groups. London A.C.*, 1946

Males	*Age*	*Females*
Premature birth (24·8)	Under 1 year	Premature birth (22·4)
Pneumonia and Influenza (14·8)		Pneumonia and Influenza (16·1)
Congenital malformations (13·1)		Congenital malformations (13·8)
Other diseases of early infancy (12·6)		Other diseases of early infancy (13·1)
Accidents (25·1)	1 to 14 years	Tuberculosis (20·0)
Tuberculosis (17·3)		Contagious diseases (16·8)
Pneumonia and Influenza (12·0)		Accidents (15·4)
Contagious diseases (8·4)		Pneumonia and Influenza (11·4)
Tuberculosis (35·5)	15 to 24 years	Tuberculosis (55·5)
Accidents (12·5)		Heart disease (6·1)
Heart disease (7·1)		Accidents (3·8)
Tuberculosis (29·1)	25 to 44 years	Tuberculosis (25·6)
Heart disease (11·2)		Cancer (16·7)
Cancer (10·1)		Heart disease (13·4)
Cancer (22.3)	45 to 64 years	Cancer (30·3)
Heart disease (21·2)		Heart disease (18·2)
Bronchitis (10·2)		Intracranial lesions of vascular origin (10·8)
Tuberculosis (9·3)		
Heart disease (33·7)	65 and over	Heart disease (37·7)
Cancer (16·1)		Cancer (14·1)
Bronchitis (10·0)		Intracranial lesions of vascular origin (12·4)
Intracranial lesions of vascular origin (9·3)		Bronchitis (7·3)

(iv) *Occupation*

The fact that there are special occupational risks hardly needs illustrating. Special dangers have been recognized in the statutory obligation to report cases of specified diseases to the Inspector of Factories. The Decennial Supplements of the Registrar-General (11) on occupational mortality contain a wealth of evidence pointing to special occupational hazards, as distinct from the more general sanitary influence of the social environment determined by the occupation—reference is made to this later. The excessive risk of respiratory tuberculosis among printers, revealed in the 1930–32 report, is one example, and underlined by the morbidity study in this trade made by Bradford Hill (12). Among metal workers, the furnacemen, rollers and their skilled assistants appear to incur a special occupational risk of cancer and respiratory disease. These are two examples taken at random. The more hackneyed examples are diabetes in grocers, and cirrhosis of the liver and cerebral haemorrhage among inn- and hotel-keepers.

The analysis of the population according to occupation is not only essential for the study of pure occupational risks, but also automatically divides the population into groups of differing economic levels.

(v) *Housing*

Housing conditions, apart from their influence as a component of the general economic standard of living, play an important part in what Greenwood has described as "epidemic and crowd diseases." The more people are crowded together the more easily are infections transmitted from one to another. It is difficult to separate this factor from the resistance to the disease which is governed by other social conditions, such as, for example, nutrition, which are themselves strongly correlated with housing conditions. Thus Daniel (13) found that incidence of rheumatic heart disease was associated with the standard of living as measured by the family income, but that independently of this effect the incidence was also related to housing density as measured by the number of persons per room. Owing to the influence of other factors the evaluation of the

distinctive role of overcrowding is often a problem in multiple regression analysis of great complexity. Commonly we are not dealing with controlled experiments, but are rather compelled, as Buckatzsch remarks (14), to regard the morbidity rates "recorded in different areas at given times as the results of vast biological experiments performed impersonally by the operation of the social system in its progress through time," and he has given a timely warning against the acceptance of partial regression which owing to the non-independence of the "explanatory" factors are not really determinate, and do not really measure the relative strength of the number of factors employed. The use of Confluence analysis or of Factor analysis to guard against these dangers is a suggestion which, if followed, may prove to be a deterrent against the construction of multiple regression equations, based upon a mixed collection of factors, which are a "good fit" to the data but have little real meaning.

Whatever the difficulties, the need to attempt an assessment of the independent effect of overcrowding is sometimes inescapable, and it is essential to have basic information relating to the density of housing.

(vi) *Social Conditions Generally*

Few will now deny that disease and poverty are closely linked. A body cannot be healthy unless it is properly nourished and exercised, and functions in an environment in which it can best thrive. Disease is not so much now regarded as the blind play of chance or punishment for sins; we are realizing more and more that a lack of balance between input and output of energy, in short, underfeeding and overwork, can produce disease, as also can the fear and anxiety of economic insecurity or the stress of bad factory conditions. A definite association between the social environment and the incidence of a particular disease is not any longer the discomforting revelation of the pioneers of social medicine, but a common expectation and an accepted basis for the organization of preventive medicine.

A broader view must be taken than that which is implied by a mere reference to poverty, for it must be borne in mind that disease can arise from dietary excess as well as from starvation, and the Registrar-General's Report on occupational mortality, 1931 (11), illustrates disease relationship to social gradient in both directions. For example, tuberculosis mortality rises steeply as the social gradient falls, while diabetes runs, in men, in the opposite direction, mortality falling more heavily on the affluent.

How are these social factors to be measured? In planned surveys it is possible to tabulate measurements of specific elements in the environment, but in most normal public health work we are faced with the circumstances cited by Buckatzsch (*loc. cit.*), who reminds us also that the material, "particularly that used to define the environmental conditions studied, consists of information collected for other, usually administrative, purposes and of only incidental relevance to the problems of social medicine."

Sources of Demographic Information

How is this classification of the population by several axes obtained? The statistician is left very much to his own resources. The full information is available only from census returns, and the last census in this country was taken in 1931. Since then there has been a war in which many persons of military age have been killed on the battlefield, and in which persons in other age-groups have been killed in air raids or had their deaths hastened by war conditions. There have been considerable changes in fertility and mortality. The following comparison for England and Wales shows how dangerous it may be to make any reference to the 1931 census figures:—

TABLE III.—*England and Wales—Total Population. Percentage in Each Age-Group*

	1931 *Census*	1945 *Estimate by Registrar-General (Statistical Review)*
0–4	7·5	7·3
5–14	16·3	13·2
15–24	17·3	14·8
25–44	29·8	31·2
45–64	21·7	23·3
65–	7·4	10·2

The present position is that at quarterly intervals the Registrar-General circulates, privately, estimates of the total civilian population in each local area. Wartime movements have hitherto made it impossible for these estimates to be given in full age distributions, but before the war such age, and sex, group estimates were provided annually, and with commendable reliability, and it is to be hoped the service will soon be restored. The Registrar-General does provide, annually, for local areas simple division of the population into the groups 0–4, 5–14, 15–, both sexes combined, and this is of great value.

Housing data must be sought from the records of surveys made by the departments responsible for meeting the present heavy demand for accommodation, and as a result of surveys made by the Valuer of the London Council of war damage and subsequent repairs and new building, it has been possible to derive estimates of the number of dwellings in each of the metropolitan boroughs of London.

Before the war the Ministry of Labour published, on a regional basis, statistics of persons employed in different occupations, so that local changes in occupational distribution could be followed. At present only national figures are published in the Digest of Statistics issued by the Central Statistical Office, and it is to be hoped that local figures will be issued as soon as possible.

Some use can be made of sample surveys made by the Social Survey or other agencies for Government Departments. These surveys have been made to obtain social information to facilitate administration or to gauge social difficulties during the war, but the results, where published, or otherwise made available, often throw valuable light on population structure. In 1946 the Social Survey were able to assist me with two important facts relating to the London area—the average size of family, and the average number of households occupying the same dwelling. Local authorities might well consider making direct use of sampling technique themselves for this purpose. With efficient organization, valuable information can be obtained at low cost.

An excellent example of the sampling method is provided by the social survey carried out by Grundy and Titmuss and others (15) in Luton in 1945. A simple demographic questionnaire was completed, at personal interview, by members of the W.V.S. in respect of every house in areas of unfit houses, one house in five in older parts of the town and one house in ten in the rest of the borough (these areas being separately coded). These questionnaires, which were analysed on punched cards (covering over 16,000 individuals), gave up-to-date information *inter alia* of housing conditions, age and sex, marital condition, birthplace, duration of residence in Luton, occupation, education (of school-children), and fertility. The cost, excluding clerical work, was just over £300. Not only was a completely detailed picture of the local population obtained, but as a by-product valuable material relating to fertility trends was subsequently analysed(16). This kind of survey might well be made by all local authorities, especially where the population is reasonably compact.

There is one aspect of population measurement in which one is on somewhat surer ground, namely, the estimation of the child population. Here it is possible to make a direct estimation from births in recent years, allowing for mortality and migration, where the latter factor can be assessed. Thus it was possible recently to produce forward estimates of the London population of school age, by individual years of age, up to 1952, to assist the Education Department to plan accommodation. Information of probable "decanting" out of the county was furnished by the Director of Housing.

Where national figures are available the statistician must use his local experience and ingenuity to decide, how far the local figures differ in constitution from the national distribution. For example, if there has been in the past a consistent relationship between local and national fertility, then the local female population of child-bearing age-group 15–45 could be estimated by reference to local births, the national figure for females in this age-group and this known difference in fertility. Similarly, if local cancer mortality is known to be at the same level as in the country as a whole, then the local population at, say, 45 and over could be estimated by direct proportion from the national figure on the basis of local cancer deaths in this age-group.

What is the State of Public Health?

When an assessment has been made of the numbers and classes of persons for which the health authority is responsible, the next objective is the measurement of their state of health or, more modestly, since health is so difficult to define, their state of ill-health.

In total the available and up-to-date information about morbidity amounts to all too little. Apart from knowledge of the incidence of certain infectious diseases which are compulsorily notifiable, and certain other very limited data, e.g. school health statistics, the Medical Officer of Health has only rates of mortality from different causes of death from which to judge the health of the community. The bulk of the data available relates, therefore, not to morbidity but to mortality; not to current ill-health but to certain degrees of past ill-health.

It is, indeed, fortunate that the mortality statistics available in this country are of such a high standard. We have, indeed, come a long way since 1837, when the General Register Office was instituted and Dr. William Farr began his monumental task of educating the medical profession to a scientific basis of death certification, in order that deaths could be equally scientifically classified by cause. In that year coroners' justices were still using phrases such as "accidental death," "visitation of God," "natural causes," and other terms equally vague. Dr. Farr took immediate action by obtaining the concurrence of the royal colleges and of the Apothecaries' Society to the issue of a joint circular, in which these authorities severally undertook to certify, in writing, the cause of death of every patient dying under their care; they further earnestly appealed to their medical brethren throughout the country to adopt the same practice, but progress was slow, and Dr. Farr had to toil ceaselessly to produce a satisfactory nosology, to impress upon practitioners "the duty of searching beyond symptoms for the actual causes of disease." Diphtheria was not distinguished from Scarlet Fever until 1855. Typhus, Enteric and simple continued fevers were first distinguished in the death returns for 1869, having been previously returned under the head of Typhus. These examples, which are a few of many gradual stages of improvement, are chosen as a reminder that until the end of the nineteenth century, when bacteriological advances were being rapidly made, the identification of a specific cause of death, particularly when the disease was of an infective type, was difficult, if not impossible, for the ordinary practitioner. These examples also illustrate the fact that any investigation of long-term trends in mortality for certain diseases is sometimes invalidated by the differing standards of diagnosis which have operated at different times, and the statistician must be continually on guard against the dangers of making unwarranted comparisons. The extremely detailed classification of causes of death which the Registrar-General is now able to make is a tremendous advantage to the public health statistician, and a debt of gratitude is owing to all those who have made this possible. One element in the policy of the General Register Office deserves comment. Successive Registrars-General have always been careful not to adopt classifications which are more detailed than that which the prevailing standard of certification would support. There is always the danger that the drive to subdivide the classes into which deaths are assigned, if carried out without regard to the ability of medical practitioners, in the present circumstances, to give certificates that would permit such subdivision, might lead to a reduction rather than an increase in accuracy. There would be a danger that, in relation to the new subdivision, current certificates of causes of death would be incompletely defined and would be pushed into a subdivision, to the head of which they would bear no statistical relationship. Pedoe (17) thinks we have already gone too far in this direction, but it is clear from a study of the statistical reviews of the Registrars-General that changes in classification do not take place until its effect has been properly studied in relation to the whole range of types of certificates submitted from up and down the country. For example, the rules of selection of joint causes were not abandoned until statistics on both the new and the old basis had been collected for a considerable period of time, and until the action appeared to be fully justified. Such precautions increase the confidence with which the final choice of classification is accepted.

In what way are the general mortality figures for the local area to be studied? The local mortality must be adequately analysed, not only by cause of death, but by age and sex, so that either standardized death-rates for each cause can be calculated, or actual rates can be derived within each sex and age-group. Unless this precaution is taken, of course, no reliable inference can be drawn from the statistics, because a change in the age and sex constitution of the population might cause large disturbances in the rates for all ages and sexes combined. An extreme illustration of this source of error arose at an inspection of the mortality amongst patients in mental institutions in the London County Council area during the war years. It was found that the crude death-rates from peptic ulcer per thousand living in the mental institutions over the years 1943–1945 was 0·434, but when standardized on the basis of the population of the County of London as a whole this rate was only 0·232. The death-rate from the same cause in the County

of London was 0·221. Thus, acceptance of the crude figures without regard to the difference in the constitution of the two populations would have suggested that the risk of death from peptic ulcer was greater among mental hospital patients whereas, in fact, the risk is no greater than in the general population.

Comparisons between places or between different local areas, especially between local areas in different countries, are complicated by the changes which have taken place in the international list of causes of death from time to time, and in the case of foreign comparisons by differences in diagnostic practice. In regard to changes in classification, one need only refer to the most recent example, namely that made in 1940, when a new international list was put into operation simultaneously with the abandonment of the rules of selection which had hitherto operated in joint causes. This change has produced a sharp discontinuity in the rates of mortality for certain of the principal causes of death, notably heart disease, bronchitis, pneumonia and other respiratory diseases, cancer and nephritis. Full details of these changes have been given in the Registrar-General's *Statistical Review* for 1940, and it is not proposed to set them out in full here, but the following table shows the combined effect of these two changes upon the death statistics:—

TABLE IV.—*Changes in Assignment of Deaths to Causes*

Cause	*Approximate change as a percentage of those formerly assigned to this cause*
Influenza	−11
Cancer	− 3
Diabetes	·30
Heart disease	−10
Other circulatory	− 6
Bronchitis	+100
Pneumonia	+ 5
Other respiratory	+50
Nephritis	+12
Diseases of pregnancy, etc.	+10

One advantage of the dual statistics tabulated in advance of this change has been that the Registrar-General has been able to provide approximate factors by which pre-1940 statistics can be redistributed on the new basis, in order to overcome the discontinuity which would otherwise operate.

With regard to differences in diagnostic practice it is, for example, impossible to compare the mortality from respiratory diseases in London, where many deaths are assigned to bronchitis, with the mortality in New York, where few deaths are assigned to this cause.

The more common trouble is that if an attempt is made to trace through a number of years the mortality from a particular disease, it may be found that the disease is sometimes shown separately and sometimes grouped with other diseases, and where grouped it may change its present class from time to time.

In the light of these difficulties Pedoe (*loc. cit.*) emphasizes Professor Bradford Hill's caution (18) that "in making comparisons between death-rates from different causes of death at different times or between one country and another it must be realized that one is dealing with a material which is in Raymond Pearl's words 'fundamentally of a dubious character' "; and forms the opinion "that mortality by causes (when dealing with statistics of a whole country) is only of significance when grouped together to give a few main classes. Even then trends must be watched closely for possible transfers among these main classes."

At the present it does seem advisable, at least for the purpose of comparing trends, to analyse deaths in broad groups.

For example, by grouping together deaths from heart disease, other diseases of the circulatory system, intracranial lesions of vascular origin, nephritis, to which is added bronchitis (owing to its frequent concurrence with heart disease, coupled with the fact that well over 90 per cent. of bronchitis deaths are in persons over the age of 45), a general picture is obtained of the trend of degenerative disease; this group covers more than 40 per cent. of the total mortality. The

ageing of the population is illustrated by the fact that the crude death-rate from these causes rose from 4·74 per 1,000 living at all ages in 1889 to 6·27 in 1947. Next in importance come deaths from cancer and malignant tumours—a simple and homogeneous group, but one requiring careful statistical treatment. It is difficult, especially in dealing with particular sites, to separate the effect of improved diagnosis from absolute changes in mortality. Thus the registered death-rate from cancer of the respiratory system of males in the County of London has risen from 0·236 per 1,000 in the period 1931–33 to 0·737 in the period 1945–47, but there is no doubt that a very large part of this increase is due to a progressive increase in the efficiency of radiological detection. The effect of improved case-finding is often cumulative. As more cases are found, so there is an increased tendency to look for them. The death-rate from tuberculosis is an important index of the average standard of living, and particularly of the balance between energy output and energy input. The long-term trend has been a steady fall with improving social conditions and methods of control. The death-rate from pulmonary tuberculosis in London has fallen from 1·04 in 1920 to 0·62 in 1947. In 1941 the death-rate from pulmonary tuberculosis in London rose to a peak at 72 per cent. above the 1938 level, mainly due to the impact of the hard conditions of war upon the existing tuberculosis population. In their study of the relationship between tuberculosis and social conditions in England, P. D'Arcy Hart and G. Payling Wright (19) found a special sensitiveness of young adults to social conditions as regards their mortality from respiratory tuberculosis, a sensitiveness which in their words " is possibly the effect of combined environmental and biological causes—the stress and strain of industrial occupation during the important formative period of adolescence and early adult life." They found that changes in general social and economic trends, e.g. the check to the rise in the standard of living which occurred about 1900 and the accentuated increase in the industrial employment of young women taking place about the same time, were reflected in the trend of tuberculosis mortality in young adults. There was evidence of " local determining factors, i.e. unsatisfactory local social conditions, rendering the effect upon the young adults in certain localities especially severe. Of these local social conditions, housing appears to be of great importance. The part played by other factors in poverty is less clear. In the London boroughs, where the poverty data available are more complete than in the county boroughs, and information is obtainable for average income less rent per head, there is some evidence that other factors in poverty, such as inadequate nutrition, have also been influential." Mortality is thus sensitive to changes in environment and physical strain, though for this disease a more sensitive index still is provided by the morbidity records. Resistance to other respiratory disease is measured by the mortality from pneumonia and influenza, but comparison with the experience of more than a few years ago is invalidated by the dramatic effects of the introduction of sulphonamide and other anti-biotic treatments. This break with the past has also to be borne in mind in surveying the mortality of important infectious diseases, such as measles and whooping-cough, where the principal death risk is from secondary pulmonary infection. The death risk from the common infections is now very low, and they might be treated as one group. Observation of the mortality (in particular age-groups) from such diseases as cerebral haemorrhage or peptic ulcer may give some guide to reactions to changes in the tempo of life (industrial or leisure) and the concomitant strain.

With regard to peptic ulcer, Nicol (20) has suggested that in the aetiology of gastric ulcer, local

TABLE V.—*Males aged 20–65—England and Wales*

Social class	*Deaths registered in 1930–32 as due to—*		*Ratio Duodenal to Gastric D/G*
	Duodenal ulcer	*Gastric ulcer*	
I. (Professional, etc.)	77	76	1·013
II. (Between I and III)	465	598	·778
III. (Skilled workers)	1,207	2,119	·570
IV. (Between III and V)	419	870	·482
V. (Unskilled workers)	473	1,001	·473

(Registrar-General's *Decennial Supplement—England and Wales*, 1931, Part IIa.)

trauma of the gastric mucosa is the important factor, while in the aetiology of duodenal ulcer psycho-physiological disturbances are to be identified as the operative factors. Nicol compares the relative incidence of the two types of ulcer, and suggests that " the swing from a preponderance of gastric ulcers to a preponderance of duodenal ulcers that has occurred since the beginning of the twentieth century " is partly due to "an improvement in the nation's diet and a spread of knowledge regarding physiological methods of eating." The figures shown in Table V are of interest.

These figures show that gastric ulcers become more preponderant with a descent in the social scale. The very marked drop in the ratio D/G between the better circumstanced and the less well circumstanced population suggests that not only are Nicol's "G factors" (local trauma to gastric mucosa) less operative in the higher social groups, but that "D factors " (psycho-physiological disturbances) are more operative in these groups. This is in accord with general impressions that the type of mental distress and intensity of living which Nicol has in mind is more within the experience of the higher income groups, while it is the lower income groups who have been driven to bad feeding habits.

Apart from peptic ulcer, other digestive diseases might be considered in one broad group.

Diabetes is of special interest, since as the Registrar-General (21) remarks, "it affects chiefly the classes provided with the financial means of over-eating and under-exercising." However, there is no suggestion that the trend of diabetes might accurately portray economic changes, or that any relationship which diabetes mortality bears to social class can be observed for more short periods of time owing to the effect upon the mortality statistics of the extended administration of insulin which came into use in 1923. The death-rate from diabetes in London continued to rise in 1923 partly as a result of a contemporary increase in the incidence of the disease as the stringent diet of the 1914–18 war receded into the past, partly as a result of the more frequent recognition of the disease as a senile condition and mention on the death certificates, and partly due to the fact that for various reasons insulin was not so commonly applied in cases of advanced age. If some of these effects are separated by referring only to deaths under 55, the effect of insulin is seen in the reduction of the average annual death-rate in London in that age-group from 153 per 1,000 in 1921–23 to 86 in 1936–38. During the recent war years the restriction of food tended (as in 1914–18) to reduce mortality from diabetes and increased the impetus of the decline in mortality. In 1945–47 the average annual deaths in London under 55 from diabetes was only 31 as compared with 86 in 1936–38—a change out of all proportion to the reduction in population.

If disease is the penalty which society pays for the mode of living it adopts, there is one other penalty which is worth recording, namely, the death-risk from accident and injury. The deaths in London in this group in 1947 were:

Suicide	354
Road accidents	319
Other violent causes	1,073
	1,746

This group formed 4·2 per cent. of the deaths from all causes. That so many lives should be lost by injuries sustained in preventable accidents is a disturbing fact.

Infant mortality must be given special attention. Mortality under one year of age has been so sensitive to environmental conditions that it has been accepted (perhaps too readily) as a guide to the standard of hygiene and the general level of economic conditions in the community. Within this group it is convenient to separate neonatal deaths, i.e. within four weeks, from those which occur during the remainder of the first year of life. The main reason for this distinction is that the dangers inherent in the transition from intra-uterine to extra-uterine existence diminish rapidly with time, and the neonatal group does conveniently class together those deaths occurring at the peak period of risk, but it is also important because the reduction in infant mortality in the past fifty years, as a result both of rising social standards and the energetic intervention of public health authorities, has mainly been effective against the causes of death (principally infectious) which do not operate until after the first few weeks of life. Deaths *within* the first four weeks of life have not been reduced to the same extent. The following figures for London illustrate this point:—

TABLE VI.—*Deaths of Infants per* 1,000 *live births*

	Under 4 *weeks*	*Between* 4 *weeks and one year*
1906–10	37	77
1947	18	16
Percentage reduction	51	79

The reason for this is that while in the first weeks of life mortality from infection has been much reduced (as at later ages), possibly as the result of improved nutrition there has remained an apparently intractable core of mortality from prematurity, congenital malformations and birth injury, i.e. conditions which represent a fatal handicap to the infant on entry into the world.

The distribution of causes of death in the first four weeks of life in 1947 was as follows:—

TABLE VII.

Cause	*Number of deaths*	% *Total*
Prematurity	501	39
Birth injury	160	12
Congenital malformations	152	12
Other diseases of early infancy	329	25
Bronchopneumonia	76	6
Diarrhoea and enteritis	9	1
All other causes	62	5
Total	1,289	100

Thus deaths from prematurity, birth injury and congenital malformation account for 63 per cent. of all neonatal deaths. Many prematurities could be prevented by greater attention to the general health of the mother and by improved ante-partum medical care; and where prevention fails and premature births do occur, many of the subsequent deaths are preventable by specialized treatment of the infant.

In London deaths in the first four weeks from *prematurity* per 1,000 live births fell from 16·1 in 1911 to 11·6 in 1938—a comparatively slow rate of progress. After a temporary rise due to wartime difficulties, the rate has been further reduced from 11·6 in 1944 to 7·0 in 1947, and this comparatively large advance is doubtless associated with the contemporary emphasis on the care of premature infants. Many birth injuries would not occur if specialized obstetrical assistance were more general. The neonatal rate from this cause was still 2·24 in 1947. The rate for congenital malformations in 1947 was 2·13.

In these circumstances infant mortality is not to be expected in the future to measure so sensitively as in the past the trend of social and economic conditions, since these very conditions have now reached a level at which the sensitive part of the index, viz. the infective mortality, is now a minor element. Infant mortality will in future measure more strictly the success of medical intervention in both the narrower obstetrical sense and the wider educational sense. The causes which now terminate life before or soon after birth involve not only the nutrition, protection and cleanliness of the infant, but the whole complex antenatal experience of the mother, and the level of obstetrical skill.

Before leaving this aspect of mortality it is important to note that recent reductions in the still-birth rate in so far as they have been achieved by the postponement of deaths until after birth have made artificial additions to the neo-natal mortality. Such fluctuations can be avoided by using a combined rate for foetal and neo-natal mortality.

Maternal mortality is an extremely small part of the total death risk, but it is an important subject of study not merely from the material view that a child-bearing woman's life is valuable to the race, but because pregnancy is not, or should not be, an illness, and the birth of a child should

not be in the least associated with death if this risk can be eliminated, as most of it can. Maternal mortality was falling only very slowly until the introduction of the sulphonamides, in the middle 'thirties, resulted in a substantial decline in the mortality from puerperal infection. Puerperal sepsis in recent years has lost its position as the most serious mortality risk of pregnancy, and has been displaced by toxaemia, haemorrhage, and other accidents (trauma of the pelvic organs, etc.), which now contribute the greater part of the mortality, but even here the trend is downward. The Ministry of Health collect reports from local authorities on all maternal deaths, and these reports are subsequently analysed to determine how many deaths would have been avoidable with better care. It would be as well if this were done generally on a local basis as well as on a national basis.

Sources of Mortality Data

The County of London is in a specially fortunate position in that the the Weekly Return of the Registrar-General contains an analysis of deaths registered during the week by cause (short list of 36 main causes) and age. These figures are naturally not corrected for residence, but long experience has provided a guide to the direction and approximate extent of the corrections for each cause, and this weekly table is of great value as an indication of the short-term trend in mortality. This information is supplemented by a return which is obtained from the Registrar-General by private arrangement giving registration particulars of all deaths from—

Smallpox.
Dysentery.
Poliomyelitis.
Encephalitis lethargica.
Enteric fever.
Cerebrospinal fever.
Meningitis.
Childbirth and death associated with childbirth.

At the end of the year the Registrar-General provides a detailed analysis by sex, age, and each subdivision of the International List of Causes, for all deaths of residents of the County. In addition deaths in the first year of life are divided for each cause into smaller intervals of age. Certain comparative material for England and Wales is also provided. For comparison with other local areas it is necessary either to correspond directly with the health department concerned, or to await the publication of the medical tables of the Registrar-General's Annual *Statistical Review.*

Infectious Disease

Persistent measures of control, health education, bacteriological advances, and a rise in nutritional standards have all contributed to the reduction of mortality from the common infections from a very high to a very low level; but as has been remarked earlier, were it not for continuing communal and individual vigilance, these pestilences which we now regard as conquered could once more return. Therefore it is important that the number of cases of infectious disease reported daily to the local authority must be closely followed, so that any upward turn may be quickly recognized and the necessary measures of control as quickly instituted.

The statistical arrangements in London so far as they affect the L.C.C. are as follows:—Local practitioners notify cases to the Medical Officers of Health of the Metropolitan Borough Councils, who summarize the details on a daily return which is sent to the L.C.C. These daily returns are coded, and from these punched cards are directly prepared. A weekly analysis is made and a review of movements in the numbers is circulated to the medical personnel concerned, in particular the Medical Officer of Health, the Council's epidemiologist, and the medical officer immediately responsible for the school health service. Changes of diagnosis in respect of earlier notifications are shown on the daily returns from the boroughs as they come to light and correction cards are punched. (Tuberculosis is included in this arrangement with two differences, (*a*) that the return from the boroughs is weekly, not daily, (*b*) that the occupation of the individual is recorded for tuberculosis but not for other infectious diseases.) Adequate space is provided on the card for addition of supplementary information if this is required for *ad hoc* study. The routine items shown are: Date of notification; number of week (in year); disease; age; sex; borough of residence.

The main use made of these records (apart from the weekly review of incidence) may be summarised as—

(*a*) Variations of incidence in subdivisions of the area of the local authority. Since those subdivisions will have different social conditions, valuable information may be gained as to the influence of economic factors. During the poliomyelitis outbreak of 1947 useful information on these lines was analysed from the normal notification records and reported elsewhere (22).

(*b*) Patterns and rapidity of spread throughout the area and the identification of foci.

(*c*) Changes in sex or age incidence, which often serve as important epidemiological clues.

It is not always safe to accept the notification figures at their face value. During epidemic outbreaks, e.g. poliomyelitis, when public fears are intensified, there is an increased demand for medical advice, and milder cases, which would not normally be detected, are notified. Improved pathological services and the more frequent and earlier examination of stools have inflated the dysentery notifications (23). Secular changes and local variations in the standards of notification occur for other reasons. There are fashions in medicine as in other aspects of life, and from time to time particular emphasis is laid on certain diseases. More important than this, there is the probability that for a particular type of infectious disease the ratio of overt reactions (i.e. easily recognized by prominent symptoms) to subliminal (mild and almost symptomless) infections varies with either the evolution of new strains of the infecting organisms or variations in the resistance of the host (perhaps as a result of nutritional improvement). That the apparently low incidence of measles since 1940 is to be explained partly on this hypothesis is at present the subject of study, and will be reported on elsewhere at a later date.

Some infectious diseases have been easier to overcome than others. With the general improvement of sanitary conditions the waterborne infections have virtually disappeared. The last cholera case in London was notified in 1911. At the turn of the century enteric fever cases were running at 3,000 a year, but the annual total has now dwindled to less than 100.

Dysentery outbreaks, in the main, occur only in institutions for the very young or very old, in whom cleanliness of habits is difficult to maintain. Typhus has been almost unknown for many years, as also has been anthrax.

These advances, so quickly passed over in few sentences, were only won after years of campaigning by the early clear-sighted epidemiologists who forcefully marshalled the facts of causation and thrust them before the attention of a reluctant administration. We are not without such battles in modern times. It was not until 1943 that scabies was added to the list of notifiable diseases; the tighter control thus obtained has been justified by a drop in the annual cases in London from 16,450 in 1944 to 5,304 in 1947. This reduction has been steadily proceeding since 1943, and cannot be merely due to the emergence from war to peace. Scabies tends to be a condition of the family rather than the individual, and since notification facilitates home visitation by the local authority, it provides an essential condition for the reduction of incidence of this social nuisance. A satisfactory antigen for diphtheria prophylaxis had been found long before the war, but to educate the public to use it has taken intensive efforts of health workers and school teachers in a co-ordinated campaign. The early reluctance has gone, and the proportion of London children under 15 immunized against diphtheria has risen from less than 5 per cent in 1938 to 75 per cent in 1947. The diphtheria statistics have been under careful scrutiny for signs of the results, and in 1946 (24) we were able to report indications of a parallelism between the falling incidence of diphtheria and the falling percentage of children still not immunized. The notification-rate in London (corrected for errors in diagnosis) has fallen from 1·13 per 1,000 in 1938 to 0·14 per 1,000 in 1947. Except in certain special areas, infantile diarrhoea is not notifiable unless it is fatal. The study of the aetiology of this disease has thus been hindered, because it has been largely confined to mortality records.

Tuberculosis has been referred to in this section for convenience; but in practice it is given special attention. A book could be written about tuberculosis statistics, and the contribution which statisticians have made, and are making, to the organization of the control of the "white scourge," which strikes over 5,000 persons in the County of London yearly and from which over 2,000 die yearly. The intensity of case-finding, the location of "black" spots or "black" occupa-

tions, and the success of isolation is a matter which must be kept under constant statistical review. The success of the treatment schemes and of after-care must be continuously and rigorously tested by adequate follow-up records. It is a social as well as a medical problem, as will be clear from earlier comment in mortality statistics.

There are many problems left for the medical epidemiologist and the statistician jointly to tackle. The case-mortality of measles has fallen dramatically, but measles epidemics are nevertheless a heavy burden on doctors and nurses. Can we contrive an adequate warning of their arrival? Owing to wartime disturbance of the child population and other factors yet to be ascertained, the prewar biennial rhythm has not been followed in recent years. What sort of rhythm *is* emerging? At any instant can an assessment be made of the proportion of the population susceptible to attack? What is the role of nutrition; of urbanization? There are similar problems in the epidemiology of whooping-cough. Poliomyelitis engages more of our attention than hitherto. As communities emerge from squalor to high degrees of sanitation their infant population ceases to be protected by the frequent mild immunizing infections of earlier times. A mild endemic disease is now replaced by periodical explosive outbreaks of a violent and damaging nature. We have to face the fact that such an outbreak as occurred in 1947 will probably recur. The discovery of an effective method of control is a matter of urgency. Professor Greenwood (25) has listed many more problems, and summarizes —"the riddle of the epidemiological Sphinx is still unread." The raw material is in the records of local health authorities, and it is for statisticians in concert with their medical advisers to see that no experience is wasted.

School Health Statistics

All children in schools maintained by local authorities are given regular medical and dental inspections. Routine medical inspections take place, on entry into school, at 7 years, at 11 years, and just before leaving school. Dental inspections are more frequent. The local authority provides the treatment for some of the minor ailments found and supervises the treatment of all defects, whether treated by itself or referred to other agencies. The statistical records of this service are available as a measure of the contemporary health of school-children.

Controversy exists as to the value of these records, and even as to the value of school routine inspections themselves. It is true that as at present conducted these inspections cover, in the main, a group of children from whom the relatively unhealthy have already been excluded by absence from school, or by earlier observations of symptoms by the teacher. It is also true, as Gordon (26) remarks, that the quality of the inspections is to be criticized on account of the short period of time devoted to each child, with the consequent failure to discover anything "not immediately apparent in five minutes." Irvine (27) has drawn attention to the difficulties of assessing child health under any circumstances, and to the necessity of distinguishing between the average and the ideal. Against the rigorous criteria which Irvine presents, school inspections may possibly appear of limited value; nevertheless the statistical indices derived from school medical records do provide some measure of the health of school children, which at least has the merit of being comparable year by year and thus of recording a trend. The data available are:

(*a*) *Inspections*

There are three different types of inspection. At the periodical *routine* inspections the following information is recorded on the main school medical record card of the pupil (and subsequently transferred to a punched card. The detailed mechanism has been described elsewhere by Gore (28)):—

Name of school (coded).
Age at examination.
Date of examination.
Prophylaxis (whether protected against smallpox, diphtheria).
Vision (Snellen index).
Cleanliness (infestation of head or body).
Condition of teeth.
General condition of pupil (good, fair, bad).
If leaving school, any contra-indications for particular types of occupation.

Presence of defects falling in the following groups:

Skin—Ringworm, head or body.
Scabies.
Impetigo.
Other.
Eyes—Blepharitis.
Conjunctivitis.
Keratitis.
Corneal opacities.
Squint.
Other.
Ear—Hearing.
Otitis media.
Other.
Nose, throat—tonsils/adenoids.
Defective speech.
Enlarged glands.
Heart—Organic.
Functional.
Lungs—Bronchial affection.
Other.
Development—Hernia.
Other.
Orthopaedic—Posture.
Flat foot.
Other.
Nervous—Epilepsy.
Chorea.
Paralysis.
Other.
Psychological—Development.
Stability.
Rheumatism.
Anaemia.
Debility.
Enuresis.
Nutrition.

In respect of each defect an indication is given of whether the disease is sufficiently severe to require treatment, or whether the condition is mild or likely to clear spontaneously and is merely to be kept under observation.

The trend of illness in the above categories is assessed by the year-to-year comparison of the percentages found (in each age-group) to require treatment or observation for a particular defect. A broad summary of these figures for the four principal age-groups (5, 7, 11, 14) combined, in London, in certain recent years is shown in Table VIII.

Behind such a simple summary lies, of course, a large number of detailed comparisons for individual defects, individual age and sex groups, and subdivisions of the county. Space does not permit illustration of more than basic principles.

When a child is referred for treatment or observation a follow-up card is at once made out on which details of the ailment(s) are noted. This card, which is of the standard type for punching for mechanical tabulation, is kept at the school for six months, when the child is given a *re-inspection*. The condition of the child and a note of the treatment given is entered on the card, which is then sent to the central office. Here these cards are coded and punched and the results of follow-up are tabulated. The cards are then returned to the school for further reinspections when additional information is noted. A continuous follow-up is thus made, the cards passing backward

TABLE VIII.

	1938	1945	1946	1947
Numbers examined	169,995	112,964	144,500	142,064
	Percentages.			
Skin diseases	0·8	1·5	1·3	1·2
External eye disease	0·5	0·6	0·5	0·7
Defective hearing	0·1	0·3	0·3	0·3
Otitis media	0·5	0·4	0·5	0·5
Enlarged tonsils and adenoids	7·5	6·8	8·2	7·7
Defective speech	0·2	0·2	0·3	0·5
Enlarged cervical glands	0·6	0·9	1·1	1·0
Heart and circulation	1·0	0·6	0·6	0·6
Lung disease (not T.B.)	1·2	1·1	1·4	1·4
Orthopaedic defects	(*a*)	(*a*)	(*a*)	4·0
Defects of nervous system	(*a*)	(*a*)	(*a*)	0·3
Psychological defects	(*a*)	(*a*)	(*a*)	0·4
Anaemia	0·3	0·3	0·4	0·3

(*a*) No exact comparison can be given. New classification introduced in 1947.

and forward between central office and school. This arrangement provides a check on the efficiency of the treatment organization. Thus in 1947 we were able to say that of children referred for treatment, for any defect, 87·1 per cent. eventually obtained it. The balance of 12·9 per cent. represents children who have left school or removed outside the area, and unco-operative parents.

A pupil does not have to wait until a routine inspection before seeing a school doctor. If the teacher thinks a child is ailing, if a certificate for juvenile employment is required, or the child, for any reason, requires an urgent examination, then such an examination is arranged. These are referred to as *special* inspections—a miscellaneous collection of little statistical value, but a count of the numbers of different types of examination (reasons) is made and of the defects found (in the same way as for routine inspections).

(*b*) *Treatment*

The London County Council maintain a number of clinics to which pupils can be referred for treatment for diseases of eye, ear, nose, throat and skin and other minor ailments. Individual clinical records are, of course, maintained, and statistical returns are made of the number of new cases, the total attendances, and the operations performed during fixed periods. At one time the volume of work done was a pointer to the prevailing extent of ill-health, but now that every child has his own private practitioner it is doubtful whether this is still valid. Children admitted to hospital are followed up and the hospitals are encouraged to submit reports on discharge, so that a record of treatment can be made in the pupil's school medical record. The statistical aspect of treatment and follow-up is largely covered by the reinspection card referred to above.

(*c*) *Height and Weight Measurements*

Pupils in all London Schools to which the school health service extends are weighed and measured twice a year in normal clothing, but without footwear. These measurements are charted on individual records, which are kept under review so that action can be taken if there is any abnormality in the growth of the child. At regular intervals a mass survey is made of these records to provide an objective assessment of the nutritional condition of London school-children. The last full survey was made in 1938 (29), and the next comprehensive analysis will be made in 1949; during the war years the lack of calibrated weighing machines and staff made it necessary to restrict the arrangements to cover only twenty schools in the north-east and twenty schools in the south-west of London, but periodical surveys have been made (30) on this limited basis and these have provided a valuable wartime nutritional index. There is scope here for considerable statistical

research. What is the optimum relationship between height and weight? Can the measurements themselves, together with age, be embodied in an index which can be used to screen the unhealthy from the healthy? Jones (31) and others have contributed inportant material to a discussion of the problem from which much controversy has arisen. Are seasonal variations in growth natural or the result of variations in health? What is the relationship between growth and the other criteria by which Irvine (*loc. cit.*) would have us judge health—"satisfactory physical activity, happiness, physical efficiency, spontaneity, resiliency"? The answer must be sought in the correlation of growth variations with all other accepted physical and psychological symptoms of deviation from health. In an interesting review of the many problems that arise Bransby (32) stresses that "there are many avenues along which work can proceed to discover how growth data can effectively be used in public health work. The present ignorance makes a strong case for an intensive research effort on growth and development in connection with the school medical service. It is wrong to believe that only by the study of large groups of children will positive results emerge. It is the significance of the variations in the growth of the individual child which needs to be understood. Careful study of small groups of children, relating clinical findings to anthropometric changes, seeking explanations for, and the meaning of, seemingly inexplicable phenomena, may throw light on aspects of growth at present clouded in mystery, until eventually sufficient knowledge accumulates to permit growth data to be used to their greatest advantage in the promotion of child welfare."

The Main School Medical Record

This record which is maintained at the school is an individual case-history of the child throughout school life, has recently been revised by the Ministry of Education, and the new form gives details of social background, educational assessment, and a complete medical history. It is being introduced for all new entrants. Arrangements have been made for the pre-school medical records of the child welfare service to be transferred to the school record when the child begins to attend school. In a few years' time these records will provide valuable material for sampling surveys of school health, and of the relation of physical and mental development to environment and disease.

Maternity and Child Welfare Records

At present we have more information about the morbidity of infants than of any other class of society. As soon as an infant is born in London three important records are instituted:—(i) the health visitor's record, on which she records the information about the infant gleaned from routine visits to the home; (ii) a consultation record at the welfare centre, to which an increasing number of mothers bring their infants for medical advice; (iii) an index card which is maintained at one of nine divisional offices through which the county service is administered and which summarizes the information contained on (i) and (ii).

The principal contents of these records are:

1. *Outline of Health Visitor's Record* (*Infants* 0–4)

This document first records background information : Name, date and place of birth; parents' ages, occupations; general home conditions, e.g. rooms, garden space, food storage, sanitary arrangements; health of mother before and after birth of infant, together with details of previous pregnancies.

Then in respect of the first visit after birth the following details of the health of the infant are given: Weight, general condition (eyes, mouth, umbilicus, skin), feeding, clothing, sleeping arrangements, and general care and mothering.

At subsequent visits notes are recorded under the following headings: Age, weight, and notes on feeding, advice given, etc.

At the end of the record there is provision for recording infectious disease or other illnesses as they occur and for details of prophylaxis.

Finally, space is given for an annual survey of the child under the following heads: Nutrition (height, weight, feeding, colour), posture (muscle tone, feet, date of first walking), skin, eyes, teeth, nose and throat, speech, hearing, mental progress, enuresis, adjustment, infestation, and parental care.

2. *Outline of Infant's Consultation Record (Kept at Welfare Centre)*

This card covers the age-period 0–4, and is arranged to dovetail into the Ministry of Education school record card, which is brought into use when the child attends school.

The card gives background information similar to that recorded on the health visitor's record (it need only be obtained once).

Details of the first medical examination at the centre include height and weight, feeding, digestive system (vomiting, stools, flatulence), general condition (cleanliness, colour, anaemia, skin, eyes, mouth and teeth, nose, throat, glands, ears, umbilicus and phimosis), congenital defects, fontanelle, bones, rickets, posture, gait, activity, physical development, mental development, heart, lungs, and general care.

Then follows the progress notes at subsequent visits.

Provision is made for periodical inspections. From 0–2 the details recorded at these inspections are similar to those recorded at the initial visit (above), some of the information being obtained from the health visitor's record, but from 2–5 the details shown are those set out on the Ministry of Education main school medical record card, thus ensuring continuity.

These are newly designed records which have been recently introduced as part of the reorganization of the service consequent upon the passing of the National Health Service Act of 1946, and it is too early to report tangible results, but the divisional index cards which summarise the information will be sampled at regular intervals, and will provide a continuous picture of changes in, and of factors influencing, infant health. A beginning has already been made by a pilot study of infant weight records in order to construct a much-needed standard growth curve as a reference scale for the paediatricians.

In respect of the maternity service considerable research material is provided by the medical records kept at the ante-natal clinics (to which both health visitors and medical officers contribute), and by the case-records of the Council's midwives. These provide the statistics for observing trends in fertility and maternal mortality and for testing the efficiency of the medical service. Some 20,000 babies are delivered by L.C.C. midwives each year, and the clinics also provide the antenatal care for many women who are booked to be confined in hospital.

Home Nursing

The district nursing associations have begun to provide the L.C.C. with routine returns of their visits to cases referred to them for home nursing. These returns give the name, age, sex, disease and treatment given to the patient. Analysed month by month these returns will provide a measure of the trend of chronic invalidism in the elderly.

The Remaining Gaps in Information as to Morbidity

Making the best of the available morbidity material is a painstaking and laborious task, demanding skill and a capacity for enduring frustration. It is not as satisfying as the analysis of wholly *ad hoc* morbidity records, but it can be moderately successful. What improvements are desired? The records available in respect of infants and school children are at least more complete than for adults, and the principal want is a good morbidity index for the adult population.

How could this be provided? Some authorities have begun to analyse the clinical records of patients treated in hospitals and other treatment centres. In so far as this analysis enables the medical officer to exercise a more efficient administration of his resources for the treatment of disease it is a very necessary part of departmental efficiency, but as a source of information of morbidity it may be misleading, and its use for this purpose should not be lightly accepted. The disease pattern of a hospital population is liable to change, not because a particular disease becomes more prevalent in the area served by the hospital, but because either a change in medical practice dictates that that disease should be given more hospital treatment than hitherto, or because a particular specialist worker or teacher in the hospital asks for more material. Many patients now have to be treated at home because there are insufficient nurses to staff the available hospital beds. The trend of admissions for a particular disease is often, therefore, quite unrelated to actual variations in incidence. Even the incidence of certain serious illnesses, which are never refused

admission, would be obscured by the variable quantity of not so serious cases, which in some years gain admission more easily than in others. The further difficulty arises that it is not possible to say that every case of a particular disease will go to a particular hospital or group of hospitals; thus the exposed-to-risk is not known and absolute numbers must be used and not rates.

Hospital statistics have a very high domestic value (for teaching or administrative purposes), but their public health value depends upon the careful examination of the figures and their integration into the complete picture of morbidity, including statistics of domiciliary treatment.

Some attempts have been made to measure morbidity, for example that of the Social Survey, by sampling individual medical histories from selected strata of the population. This method represents a real advance in so far as it is an attempt to focus attention on the summation of contemporary variations in the personal health of typical members of the community; it is a real social measure of illness. There is, however, the serious defect, even when considerably mitigated by the careful methods of control and verification adopted by the Social Survey, that the information as to illness comes from the patient himself, and even when endowed with maximum of sincerity the patient is not the best observer of his own health of mind and body and of the relationship between them.

Use could be made of the certificates which the general practitioner issues as part of the machinery of sickness insurance, but it would be difficult to assess severity, and the information will be out of date before it becomes accessible to public health workers.

It is probable that records of sickness absences in large public or industrial organisations, if adequately maintained and properly analysed, could throw much light on current trends in morbidity. A great deal depends on the manner in which the recording is organized. In many cases the records are furnished to the centre by departmental staffs, it being rarely possible to submit original medical certificates, since these are passed elsewhere for insurance and sick pay purposes. In the circumstances the copying of the diagnosis would need to be done intelligently, and with some knowledge (at least) of terminology. Skilled medical advice and supervision would be needed in the classification of the absences to groups of diseases. It is doubtful whether the standard of recording has reached this level in more than a very small number of organizations (if any); many attempt a breakdown of sickness records by age and grade of employee, but only rarely are the different diseases classified. There is no doubt that the continuous survey of such records (with due reservations as to their application to certain occupations only) could be of great assistance to health authorities.

A more intimate and comprehensive record is desirable, and it is probable that when the Health Centres become a reality we shall have all the necessary conditions—doctor-patient-family relationship, clerical assistance, equipment, accommodation—for routine examination and complete morbidity measurement of the population. Even if it could be achieved for a small representative sample it would be a tremendous stride forward.

Examination of the Results of the Services

No scheme of inspection, supervision or treatment ought to be operated unless there is simultaneously a sincere attempt to assess its efficiency. Complacency at the absence of major epidemics is dangerous. Personal impressions of increased well-being and lessened morbidity are too often misleading. Satisfaction with new low records in mortality is not justified while manifestly preventable deaths still occur. There must be a target and a scientific measurement of progress towards that target within every division of the public health service, and as between divisions, the correct balance of emphasis must be secured. This implies careful fact-finding.

If morbidity fails to fall or actually rises, is this because the invasive organism or the inimical conditions (whichever applies) are more powerful? Have the individual members of the community become more susceptible to the disease? Are the preventive forces inadequate in quality, or in quantity, or in both? If the morbidity from a particular disease falls, is this the outcome of some specific element in the individual or his environment, or of some intervention by society either in curbing or changing habits of behaviour, i.e. by law, or of intervention by medicine in providing new means of combating the disease? Such questions can only be answered adequately if all the variables are identifiable and measurable—an ideal almost impossible to attain. Commonly the recognition of important factors determining the trend of morbidity is a gradual process,

and even when recognized, some factors are difficult to measure within the limits of normal resources; but an attempt must be made. Twenty-seven years ago Sir Allen Daley (33) emphasized that "the great need . . . is to decide which public health work is of proved value, which has its value unproven, and which, if any, may be regarded as comparatively ineffective." This need is still as urgent, and still not sufficiently widely recognized.

The normal method of assessing the value of a special process is to conduct a controlled experiment in which the process is applied to one only of two groups, alike in all other respects, and the effects of the process are then assessed by comparing the "test" group with the "controls." In public health administration, however, this can rarely be done because there are no "controls"; patients cannot be deprived of treatment or sanitary protection in order to see whether they will die without it, and those who intentionally do not avail themselves of services are rarely comparable in environmental qualities with those who do. Much of the assessment is necessarily based upon the presumption that the long-term trend of morbidity from a particular disease will indicate the cumulative effect of intervention by the health authorities, and that careful analysis will enable the association to be established, i.e. that the contribution of all other factors can be separated and the value of public health administration thus isolated.

There are usually two main problems arising from the consideration of a particular health project: (i) is it of any value to all, i.e. does it save any lives or reduce periods of sickness? or (ii) even if it does, is it the most efficient means of achieving this end, i.e. could the same result be achieved by some other method that is less expensive in equipment and manpower or involves less interference with personal freedom?

It is desirable to incorporate in the day-to-day administration of each part of the health services records from which its efficiency can be directly assessed. In certain services this is simple to arrange. In the maternity service it is not difficult to analyse maternal and foetal deaths by cause to determine which might have been prevented if the service was of a higher standard. Again, in the infant welfare centres and in the school health service, routine individual clinical records are available which, if carefully kept, will show the benefit of treatment or observation. Sometimes it is necessary and convenient to provide an *ad hoc* record from which some assessment of the value of the service may be obtained. In the treatment of tuberculosis and cancer it has long been the practice to measure survival rates for particular types of treatment by routine follow-up with special records for the purpose.

Unfortunately the local health authority has now been deprived of most of its responsibility for treatment, which has been transferred to the regional hospital boards, but there is scope for co-operation between the two types of authority in organizing follow-up, and there is scope in the "after care" functions now imposed on the health authority for individual effort on the lines which have been suggested. Little has yet been done, but there is an increasing interest in this type of study.

Special Studies

If what has been described is not enough for the public health statistician to do, he need not look for work. As soon as his presence is felt by the medical staff a plethora of statistical problems is thrust at him. The following is a selection from over 70 special investigation files of the statistical section of the L.C.C. public health department opened in the last two years. In these studies the statistician is not the director but a member of a team. The clinician conducts the experiment, provides the material and poses the question; the statistician attempts to find a statistical answer in the material. Even if the statistician is successful, the clinician has to provide the biological interpretation.

Atmospheric pollution—an attempt to assess effect upon respiratory mortality of smoke pollution from power stations, etc. Inconclusive.

Strangulated bowel—a study of pooled surgical notes on operations at L.C.C. hospitals in order to codify criteria for assessing viability of the strangulated bowel. Interim report published *Brit. Med. J.*, 1947, **1**, 43.

Carcinoma of the lung—study of post-mortem examinations and associated medical histories. Not yet complete.

Mortality in mental hospitals—a survey of causes of death in L.C.C. mental hospitals, and a comparison with other mental institutions and with the ordinary population.

Congenital deafness—a study of the relationship between congenital deafness and rubella, and other infections in pregnancy of mother.

Electro-encephalography—association between personality traits and E.E.G. rhythms in patients attending psychiatric clinics.

Enteritis—study of a series of cases treated at a special L.C.C. enteritis unit.

Mass-radiography—statistical analysis and inter-comparison of a number of special X-ray surveys.

Diarrhoea—assistance to a research worker on incidence of Giardia Lamblia in an L.C.C. Residential Nursery. See *Archives of Disease in Childhood*, 1948, **23**, 119.

Schoolchildren in Switzerland—effect of visits of children to Switzerland upon their growth.

Pleural effusion—statistical analysis of case-records at special units for the study of early tuberculosis.

Weight changes in pregnancy—analysis of weight records at antenatal clinics. See *Lancet*, 1948, I, 550.

Pediculosis, ringworm in London—trend of incidence during war years.

Rheumatism—a review of incidence of chronic rheumatism in London.

Orthopaedics—a test of the curative effect of exercises for correcting foot valgus.

Respiratory disease and temperature—examination of effect of severe conditions of March–April, 1947, upon respiratory mortality.

Of the many inquiries made, some are trivial and some important, and many of the results are buried in interdepartmental files, but the cumulative effect of the statistical contribution appears to be valued—at least there is no relaxation of the demand !

Cost

The annual salaries and wages bill of the statistical section of the L.C.C. Public Health Department is £8,000, to which £500 may be added for amortization of capital expenditure on machinery. The total expenditure of the Public Health services (before grant) is about £3m. Readers can decide whether the cost of the statistical service is exorbitant or modest.

Conclusion

The greater part of this paper has been devoted to the public health statistics. Inevitably less space has been devoted to such examples as spring to mind of the application of statistical operations to other aspects of local government—there are others more expert here than the author. In public health there is a movement toward wider recognition of the value of statistics, and as Sir Andrew Davidson (8) remarks, much has to be done " before the statistician is regarded as a member of each medical officer's staff appointed with as little hesitation by local authorities as is the medical officer himself. But undoubtedly the trend is in that direction, and is a trend of growing magnitude. Undoubtedly, too, it is a trend that should be encouraged by all thoughtful members of the community." One or two town and county councils outside London have already appointed statisticians in their public health departments; others have made arrangements for statistical work to be undertaken by the local university.

There is a widespread misunderstanding in local government departments as to the treatment of statistics. Many administrators seem to think that it is sufficient if statistical tables are drawn up without any critical analysis as to the reliability of the figures or as to interpretation. Often the tables are dangerously misleading in the concealment of heterogeneity, or incomparability with other tables and of sources of unreliability; they are found difficult to assimilate by officials and committee members, who either ignore them or condemn them because they do not understand them; inevitably they bring discredit upon statistical practice. The stock gibe that "statistics can prove anything" dies hard in these circumstances. Others think they have improved on this situation if they delegate the work to unqualified clerks who claim to understand statistical methods. Unless these officials have been properly trained there is a danger that they will be unable to distinguish between methods which are appropriate and those which are inappropriate; between

statistical tests which are valid and those which are invalid for the particular material being analysed. More harm than good might come from such an arrangement.

To keep records which are not properly used and to produce statistics which are not worth the cost of production is an unforgivable waste of human resources. Statistical activity should not be attempted except under skilled direction. If the local authority is not large enough to afford a statistician then it might well consider whether an arrangement for part-time assistance from the local university might not be a practical solution to the problem of cost.

The appointment of a statistician in the public health department of the large authorities is at present an economical arrangement, because it is mainly in this department that statistical work is regarded as essential, but if the advantages of statistical advice are to be made available to *all* departments, as has been urged in this paper, it will be more economical in cost and will promote greater co-ordination of effort if a central statistical office were to be established, as has already been done, experimentally, at Birmingham, rather than creating separate statistical sections in departments. The departments would still have individual care, because members of the statistical service staff could be specifically attached to departments, but the higher level of skill, of a more consultant type, and expensive machinery would be pooled, and the general direction, by seeing farther than the confines of any one department, could secure greater efficiency and a more extensive interchange of knowledge.

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Discussion on Mr. Benjamin's Paper

Dr. Enid Charles: Those of us who are working in the field of local government statistics will find much that is useful and stimulating in Mr. Benjamin's paper. He has covered an enormous field, and rightly so, since a local authority concerned with the welfare of its citizens will find something of interest in every facet of social statistics. While it may seem invidious to criticize the scope of a paper, I personally would have liked to see rather more emphasis on the functional aspects of local government work.

Problems of devolution and integration of function have been prominent in the Office to which Mr. Benjamin referred at the close of his paper. The Birmingham experiment has been in operation for a brief and exploratory period only, and while what has been done has sufficed to justify its establishment on a permanent basis with an appropriation adequate for an initial period of growth, my contribution to a general discussion such as this must necessarily refer to problems awaiting solution rather than to work accomplished.

A major activity of a central statistical office is to act as a service department to deal with statistical problems presented to it by other departments. For reasons given by Mr. Benjamin, the Public Health Department has hitherto been the focus of local government statistics, but it is important to realize that this will no longer be true. Increasingly, the planning activities and social services of municipal authorities seek guidance from relevant statistical data quite outside a field in which the regional authorities of the new Health Act will share the pre-existing prerogatives of the Chief Medical Officer. Meanwhile the activities of a city public health department raise new liaison problems.

Intimately concerned with the statistical aspects of a baby's first week of life we have the Registrar-General, the Ministry of Health, the local authority—Public Health, Education, and Estates Departments—the Regional Hospitals Board, the Local Executive Council, at least three University departments, and doubtless other bodies. Locally, my Office is fortunate in having machinery for close co-operation between most of the bodies concerned, and has been able to assist the Maternity and Child Welfare Department to systematize its records, with a view to salvaging valuable material in a form more amenable for analysis, so that it may contribute to an integrated picture of everything that affects the welfare of mother and child.

For reasons I have stated, much of this material may eventually—perhaps in the very near future—cease to be our concern in so far as its scope is specifically clinical, but our interest in the records of the Health Visitor has other aspects. They provide the most complete coverage in respect of birth; and it seems likely that the future may see greater emphasis on the domiciliary function of the local authority in respect of child care. Thus the residual health responsibilities of a local authority link up with Mr. Benjamin's first objective, a study of the constitution of the population, an objective which is also basic to the work of many Corporation departments other than Public Health.

Arising out of this part of the paper, a minor issue invites verbal criticism. Under the heading "Race" Mr. Benjamin describes some differences between immigrants and the native population which may be relevant in the sphere of public health. None of these differences, however, have anything to do with the term "race" as it is used by biologists. I should like to suggest that Mr. Benjamin finds a less tendentious word to describe groups of people differentiated by place of birth.

His discussion of sources of demographic information raises anew the problem of demarcation of function mentioned earlier. It is so long since the last census that we perhaps tend to overlook the fact that for some types of data the census is the prime and perhaps the only possible source. In this connection there seems to be some confusion in Mr. Benjamin's reference to occupational statistics. Accepting the customary census dichotomy of classification of the gainfully employed by industry and by occupation, the Ministry of Labour figures published in the *Digest of Statistics* are based on industry. To the social statistician the alternative classification is at least equally essential, and as far as I am aware, the only complete occupational description of the population of any area is that provided by a national census.

That being so, it is of some moment that the best use should be made of the provisions of the Census Act, 1920, which empower the Registrar-General to prepare and supply statistics in greater detail than could be published either in the national or county volumes. Since greater detail would be welcome, not only for occupations but for other census topics, possibly local government statisticians could confer, in advance of the census, with a view to agreeing a list of detailed tabulations of interest to most local authorities. From this request list those tables which the census machinery can most conveniently turn out would also have the widest utility.

While the paper control urged by Mr. Benjamin—to which might be added, question control—demands making the fullest use of national statistical machinery, the life of the local government statistician would be barren if an equally strong case could not be made out for local statistical autonomy in many other fields. The paper has drawn attention to a number of lacunae which it is both necessary and practicable for at least the larger local authorities to attempt to meet; and it seems most fitting that authorities which have played a pioneer role in what are now State activities should continue to do so in newly or rapidly developing fields of social statistics. But in addition to exploring the new, the old may require redefinition based on detailed knowledge of a kind best obtained locally.

One such topic is suggested by a reference in the paper to social class. One of the commonest

exhibitions of this ambiguous concept is a classification of occupations by social and economic status, initiated by the census of 1911. At no time were the underlying ideas precisely defined, and social changes which have already taken place or are continuing to take place have largely vitiated their claim to survive as a basis for chronological comparisons. I have approached this topic elsewhere through an index based on normalized scores in average earnings and average educational status, due account being taken of age composition; but this is obviously only one of many taxonomies more or less valuable with one or other end in view. A local authority has occasion to record occupations in many contexts, and greater precision in describing and coding them is therefore an important function of my Office.

If my comments on the very interesting paper we have heard have perhaps been somewhat pedestrian, it suffices to say this. Mathematical analysis is the legitimate goal and most congenial objective of the statistician, but it is first necessary to have material that merits analysis. Such a task is, inescapably, my immediate preoccupation, as it is to some extent that of most statisticians in the public service, and for Mr. Benjamin's welcome contribution to both means and goals I have much pleasure in moving a hearty vote of thanks.

Mr. Ernest Long: I am glad to pay tribute to Mr. Benjamin for introducing this subject to the Society. As one who has had some experience of local government on the financial side in a number of authorities I support his plea for the greater application of statistical theory and practice to the improvement of the efficiency of local government. I do so, however, with some reservation. I cannot claim much familiarity with Mr. Benjamin's main field, the public health service, and I must leave that to others; but I cannot resist a comment on his suggestion regarding the National Health Service that "all will agree that the preparatory statistical work was of high quality." That I find rather difficult to reconcile with an error of about 40 per cent. in the estimated cost of the service during its first nine months. If, on the other hand, Mr. Benjamin can show that the services of the statistician were not fully used in arriving at that estimate of cost, it would give point to his general argument that too often public bodies neglect the aid of the statistician.

Laymen are notoriously suspicious of the expert, but I think it can be shown that there is a great deal of useful statistical activity in local government to-day—much more than might be gathered from Mr. Benjamin's rather sweeping criticisms, especially his statement that very little progress has been made since 1915.

No reference to local government statistics would be complete without mention of the work of the Ministry of Health, particularly during the '30's under the leadership of the late Sir Gwilym Gibbon. Those years saw an expanding range of comparative costs and statistics gathered from all types of local authorities, covering hospital and institutional costs, refuse collection, sewage disposal, highways costs, and other services. The war stopped that good work, and unfortunately the hard-pressed Ministry has not been able to take it up again. But those pre-war efforts did achieve one thing of lasting value. They made local financial officers more statistically conscious, and stimulated them to develop and apply the statistical knowledge they had gathered in the course of their professional training as municipal treasurers and accountants.

Mr. Benjamin refers to the value of statistics to the finance department. Incidentally, the block grants of which he writes have now been replaced by the Exchequer equalization grants introduced last year, and the first sentence of that section of the paper should now, as it were, be read in reverse: "The financial burden of local services is now *less* in the more necessitous areas, as one result of these new grants."

Mr. Benjamin mentions a number of functions of estimating, financing, accounting and auditing in which the statistician might help. On that score, there are few finance departments where some degree of statistical knowledge is not brought to bear already on the type of problem instanced. The production of monthly statistics as a by-product of financial administration and accounting—of changes in rateable value, collection of rates and other revenues, bank balances and so forth—has long been a regular feature of the local treasurer's report to finance and other committees. The treasurer's annual abstract of accounts is a mine of statistical information, although I remember that the Ray Committee of 1932 considered the figures too "elaborate." Recently these abstracts have been supplemented by a spate of "potted abstracts" relating to the town and its finances. These are informative alike to the scientific enquirer and to the ordinary ratepayer. On a rather more ambitious plane are the county booklets of financial statistics, another recent development in which the principal facts relating to the county council and its constituent authorities are assembled. These are becoming very popular. They are in more or less standard form, up to date, and are published soon after the financial year ends.

I may be forgiven, perhaps, for referring to the Institute of Municipal Treasurers and Accountants, and the part that it has taken in promoting the study of statistics and the preparation of

statistical returns. In conjunction with the County Accountants Society, which itself issues a number of county statistical returns, the Institute has helped to co-ordinate the form and content of the county booklets. It has, in addition, undertaken the publication of a number of returns of aggregate statistics. Before the war, for the purpose of promoting uniformity in accounting analysis, the Institute had issued standard forms of Abstracts of Accounts and Summary of Rate Estimates. The Institute has also taken over from a few pioneers among local authorities some returns of a national character. A return of rates levied by the principal local authorities in England and Wales, published each year soon after the rate-making season is over, contains a detailed analysis of rate poundages as well as rates levied per head of population.

Another fairly new return is the analytical return of outstanding debt of local authorities. I should also refer to the part the Institute has taken in the collection of statistics for the National Institute of Economic and Social Research in connection with studies of the national income and expenditure.

These returns are gathered on a voluntary basis, and I am not here to claim that they are perfect statistical compilations. I should readily agree that they would be improved if the mind of a trained statistician were brought to bear upon them. The Institute, however, is a professional body with limited resources. If its funds were larger it would give high priority to the employment of a trained statistician on its staff.

As Mr. Benjamin points out, "there is no room for wasted man-power" in the employment of statisticians. Not every local authority has scope for their employment. Some have prior needs--for example, the county council which has within its area three or four new towns. But local government as a whole needs a statistical service. We have no counterpart to the American Municipal Bureau of Statistics. There is no central organization for the collection and interpretation of local government statistics. There is no body with statistical information readily available to the associations of local authorities when they wage their frequent, and losing, battles with the central Government over grants, and latterly, services.

There is a case for the constitution under formal auspices of a local government bureau of statistics, which could be coupled, perhaps, with research and provide an advisory service with specialist advice. That, it seems to me, would be the primary place for the employment of statisticians in local government. Meanwhile the establishment experimentally of a central statistical office in Birmingham, to which Mr. Benjamin and Dr. Enid Charles have referred, is of great interest.

I have much pleasure in seconding the vote of thanks.

Mr. Lythgoe expressed his appreciation and indebtedness to Mr. Benjamin for bringing this important question before them. He thought that those who were associated with local government would all agree that there was need for considerable development in this field. He would add that there was also need for co-operation between departments. He was glad to hear Dr. Charles refer to that because, from his experience of trying to do some little pioneer work in this direction, he thought there was a real danger of the statistician becoming very busy in his own department without fully appreciating the real needs of the operating departments, and also, perhaps, without having a measure of detailed working knowledge of the departments, which should direct much of the statistical research. That question of co-ordination was very important.

He agreed with Mr. Long that Mr. Benjamin's paper might give the impression that not much was done in the way of collecting statistics among local authorities. More was done than might be supposed. Unit costs were taken out very extensively by local authorities, but he agreed that more might be done. The statistics were, however, used very largely as guides to efficiency, and some individual authorities went considerably beyond what appeared in the published statistics of the Ministry of Health. If time were not limited, he could have given a number of illustrations of the way in which statistics of this kind were very effectively used. They were most decidedly used as over-all guides to the business of budgeting, and were also used by financial officers as tests in relation to audits.

A great deal of work was being done in other directions. Statistics were being used as guides to the direction of public expenditure, and here one bordered on the field of the relationship between the operating departments and the finance department and the statistician. An endeavour was being made to show the values of money spent, but there were certain snags. He was looking at some figures, some years ago, in regard to the decline of enteric fever in his authority, and was shown a lot of statistics to indicate how this related inversely to the cumulative figure of expenditure on the substitution of pail closets by the water carriage sewage system. The relation to the development of these services was striking, but he found that there had been a large development at the time in the cleansing of offensive stables and in the control of the sale of shellfish, and it

was well known that the consumption of infected shellfish had an important bearing on the incidence of enteric fever. Thus the deduction to be drawn from the development of the water carriage sewage system had to be qualified. That was merely one illustration of the need for co-ordination between the statistical officer and the other departments.

Mr. Benjamin had referred to the use of statistics in connection with planning. He would utter a word of warning there and he was sure that local government would welcome any help Dr. Charles could give in the course of time. He had found that the relative statistics even after five years were usually out of date. One of the difficulties in local authorities was in dealing with imponderables, such as changes in social habits. He was quite certain that this kind of statistical analysis could be a snare and a delusion unless it was carefully cross-checked by repeated up-to-date social surveys. There were limits to the possibility of forecasting the future on the basis of statistical evidence from the past.

One other direction was the attempt by local authorities to pursue the line of social accounting, giving not only the objective costs for comparison, but also the social value of their expenditure. Mr. Long had said that a great deal of work was being done in the initial stages in trying to break down local authorities' expenditure and to link it up with the national body of income and outgoings, and any help that statisticians could give them in that direction would be welcome.

There were a number of directions in which he thought there was need for advance. Undoubtedly there was need for more central co-ordination with a view to a greater degree of uniformity between authorities. There was also need for much greater information about unit costs of service as a basis for grants. The local authorities were, generally speaking, anxious to get away from dependence upon grants which varied with the expenditure incurred, and this development of accurate unit costs which were a real measure of quantum of service as well as of expenditure was a very important matter, and gave emphasis to the importance of co-ordination between statistical officers, accountants, and the departments.

Mention had been made of the value of statistics as a measure of the relative wealth of local authorities. He would suggest that in this connection the need to-day was for statistics concerning the income per head of population.

In conclusion he would suggest that the needs were a closer link between accountants, statisticians and service departments; some central co-ordinated effort, and for somebody to undertake research into the whole field of statistical development and the relation of expenditure to quantum of service.

Dr. E. L. Huppert said that Mr. Benjamin mentioned that one or two County Councils had set up statistical departments. The Health Department of Kent County Council was at present setting up such an office, and he hoped this paper would encourage other County Councils to do the same. Other fields not already mentioned where statistics would be very useful included the Fire service of local authorities and the Police service. The keeping of detailed accident-statistics—as was done in Kent—was very important. These statistics should go before Safety committees to help them in planning their work. Another field for statistical investigation was the County Pathological Laboratory. The great number of specimens brought for examination supplied very useful statistical material for a number of purposes in the health department.

Health Departments were now obliged to set up new services under the new Health Service Act, and statistics for the guidance of such services would be most useful. He would mention in particular the Domestic Help service. A survey in Kent showed that about one-third of the cases dealt with by that service were in connection with old age. The Mental Health services opened possibilities for interesting statistics.

Local health authorities often objected to employing a statistician on the grounds of cost; they failed to realize that it was an economical service, and seemed almost to be afraid that mechanical tabulation and so forth would cost too much. But the apparatus which the statistician used could to some extent be shared by other departments of the local authority and the tabulating could be done centrally. A number of statistics were collected mainly in order to render some return to the Central Government, as, for example, in the case of the Health department, to the Ministry of Health. It would be a great advantage if the returns were asked for in good time before the beginning of the period for which they are required. Very often a request for a return for 1948 was made some time in 1949. Local authorities were interested in having standard forms, but he thought that Central Government Departments often did not consult a statistician who could advise them on the best standard form to use.

Another point he would mention was the wide variation in the population densities in urban and rural districts of County Councils compared with the density-figures for the districts of County Boroughs. This fact made problems much more difficult. One urban district in Kent had a population of 13 per acre, but in one rural district the density was only 0·13.

He hoped that Mr. Benjamin's paper would awaken interest in the much neglected field of local government statistics.

Mr. S. LEGGATT thought the Society might be interested to hear of a case which fully demonstrated the necessity for giving at least some attention to the co-ordination between the various authorities throughout the country in the production of statistics. Not long ago he was called upon to attend a local enquiry initiated by the Ministry of Town and Country Planning. It was worthy of attention that this enquiry was held at the offices of the local authority, and there were present the town clerk, the county surveyor, the local surveyor and many other officials. In the course of the enquiry certain questions were asked about the number of road accidents which had occurred in the district over a certain period. The chairman of the enquiry asked if any of those present could give the requisite information. There was absolute silence, and the enquiry proceeded without this necessary information, which was vital to its subject, being forthcoming. Eventually, some ten days or a fortnight later he received a copy of the required information, which came from the local county constabulary office.

It seemed to him very remarkable that in these days, especially when in London, there were, outside every town hall, large notices displaying the number of road accidents in the locality, at this well-known town in the home counties no information whatever was obtainable in the office of this particular local authority. That was one concrete example which demonstrated that the time was ripe for all local authorities throughout the country to have a properly constituted statistical department.

Professor MAJOR GREENWOOD said that many speakers in that discussion had illustrated the truth of the Biblical saying that "We are members one of another." There would be no success in this field without co-operation. In Essex, with his colleague, Mr. W. H. Leak, he had been enquiring into the fate of persons suffering from tuberculosis. Since tuberculosis was a notifiable disease and a county has an expert service of tuberculosis officers, this might seem an easy matter. Notification was, however, far from complete. Of the 505 deaths from tuberculosis registered in the County in 1938, in 94 instances no notification had been received or had been received *after* the death had occurred, and another 94 notifications were received within 3 months of the death. There were many reasons why, except in a police state, notification could not be complete. It was found, too, that of the patients who did reach the tuberculosis officers the proportion whose illness was in the earliest and most curable stage was disappointingly small. This might be partly due to over-emphasis in the past of the fact that tuberculosis was an infectious disease; sensitive people might fear that, were it known they "had T.B.," they would be shunned. These difficulties could only be removed by education, which meant co-operation. Then many people did not live all their lives in the county in which they first fell ill; once more, co-operation was important. Some patients went into hospitals; again co-operation, now between different authorities, was essential if the story were to be complete. All these difficulties would be overcome if we did remember, officials and patients, that we were members one of another.

One last reference to a technical point might be permitted. Mr. Benjamin had spoken of rash applications of the method of multiple regression. Some of these could be avoided, not by preferring a different technique, but by using a little common sense. No old-fashioned biometrician who used a multiple regression equation to express, say, one anatomical or physiological measurement in terms of several other measurements correlated with it would have dreamed of using the equation as an instrument of extrapolation, by inserting in the right-hand side of the equation values of the "independent" variables beyond the range of the observations used to compute the equation. This, of course, applied to regression equations connecting so-called social indices. With social indices, as Mr. Benjamin noted, there were also difficulties not found in the anatomical or physiological case, but ordinary common sense should prevent gross errors.

Mr. R. F. GEORGE said he had been privileged on several recent occasions to act as examiner in statistics for the younger members of Local Government staff who aspired to qualify as Borough Treasurers. In the last paper he set there was an innocent question asking candidates to state the arguments for and against a centrally-controlled statistical office in a major Local Authority—that was, an office designed to serve the functional departments impartially and generally to co-ordinate the Authority's statistics. This question proved extremely popular—much to his surprise until he realized that it was the only question in the paper which could conceivably be answered without the slightest knowledge of statistics.

In their effort to pay tribute to what they assumed were the susceptibilities of the examiner, the candidates said how highly desirable such an organization would be, and that such reform in Local Government administration was long overdue. Every major Authority ought to have a

centrally-controlled statistical service in order that functional departments could look to one source for their figures, and so be relieved of their own independent work of figure collecting. But half-way through their answers it was clear that a number of the examinees were assailed by doubts, as they gradually came to realize that such a central statistical service might possibly involve the surrender of statistical work by the Borough Treasurer, and this, of course, might imply a measure of disloyalty to the particular department in which they were employed.

In the result they compromised. "By all means," they said, "let there be a highly developed co-ordinating statistical office, but it must remain within the control of the Borough Treasurer."

This attitude of mind led him to the conclusion that there was a fundamental difficulty in the way of establishing anything like a central statistical office in major Local Authorities. To do so would necessarily mean the surrender of existing functional interests. The Medical Officer of Health, the Education Officer and the Borough Treasurer, to name only three senior officers, would be required to look elsewhere for material which hitherto they had provided for themselves. Not only was it very difficult for us to give up old-established habits, but few of us really enjoyed giving up responsibilities which we had hitherto, with justification, regarded as our own. He was sure that any centrally placed service designed to co-ordinate the statistics of a local authority would have to face this difficulty before it could be securely established and really effective.

Mr. E. R. KNIGHT, speaking of statistics of road accidents, said that in some areas these were maintained by the police, in other areas by the highway authorities and in yet other areas by both. He had in mind Surveyors' offices within the County of London where the whole of the accidents which had occurred at a given point over the last ten years could be asked for and obtained within ten minutes, and a fairly full summary from any particular angle could be produced within ten days. If statistics were not obtainable in the case already cited by another speaker, it was because for some reason it had not been realized that the figures were wanted for the purposes of the enquiry. He would ascribe the difficulty to the fact that the brief was not properly prepared.

It was only the larger local authorities which could run an adequate statistical department; the same thing could not apply to the smaller local authorities. The further one got away from the source of the data the more unreliable were the conclusions likely to be. His own accident figures would differ in certain places from those of the Commissioner of Police. Over the total they would not differ significantly, but the Commissioner of Police, for example, would include as a "car" accident a man who slipped on a banana skin and hit his head against a stationary motor car. He would do so because a car was concerned in the occurrence and therefore, in a sense, it was a motor accident, though not from the point of view of the Highway Authority.

The following comment was received in writing:—

Dr. PERCY STOCKS: Mr. Benjamin has produced a most thoughtful and stimulating paper, and there are many points which invite comment. The need to disentangle effects upon health of overcrowding from those of associated economic factors is mentioned, and it seems to me that is really a piece of research for Medical Officers of Health. Death certificates can be grouped by social class; but they don't tell us what were the conditions of the house where the decedent lived. The Local Authority will have that information, and could classify addresses by density per room so that deaths could be grouped by social class and housing density simultaneously. For my part I should find a few such square tables of death-rates (corrected for age) more convincing than partial regression, confluence or factor analysis. The social class gradient of diabetes is mentioned, and this reminds me of a curious fact. If the war did one useful thing, it was to induce young women to eat substantial meals instead of living on tea and cakes; and I feel sure it was no accident that, in sharp contrast with all other groups of the population, young women between 20 and 35 showed no fall in diabetes death rate between 1938 and 1945.

Concerning local populations, Mr. Benjamin suggests that the General Register Office provided each Local Authority before the war with regular estimates by sex and age. But here he must be speaking from the ivory tower of the County Hall, for the only sex-age estimates made for inter-censal years were for regions, national density aggregates and London county. Absence of a census has been a great deprivation in this matter, and even regional estimates by sex and age had to be dropped after 1941. In producing regional studies of some diseases at adult ages we have had to fall back on that dangerous instrument proportionate mortality, using total non-violent deaths as denominator. I should doubt the advantage of cancer deaths for such a purpose. Estimates of civilian populations of all ages in 1948 were published in the Quarterly Return for the September quarter for each great and small town. It is hoped to be able to issue local populations by sex and age derived from a count of the National Register at the end of 1947, in order to fill the gap before the next census.

Attempts to use too fine a classification of diseases are rightly deprecated for national statistics. The naïve faith of specialists that diagnostic niceties to which they are accustomed are in common use amongst the profession as a whole is always touching, and I recall Professor Greenwood telling over-enthusiastic members of a Committee that of course it was possible to make a classification with a separate box for every death, but statisticians would not be greatly helped by that.

For areas such as London the Intermediate List of 150 groups in the 1948 *Manual of the International List of Diseases, Injuries and Causes of Death* should be suitable; and for small areas the Abbreviated List of 50 grouped causes of death. On the question of comparability between New York and London, I can say that at last the same method of joint cause assignment is being used on both sides of the Atlantic, and that published statistics for 1950 ought to be free from that source of serious discrepancy.

Mr. Benjamin is, I believe, unduly pessimistic about the utility of hospital and survey statistics. We hope, for example, to know this year the causes of admission to all hospitals in at least one hospital region, in one complete county and in one large city; how many died in and out of hospital in every area, and from the Survey of Sickness what proportion of the adult population suffering from particular diseases became in-patients. By piecing together these we ought to know much more about morbidity and fatality rates than hitherto. Whilst each one of these sources of statistics has only a restricted value if taken alone, together they will provide sense, we hope. Tests of the Survey rates against other sources of information are showing, as they did in America, that for diseases where the doctor had no reason to withhold the diagnosis from the patient, agreement between rates based on the patients' story and the doctor's story is surprisingly good.

Mr. Benjamin said that he would reply in more detail in the *Journal*.. If the Paper had stimulated discussion, the discussion had also stimulated him, and as a mere junior in the statistical field he had learned a great deal from those who had spoken, especially from Professor Greenwood, whose insistence on co-operation, and commonsense in the interpretation of statistics would, he hoped, be remembered by all present.

Mr. Benjamin subsequently wrote as follows:

Dr. Charles's remarks are especially valuable to me personally in widening my outlook still further. The liaison problems to which she refers present urgent difficulties which once solved will do much to improve the efficiency of social administration generally. I agree that my use of the terms "race" and "occupation" were inexact, though I hope my meaning was reasonably clear. I strongly support what Dr. Charles has to say about Census material. I am glad also that she laid emphasis on the importance of obtaining material of a quality that merits analysis. Too often one meets what is in effect a request to make bricks without straw and local authorities should be reminded that it is useless to employ a statistician unless they are prepared to allow him reasonable powers to obtain the basic data necessary for the solution of problems he is expected to solve. Mr. Long and I perhaps mean something different when we use the word "statistics." Mr. Long cannot reconcile my praise of the statistical work associated with the planning of the Health Service with errors in the estimates of cost. The ultimate cost of a service is, of course, at the mercy of the later development of economic trends, salary negotiations and political compromise. I still consider. that the comprehensive health service was founded on some well executed statistical surveys, *e.g.* the Ministry of Health Regional Hospital Surveys. I am familiar with the Ministry of Health reports on Local Government Financial Statistics and I have also had eight years' experience in the financial department of the L.C.C. The figures to which Mr. Long refers show how much is spent on each service by each local authority, but they do not indicate the varying *needs* of the areas or what value was obtained for the money spent. I am appreciative of what has been done in the field of unit costs, but am not satisfied that the choice of units is always as realistic as it should be. I know of one instance where the cost of a new Health Centre was compared with that of a block of flats regardless of the medical equipment, which was not common to both buildings. It is, of course, foolish to generalize from the particular and to ignore also that many units have to be accepted *faux de mieux*. Mr. Lythgoe has rightly mentioned some of the important difficulties. If I seem impatient for a good deal more to be done I still sincerely acknowledge the valuable work which the Institute of Municipal Treasurers has done and I am content to leave the matter at Mr. Lythgoe's plea for a closer link between accountants and statisticians. Dr. Stocks's constructive criticism is valuable. I regret that I was unaware that the estimate of local population was yet another example of the generous treatment meted out to the L.C.C. by Dr. Stocks and his colleagues at the General Register Office. I can only say that I am even more deeply indebted. If I am pessimistic about the value of hospital statistics for morbidity survey purposes I am certainly not

unappreciative of the invaluable experiments which Dr. Stocks and his colleagues are constantly making to fill the gaps in our knowledge; future statisticians will inherit a tremendous legacy.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

Mohamad H. M. Abdulcader.
Frank Brickman.
Jacob Bronowski.
The Hon. R. E. Denison-Pender.
Horace Gray Forward.
D. C. R. Grailcourt.
William Granger.
Robert Henry Ewen Inkson.
Joan Margaret Lesford.
Donald Albert Lindsell.
Margaret Anne Mellone.
Jean Dorothy Rogansky.
Albert Senior.
Herbert Simon Sichel.

Corporate Representative

William Deneen, representing The National Coal Board (South Western Division).

TABLES OF SEQUENTIAL INSPECTION SCHEMES TO CONTROL FRACTION DEFECTIVE

By F. J. ANSCOMBE

(*Statistical Laboratory, University of Cambridge*)

1. *Introduction*

The object of this paper is to present tables of sequential inspection schemes for controlling the fraction defective of batches of manufactured articles—or, more generally, for controlling the frequency-parameter of binomial populations. The fraction defective (or binomial frequency) is assumed to be small and the size of batch large. Two sorts of inspection scheme are considered: (1) non-rectifying, in which only a small proportion of the batch is inspected, and a decision is then made to "accept" or to "reject" the batch, according to whether or not it appears to meet a required standard of quality; and (2) rectifying, in which possibly a large part or the whole of the batch is inspected, any faulty articles found are removed, corrected, or replaced by good ones, and the inspection serves to guarantee that the final quality of the batch will be satisfactory. Rectifying inspection must necessarily be non-destructive; non-rectifying inspection may be either destructive or non-destructive.

The non-rectifying schemes tabulated are due to Bartky (1943), Wald (1945, 1947), and Barnard (1946). A modification imposing an upper limit to the sample size is considered in addition to unclosed schemes. The formulae used in computing the tables are exact ones due to Burman (1946) and Stockman and Armitage (1946), taken in the limit as the fraction defective $p \to 0$.

The rectifying schemes have already been described by me elsewhere (1946). The formulae again are exact ones, taken in the limit as the fraction defective $p \to 0$.

It is hoped that the tables will prove useful in three ways: (1) They will cover, in a convenient form, a good proportion of the schemes needed in practice. (2) Approximate formulae, such as those of Wald, Barnard, and others, for non-rectifying open schemes, may be checked against the exact values given in the tables. (3) Comparisons of different sorts of schemes that achieve approximately the same ends can be made.

The tables have been compiled with the needs of industrial inspection in mind. Such inspection problems seem to occur less frequently in other fields of work, and it will be convenient to use the language of industrial inspection throughout.

The present paper may be compared with the book on Sampling Inspection issued by the Statistical Research Group of Columbia University (1948). The latter is wider in scope, since it covers single and double sampling as well as sequential sampling; otherwise, the intention is similar. The present tables give more fundamental information about the schemes tabulated than the Columbia tables do, and leave the user freer in his choice of an inspection scheme suited to his special requirements.

A reader who is not interested in mathematical theory should omit sections marked with a †.

2. *Classification of industrial inspection problems*

It may be helpful, before considering specific inspection schemes, to distinguish some different types of inspection problem. Let us consider a factory, or unit in a factory, which receives goods from a supplier, performs certain operations on them, and then despatches them to a consumer. (There may indeed be more than one supplier and more than one consumer. Both "supplier" and "consumer" are sometimes merely other units in the same organization.) There can be inspection at three stages: (1) *acceptance inspection*, i.e. inspection of the goods received from the supplier before they are fed into the production line, (2) *process inspection*, i.e. current inspection of the production line, (3) *final inspection* of completed goods before dispatch to the consumer. The second of these is concerned with whether the machines are correctly adjusted, etc., and usually involves some kind of check inspection at intervals. It is essentially *continuous* inspection, not directly dependent on the size of batches in which the raw material is received or the finished goods dispatched.

Acceptance inspection may be non-rectifying or rectifying. In the first case, the manufacturer decides whether to accept the goods offered or to reject them. Rejected goods are returned to the supplier. If the goods are received in a continuous flow, in the order of their production, the inspection may be of a continuous type; while if the goods are received in batches without anything being known of the order in which they were produced, or whether they were produced consecutively, the inspection will refer to each batch separately. It will again refer to the batches separately even if they are received in a continuous flow, if it is known that the batches form units of uniform quality, such as the product of a single cast of metal. On the other hand, the inspection may be rectifying. This is so when the manufacturer cannot reject goods that he is dissatisfied with (perhaps because he has already bought them), but tries to safeguard his production line against receiving material of poor quality by, if necessary, weeding out defective articles first. He will probably require a guarantee of "average outgoing quality" (AOQ) from his inspection, whether the product is received in separate batches or in a continuous flow.

Final inspection, again, may be either non-rectifying or rectifying. It will be non-rectifying, in particular, if the inspection is destructive. The decision to be made in non-rectifying inspection is between passing the goods as satisfactory and withdrawing them for correction by a further process, or for degrading or scrapping. If, on the other hand, the inspection is rectifying, the manufacturer will be giving a quality guarantee to the consumer, the nature of which will depend on the rate and conditions of delivery. Two extreme cases are: (i) the manufacturer sells his product in separate batches, to various consumers, and guarantees the quality of each batch, (ii) the manufacturer sells his whole output (or long consecutive stretches of it) to one consumer In the former case a batch inspection scheme with a "lot tolerance" guarantee is called for, in the latter a continuous inspection scheme with, probably, an AOQ guarantee.

The above classification is not exhaustive, but rather illustrative. Confining attention to cases where quality is measured by a simple fraction defective, we see that there are four main types of inspection scheme to be considered, given by the combinations of "rectifying" or "non-rectifying" and "continuous" or "in batches". Rectifying batch inspection is catered for by Dodge and Romig (1944), SRG Columbia (1948), Greb and Berrettoni (1949), and by the rectifying schemes tabulated below. Non-rectifying batch inspection is catered for by SRG Columbia (1945, 1948), Vajda (1946), and the non-rectifying schemes tabulated below. Rectifying continuous inspection has been considered by Dodge (1943) and Wald and Wolfowitz (1945), non-rectifying continuous inspection by Anscombe *et al.* (1947).

An aggregate of articles inspected is referred to as a "batch". The word has been thought to be slightly preferable to "lot", since the latter may already be in use to denote a set of articles aggregated and labelled for some other purpose, e.g. a "production-lot". A batch offered for inspection may perhaps consist of several production-lots. But the customary expression "lot tolerance" has been retained, though "batch tolerance" would have been more consistent.

Non-Rectifying Inspection

3. *Definition of schemes*

As Barnard (1946) has remarked, Wald's sequential procedure for testing the fraction defective of a batch of articles can be conveniently expressed in terms of a scoring system, as follows:—

> *Sample the batch by drawing articles randomly from it one by one. Count* $+1$ *for a non-defective article,* $-b$ *for a defective. With starting score zero, accept the batch if the score reaches or exceeds* $+H_1$*, reject it if the score falls to or below* $-H_2$*. Continue sampling until one or other decision is reached.*

H_1, H_2, and b, are given constants, usually integers. We write

$$R_1 = \frac{H_1}{b+1}, \qquad R_2 = \frac{H_2}{b+1}. \qquad (3.1)$$

In practice, unless an automatic score-reckoner is available, it will be convenient to use only schemes in which R_1 and R_2 are both integers. Then the scoring can be replaced by simple stopping conditions expressed in terms of "samples" of $b + 1$ articles, thus:—

Conditions for acceptance				*Conditions for rejection*			
0 defective(s) in		R_1	samples	$R_2 + 1$	defectives in	1	sample(s)
1	,,	$R_1 + 1$	,,	$R_2 + 2$	,,	2	,,
2	,,	$R_1 + 2$	,,	$R_2 + 3$	,,	3	,,
3	,,	$R_1 + 3$	,,	$R_2 + 4$	,,	4	,,
. . .				. . .			

Acceptance can occur only after a whole number of "samples" have been inspected. Rejection however, may occur at any stage, as soon as sufficient defectives have been found.

If R_1 and R_2 are not integers, but their sum is an integer, the stopping conditions still take a simple form, the first "sample" being one of size $\theta(b + 1)$, where θ is the fractional part of R_1, and subsequent "samples" being all of size $b + 1$.

Wald's inspection procedures are "open", in the sense that no upper limit is placed on the number of articles inspected. (The total number of articles inspected will be termed the *sample size*, the word "sample" being used in a different sense from in the stopping conditions.) A closing, condition may be introduced, however, the one considered here being:—

If no decision has been reached when $K(b + 1)$ articles have been inspected, accept the batch if the score is positive or zero, reject it if the score is negative.

K is a given constant. The rejection conditions may be contracted, by rejecting as soon as so many defectives have been found that even if no more are found the score will be negative when $K(b + 1)$ articles have been inspected.

For example, the scheme $R_1 = 2$, $R_2 = 3$, $K = 6$, has the following stopping conditions, in terms of "samples" of $b + 1$ articles:—

Conditions for acceptance				*Conditions for rejection*			
				4	defectives in	1	sample(s)
0	defective(s) in	2	samples	5	,,	2	,,
1	,,	3	,,	6	,,	3	,,
2	,,	4	,,	7	,,	4	,,
3	,,	5	,,	7	,,	5	,,
4, 5, or 6	,,	6	,,	7	,,	6	,,

Note that in the last line the number of defectives permissible for acceptance is equal to the number of "samples" inspected (6), and the maximum number of defectives appearing in the rejection conditions is one more than this (7).*

To illustrate stopping conditions when R_1 and R_2 are not both integers, conditions for (i) $R_1 = 1\frac{1}{4}$, $R_2 = 3\frac{3}{4}$, open, and for (ii) $R_1 = 1\frac{1}{4}$, $R_2 = 2\frac{1}{2}$, open, are given. In the first $R_1 + R_2$ is an integer, in the second it is not. All "samples" are of $b + 1$ articles.

	Conditions for acceptance				*Conditions for rejection*			
(i)					4	defectives in	$\frac{1}{4}$	sample(s)
	0 defective(s) in		$1\frac{1}{4}$	samples	5	,,	$1\frac{1}{4}$	,,
	1	,,	$2\frac{1}{4}$	,,	6	,,	$2\frac{1}{4}$	,,
	2	,,	$3\frac{1}{4}$	,,	7	,,	$3\frac{1}{4}$	,,
	. . .				. . .			
(ii)					3	defectives in	$\frac{1}{2}$	sample(s)
	0 defective(s) in		$1\frac{1}{4}$	samples				
					4	,,	$1\frac{1}{2}$	,,
	1	,,	$2\frac{1}{4}$	,,				
					5	,,	$2\frac{1}{2}$	,,
	2	,,	$3\frac{1}{4}$	,,				
					6	,,	$3\frac{1}{2}$	,,
	. . .				. . .			

* I regret that in a previous paper (Anscombe *et al.*, 1947) the stopping conditions for one of these schemes were quoted incorrectly.

The notation here used follows Barnard. For present purposes it would have been more convenient to have used a single symbol in place of $b + 1$. Barnard's definition of b accords with his preference of odds to probabilities. A comparison with Wald's notation may be useful:—

Present notation	*Wald's notation*
R_1	$-h_0$
R_2	h_1
$b + 1$	$1/s$

4. *The tables*

The upper portions of Tables I–III below refer to a selection of open schemes, in four sets, (*a*) schemes with $R_2 = R_1$, (*b*) schemes with $R_2 = 2R_1$, (*c*) schemes with $R_2 = 3R_1$, (*d*) three other schemes. The lower portions refer to a selection of closed schemes. Table I indicates the values of the fraction defective p for which the chance that the batch will be accepted is 99%, 90%, 50%, 10%, and 1%; the quantity actually tabulated is the product $p\ (b + 1)$. Table II gives ratios of these percentage points of p. Table III indicates the average sample size A (i.e. the average number of articles inspected) when p has any of the percentage-point values, and also the maximum value of A, which usually occurs for a value of p not far from $1/(b + 1)$; the quantity actually tabulated is $A/(b + 1)$. It is assumed that b is fairly large.

If, for example, we require an open scheme with $R_2 = 2R_1$ which gives roughly a 99% chance of acceptance if $p = 0{\cdot}01$ and only a 10% chance of acceptance if $p = 0{\cdot}03$, we see from Table II, looking in the column headed 99%/10% and among the open schemes for which $R_2 = 2R_1$, that the desired ratio of 0·3333 is approximately reached for $R_1 = 2$, $R_2 = 4$ (when it is 0·3396). Turning now to the first column of Table I we see that we should set $0{\cdot}01\ (b + 1) = 0{\cdot}5710$, i.e. $b + 1 = 57$; or, from the fourth column, $0{\cdot}03\ (b + 1) = 1{\cdot}6816$, i.e. $b + 1 = 56$. Let us take the latter. Then an open scheme with $b + 1 = 56$, $H_1 = 112$, $H_2 = 224$, approximately meets the requirement, and its stopping conditions can be expressed conveniently in terms of "samples" of size 56:

Conditions for acceptance	*Conditions for rejection*
	5 defectives in 1 sample(s)
0 defective(s) in 2 samples	6 „ 2 „
1 „ 3 „	7 „ 3 „
. . .	. . .

The 99%, 90%, 50%, 10%, and 1% points of this scheme are $p = 0{\cdot}010, 0{\cdot}014, 0{\cdot}020, 0{\cdot}030, 0{\cdot}046$; and from Table III we see that the average sample sizes at these points are 253, 395, 489, 306, 155. The maximum average sample size is 494, and for $p = 0$ it is H_1, i.e. 112. A rough graph of the average sample size may now be drawn.

It will be observed that in all the schemes tabulated $R_2 > R_1$ (or $H_2 > H_1$). It appears that in normal industrial inspection it is necessary to be almost certain of accepting material in some range of "good" quality, while material in some range of "bad" quality should be rejected with a substantial probability, perhaps with a high probability. The acceptance of "good" material is at least as pressing a requirement as the rejection of "bad" material. In these circumstances, a scheme with $R_2 > R_1$ is suitable, for reasons given below, §§6 and 7. In other fields of work the situation might be different. In testing for presence of a desirable rare character, a sequential scheme of the above type could be used, with "acceptance" and "rejection" interchanged, and if acceptance of "good" material were more important than rejection of "bad" a scheme with $R_2 < R_1$ would be needed. But for industrial purposes, the selection of open schemes tabulated should cover most needs for such schemes. The schemes with fractional values of R_1 and R_2 have been included in order to permit of interpolation, as required for the investigation described in §6. It may be noticed that differences in the columns of Tables I and II do not run very smoothly. This is because of the peculiar mathematical form of the operating characteristic (Burman, 1946). As to the closed schemes, it cannot be claimed that the selection tabulated will cover most needs, but some common needs will be met.

From Table I operating characteristics can be roughly sketched. If any accuracy is required, the curves are best drawn on logarithmic probability graph paper (logarithm of fraction defective

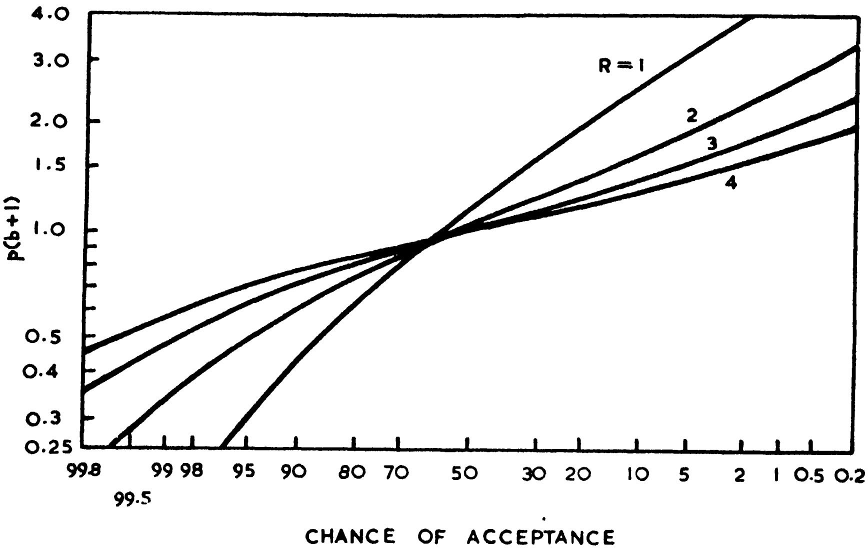

FIG. 1.—Non-rectifying inspection. Operating characteristics of open sequential schemes, for $R_1 = R_2 = 1, 2, 3, 4$.

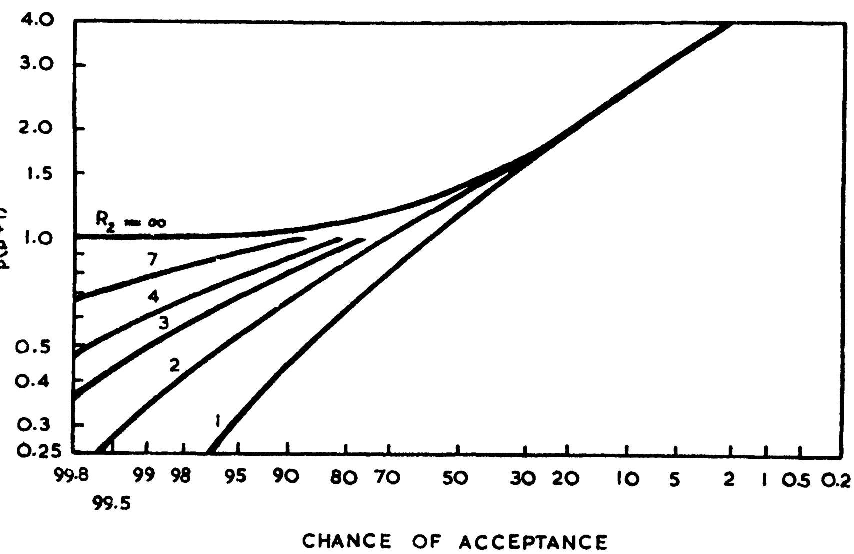

FIG. 2.—Non-rectifying inspection. Operating characteristics of open sequential schemes, for $R_1 = 1$, $R_2 = 1, 2, 3, 4, 7, \infty$.

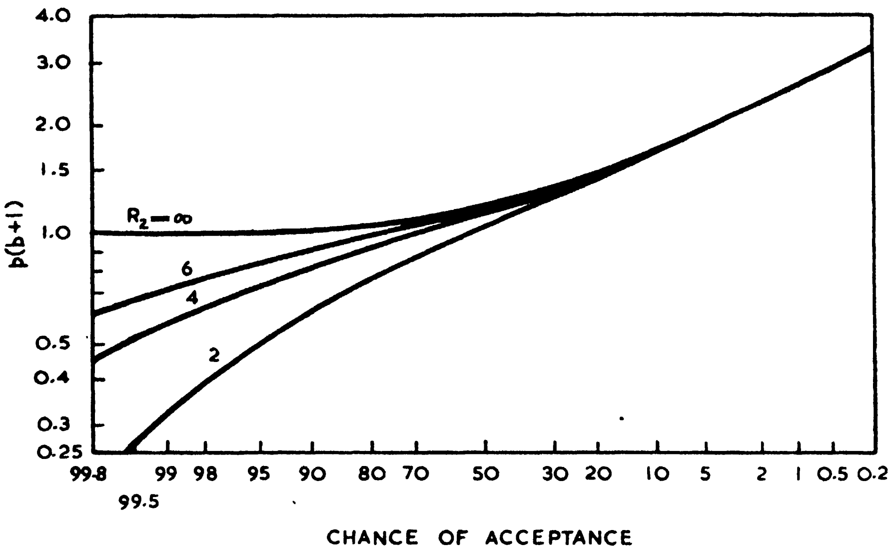

FIG. 3.—Non-rectifying inspection. Operating characteristics of open sequential schemes, for $R_1 = 2$, $R_2 = 2, 4, 6, \infty$.

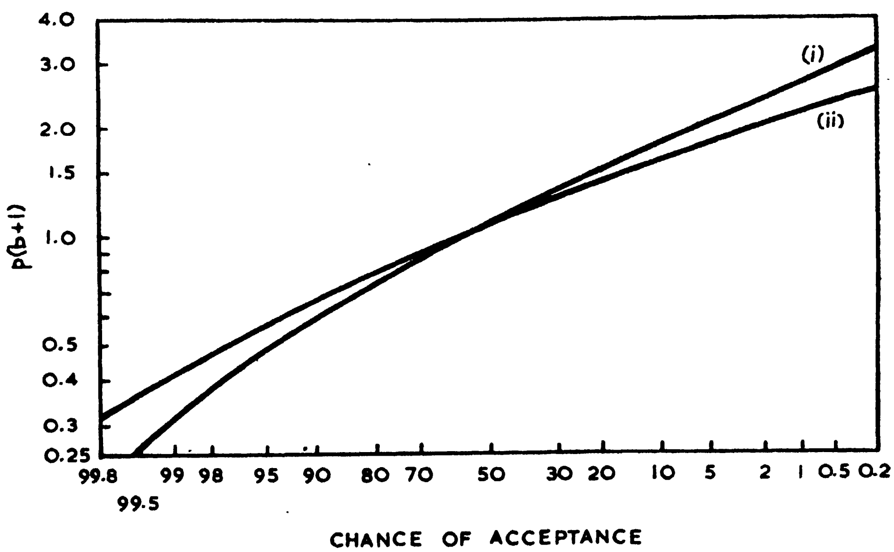

FIG. 4.—Non-rectifying inspection. Operating characteristics of closed sequential schemes, for
(i) $R_1 = R_2 = 2$ $K = 6$,
(ii) $R_1 = R_2 = 3$, $K = 8$.

plotted against equivalent normal deviate to the probability of acceptance). Some specimen curves are given in Figs. 1–4. The curves for $R_2 = \infty$ in Figs. 2 and 3 have been included to indicate the direction in which the curves change as R_2 increases. As the average sample size is infinite whenever the chance of acceptance is less than 100%, schemes with R_2 infinite are not practicable.

Fig. 5 is a demonstration piece. It refers to a scheme with $R_1 = 2$, $R_2 = 3$, $K = 5$, and $b+1 = 30$. This scheme has almost exactly the same operating characteristic, shown in Fig. 5(*a*), as a single-sampling scheme in which the batch is accepted if not more than 4 defectives are found in 126 articles inspected, and rejected as soon as 5 are found. Sample sizes for both schemes

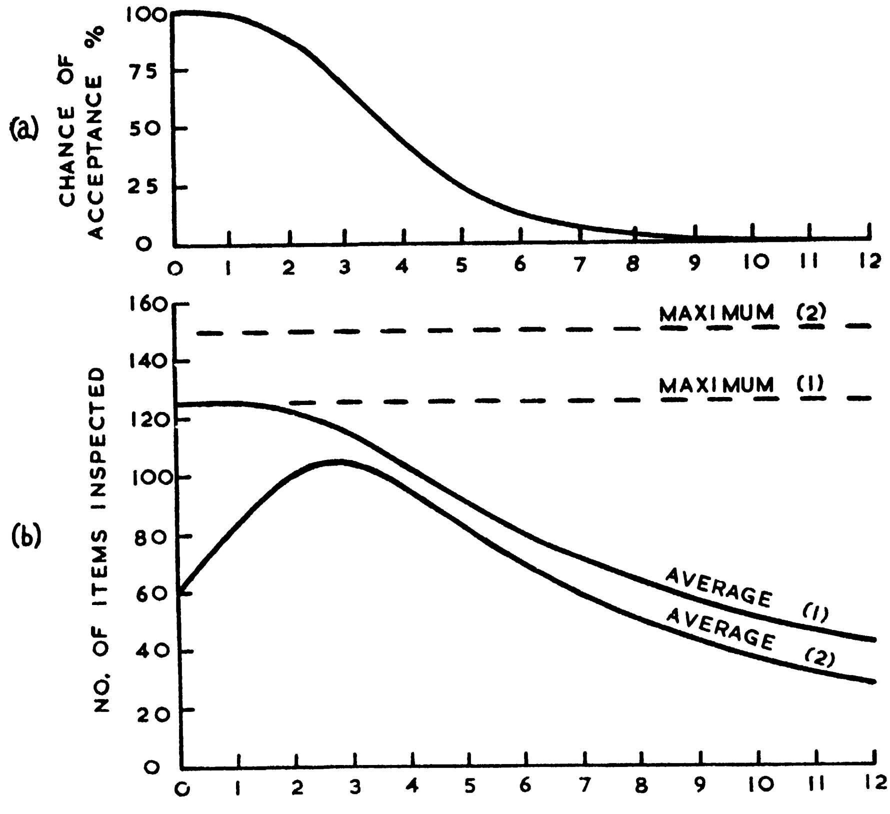

FIG. 5.—Non-rectifying inspection. Comparison of (1) a single-sampling scheme with (2) a closed sequential scheme.
(*a*) Operating characteristic.
(*b*) Average and maximum sample sizes.

are shown in Fig. 5 (*b*), the average sample sizes by continuous lines, the maximum sample sizes by broken lines. It will be seen that for all values of the fraction defective the sequential scheme has a lower average sample size than the single-sampling scheme, but a higher maximum sample size (150 as compared with 126).

†5. *Approximate formulae*

Wald, Barnard, and Bartlett (1946), have given simple asymptotic formulae for the chance of acceptance and average sample size of an open scheme. Setting $X = p\,(b + 1)$, we consider

here the limit $p \to 0$, with X constant. Wald's formula for the chance of acceptance P then becomes

$$P = \frac{\exp(R_2T) - 1}{\exp(R_2T) - \exp(-R_1T)}, \qquad (5.1)$$

where T is defined by

$$X = \frac{T}{\exp T - 1}. \qquad (5.2)$$

This gives P for $X \neq 1$. When $X = 1$, we have

$$P = \frac{R_2}{R_1 + R_2}. \qquad (5.3)$$

Barnard's and Bartlett's formulae lead to the same result except that (5.2) is replaced by

$$X = \frac{2}{T + 2}, \qquad (5.4)$$

which is equivalent to (5.2) when X is near 1. Wald and Bartlett give formulae for the average sample size A, which reduce to

$$\frac{A}{b+1} = \frac{R_1P - R_2(1-P)}{1-X}, \qquad (5.5)$$

provided $X \neq 1$, while for $X = 1$

$$\frac{A}{b+1} = R_1R_2. \qquad (5.6)$$

(As $p \to 0$, A and $b + 1 \to \infty$ and their ratio tends to the limit shown.)

When we compare Wald's formula for P, (5.1) and (5.2), with the correct values given in Table I, we find the formula is good for X above 1, being asymptotically correct for large X. But for X small, P is underestimated, as illustrated in Text-table 1, where for several schemes the approximate value of P is shown when X is such that the true value of P is ·99.

TEXT-TABLE 1

R_1	R_2	X	*True* P	*Approximate* P
1	1	·1388	·99	·9598
2	2	·3213	,,	·9805
3	3	·4642	,,	·9843
4	4	·5619	,,	·9858
1	3	·4907	,,	·9847
2	6	·6978	,,	·9874

The approximation to P given by (5.1) and (5.4), like Wald's, is asymptotically accurate for large R_1 and R_2, but for the values considered in the Tables it is seriously in error for X both small and large, and gives an operating characteristic of quite wrong shape. There is, however, a range of values of X around $X = \cdot 4$ where it is more accurate than Wald's formula.

As to the average sample size formula (5.5), if it is used in conjunction with (5.1) and (5.2) A seems always to be under-estimated, substantially so if R_1 and R_2 are small. In particular (5.6) gives an underestimate for all the schemes tabulated. But (5.5) gives quite an accurate value of A at the 99% acceptance point if the true operating characteristic, and not either of the approximate ones, is used.

6. *Comparison of schemes*

So far we have considered the properties of particular specified schemes. We turn now to the principles by which a scheme should be chosen to meet a given inspection situation. The sequential

schemes have four adjustable constants, R_1, R_2, K, and b. It is usual to begin by specifying two points on the operating characteristic of the scheme, one at a high chance of acceptance and one at a low, such as the 99% and 1% acceptance points. The two values of the fraction defective are termed by Wald p_0 and p_1, and by Barnard the safe-point and risk-point, respectively. There are then two degrees of freedom left, which may be taken as K and the ratio R_2/R_1. In other words, given the latter quantities a scheme will be completely specified by the two points on its operating characteristic. We therefore ask, how should K and R_2/R_1 be chosen?

Wald's answer to this question is as follows. He defines a unique likelihood ratio test in terms of the two specified points on the operating characteristic. This is an unclosed scheme of the type we are considering. Thus K should be infinite, except when circumstances forbid. The effect of closure must in fact be considered, but as a concession to human frailty rather than as a device possibly interesting in its own right. Approximate (asymptotic) formulae are given for determining the three remaining constants, R_1, R_2, and b. For small p_0 and p_1 the formulae are

$$b + 1 = \frac{\ln(p_1/p_0)}{p_1 - p_0}, \quad R_1 = \frac{\ln(P_0/P_1)}{\ln(p_1/p_0)}, \quad R_2 = \frac{\ln[(1 - P_1)/(1 - P_0)]}{\ln(p_1/p_0)}, \quad . \quad . \quad . \quad (6.1)$$

where P_0 is the chance of acceptance at the safe-point p_0, and P_1 is the chance of acceptance at the risk-point p_1. In particular, the ratio R_2/R_1 depends on P_0 and P_1 only, as illustrated in Text-Table 2. (Lines 5 and 8 refer to the limit as safe and risk points coincide and specify the gradient of the operating characteristic at the point.)

TEXT-TABLE 2

Chance of acceptance at p_0	p_1		R_2/R_1
99%	1%	.	1·00
,,	10%	.	1·96
,,	50%	.	5·73
,,	90%	.	24·16
,,	99%	.	99·00
90%	10%	.	1·00
,,	50%	.	2·74
,,	90%	.	9·00

Wald's likelihood ratio sequential test has an optimum property (Wald and Wolfowitz, 1948), namely, that if the fraction defective of all batches submitted is either p_0 or p_1 the smallest possible number of articles will be inspected, in the long run, with this test, as compared with what would be needed if any other inspection scheme were used having the same two fixed points on its operating characteristic. But striking and elegant though this theorem is, it is not necessarily decisive, since (1) we do not usually know that the batches inspected will have fractions defective equal to either p_0 or p_1, and (2) the cost of inspection, in the long run, may not be simply proportional to the total number of items inspected. A more realistic approach is due to Barnard (1946).

From the tables of operating characteristics and average sample sizes that have been computed (and are summarized in the Tables below), it has been possible to investigate, numerically, the relative merits of sequential schemes having two given fixed percentage points. Confining attention first of all to open schemes ($K = \infty$), we choose a pair of percentage points, and then for selected values of R_2/R_1 we find the constants of the scheme and its average sample size curve, by interpolation between the tabulated schemes. Some typical schemes found in this way are given in Text-table 3. Since interpolation between tabulated schemes having constant R_2/R_1 is only possible for $R_2/R_1 = 1$, 2, or 3, one other scheme, here $R_1 = 1$, $R_2 = 4$, may be compared with schemes having these three ratios R_2/R_1. p_0 denotes the fraction defective at the percentage point with the higher chance of acceptance, i.e. the 99% point for the first four columns of the interior of the table, and the 90% point for the fifth column. A_0 is the average sample size at $p = p_0$, A_m is the maximum value of A as p varies. The value of p_0 is not specified, and other quantities are found explicitly on dividing the numerical entries by p_0.

TEXT-TABLE 3

R_2/R_1 or H_2/H_1	Chance of acceptance at p_0 and at p_1	99% 90%	99% 50%	99% 10%	99% 1%	90% 10%
	p_0/p_1	0·6819	0·4371	0·2339	0·1286	0·3429
4	$(b+1)p_0$	0·598	0·598	0·598	0·598	0·877
	H_1p_0	0·598	0·598	0·598	0·598	0·877
	A_0p_0	1·41	1·41	1·41	1·41	3·34
	A_mp_0	2·80	2·80	2·80	2·80	4·11
3	$(b+1)p_0$	0·571	0·558	0·540	0·533	0·817
	H_1p_0	0·729	0·685	0·624	0·603	0·902
	A_0p_0	1·63	1·49	1·30	1·24	2·81
	A_mp_0	3·19	2·88	2·48	2·35	3·43
2	$(b+1)p_0$	0·538	0·509	0·465	0·445	0·728
	H_1p_0	0·971	0·844	0·677	0·614	0·952
	A_0p_0	2·04	1·67	1·23	1·07	2·37
	A_mp_0	3·89	3·12	2·23	1·92	2·84
1	$(b+1)p_0$	0·499	0·445	0·366	0·326	0·590
	H_1p_0	1·655	1·265	0·838	0·660	1·095
	A_0p_0	3·24	2·23	1·29	0·96	2·09
	A_mp_0	6·07	4·04	2·22	1·58	2·44

From these and other comparisons one may draw the following conclusions. If the 99% and 1% points are fixed (or the 90% and 10% points), the scheme with $R_2 = R_1$ is nearly optimum for all values of the fraction defective, in the sense that it gives nearly minimum average sample sizes when compared with sequential schemes having other values of R_2/R_1. If the 99% and 10% points are fixed, the optimum value of R_2/R_1 is about 2; if the 99% and 50% points are fixed, it seems to be about 4 or 5; while if the 99% and 90% points are fixed, some still higher value of R_2/R_1 is called for.

These results (except the last, which is too vague) may be regarded as supporting and extending the Wald-Wolfowitz theorem, since the ratio R_2/R_1 suggested is roughly that given by Wald's rule (Text-table 2), and the property of minimum average sample size which, according to the theorem, holds at p_0 and p_1 when the scheme is compared with all other possible inspection schemes, holds *approximately* also for all fractions defective p when comparisons are restricted to other open sequential schemes. "Approximately" is necessary here. If we knew that all batches inspected would actually have a fraction defective much less than p_0, it would be advantageous to take a higher value of R_2/R_1 than that just suggested, as this would lead to a lower H_1 and lower average sample size at the values of p that we were interested in. The effect is small, however, and it is unlikely that considerations of what fraction defective is expected would weigh directly in practice in the choice of R_2R_1—important though such considerations are in determining other features of an inspection scheme. The overriding fact is the dependence of the shape of the operating characteristic on R_2/R_1. If all operating characteristics had the same shape (e.g. were straight lines when plotted as in Figs. 1–4), the first four columns of Text-table 3 would be identical. Actually they are very different.

It may be noted that just as the operating characteristics of open sequential schemes differ from one another in shape according to the value of R_2/R_1, they all differ from the operating characteristics of single-sampling schemes, which, when plotted as in Figs. 1–4, are uniformly concave from below. The latter do in fact resemble the operating characteristics of some of the closed sequential schemes. This is not entirely surprising, since if a sequential scheme is closed early enough it degenerates into a single-sampling scheme (i.e. when $K = R_1 \leqslant R_2$). The

unambiguous comparison of a single-sampling scheme with a closed sequential scheme in Fig. 5 was made possible by their having almost identical operating characteristics. The comparison of single-sampling schemes with open sequential schemes depends on which points of the operating characteristic are fixed. If the 99% and 1% points are fixed, the average sample size curves look much like those of Fig. 5, except that the two curves cross over for a short interval round about the 50% acceptance point. If the 99% and 50% points are fixed, the curve for the optimum sequential scheme is everywhere considerably lower than the other. The difference is connected with the fact that the higher the value of R_2/R_1 the less does the operating characteristic of an open sequential scheme resemble that of a single-sampling scheme.

Let us now consider the properties of closed sequential schemes, with a view to the optimum choice of K. Not many closed schemes have been tabulated, in particular none with high R_2/R_1, and any conclusions must be tentative. If K is large enough the scheme will have much the same operating characteristic and average sample size curve as if it were unclosed. It appears that K must be considerably in excess of $A_m/(b+1)$ for the open scheme having the same R_1, R_2, and b, for this to be so. Otherwise, closing the scheme has quite a pronounced effect on the operating characteristic (slope and shape) and the average sample size curve. If we fix two points on the operating characteristic, select the optimum value of R_2/R_1 as suggested above, and compare closed and open schemes having this value of R_2/R_1, we find (for $R_2/R_1 = 1$ or 2) that the average sample size curve for a closed scheme is above that for the corresponding open scheme for large and for small fractions defective, and below it in a middle range. A few specimen comparisons are shown in Text-table 4; each column compares a closed scheme with an open one. It is likely (but has not been checked) that the comparison of average sample size becomes progressively less favourable to closed schemes as R_2/R_1 is increased, in view of the fact that closed schemes, unless K is quite large, have operating characteristics rather like those of single-sampling schemes.

TEXT-TABLE 4

Chance of acceptance at p_0 and at p_1	99% 1%	99% 1%	99% 10%	99% 10%
p_0/p_1	0·1885	0·2224	0·2145	0·2821
Closed scheme				
R_2/R_1	1	1	2	2
K	8	12	6	10
$(b+1)p_0$	0·417	0·450	0·396	0·488
H_1p_0	1·251	1·351	0·793	0·977
A_0p_0	2·01	2·38	1·24	1·84
A_mp_0	2·57	3·46	1·74	2·99
Open scheme				
R_2/R_1	1	1	2	2
$(b+1)p_0$	0·407	0·447	0·442	0·516
H_1p_0	1·039	1·279	0·606	0·872
A_0p_0	1·71	2·26	1·05	1·75
A_mp_0	3·02	4·11	1·89	3·29

7. *Fixing standards and choosing an inspection scheme*

In planning an inspection scheme, it is desirable to have information on (1) the importance of accepting "good" material and rejecting "bad", or, more completely, the cost of accepting or rejecting material of any given quality; (2) the "process curve", i.e. the relative frequency with which batches of various qualities will be offered for inspection; (3) the way in which the cost of inspecting a batch depends on the number of articles sampled. In principle, if we had adequate information on these, it would be possible to evaluate the overall running cost of the production process for any proposed inspection scheme, so that of all schemes considered we could select one that minimized the cost. But such complete information is not normally available, and the

best we can hope for is to choose an inspection procedure that roughly approximates to the optimum. We require some simple rules for doing so.

Let us first consider how to specify the operating characteristic of the scheme. It is convenient to state a safe-point and a risk-point, which are standards of good and bad quality, such that we wish to accept material offered that is as good as or better than the good standard and not to accept material that is as bad as or worse than the bad standard. The determination of these standards will depend on (among other things) the economic relation of producer and consumer, and much variation is possible. To illustrate the considerations that may be involved, let us consider an acceptance inspection carried out by a "consumer" who receives the whole or a large part of the output of a "producer", or of several independent producers. The process curve of the submitted batches will probably be J-shaped, or single humped with a long upper tail; in other words, most batches will have a low fraction defective, but a few will have a considerably higher fraction defective. The consumer would no doubt prefer to accept only batches with zero fraction defective, but will in fact be forced to accept most of what is offered to him. If the bulk of what is offered is not acceptable, the consumer must change either his requirements or his supplier. The safe-point (standard of good quality) is therefore fixed so that most of the batches will have a fraction defective below it; and a high probability of acceptance at the safe-point, such as 99%, will be required.

In choosing the risk-point there is a temptation to set it lower, and with a lower chance of acceptance, than necessary. Provided the quality of the batches remains as in the process curve being considered, bad batches will only be offered rather rarely, and if most of these are rejected the actual proportion of batches accepted that are bad will be very small. There may be no advantage in increasing the amount of inspection to make it almost certain that bad batches offered will be rejected. Thus if 5% of batches offered are bad, and 90% of these are rejected, while nearly all the others are accepted, only about 1 in 200 of accepted batches will be bad, and this may well be a low enough proportion. Sometimes the mere existence of an inspection procedure has a useful moral effect on the producer, so that he offers fewer bad batches than he otherwise might. If, however, the quality of production deteriorates so that a substantial proportion (such as 50%) of the batches are rejected, the inspection scheme will still have a powerful protective effect, since the steady rejection of a substantial part of his output would soon force the producer out of business. Thus, it may well be inadvisable to attempt to guarantee that no bad batches will ever be accepted, and more important to provide that batches not much worse than the good standard shall have an appreciable chance of rejection, so that their production is discouraged. This is done by specifying a risk-point with a not-too-low chance of acceptance, such as 10% or 50% or even higher, and using a sequential scheme with R_2 greater than R_1, in accordance with the recommendations of §6.

Having chosen the safe-point and risk-point, with their associated chances of acceptance, and hence R_2/R_1, we now consider whether the scheme should be closed, and if so how early. The dominant consideration is how the cost of inspection depends on the number of articles inspected. If the cost is simply proportional to the number inspected, an unclosed scheme will be roughly the most economical. But sometimes the cost depends on other factors. For an open scheme, the frequency distribution of sample size, for any given fraction defective, is usually skew with a long upper tail, so that sample sizes of two or three times the average for that fraction defective will occur occasionally. Such variability in sample size can sometimes cause storage difficulties or inconvenience arising from the irregular flow of released (accepted) batches. Again, it is sometimes necessary to select the articles for inspection before the inspection begins, so that the maximum number of articles that may be needed is an important factor in the cost. This happens, for example, where the articles are subjected to a lengthy endurance test, or have to be dispatched somewhere else for testing, or where the actual sampling of the batch is laborious and requires manhandling of a great weight of material to obtain a fair sample. If a decision has not been reached when all the articles selected have been inspected, trouble and delay are involved in sending for more. These difficulties are mitigated or removed by using a closed scheme. The more important they are, the earlier the scheme should be closed. If they are so important that the marginal cost of inspecting the articles is negligible, a single-sampling scheme should be used (in which no attempt is made to economize on average sample size at the expense of maximum sample size).

8. *Miscellaneous remarks*

(1) If the above recommendations are compared with those of SRG Columbia (1948), it will be seen that they are less precise. The Columbia tables are designed for immediate use, and leave as little as possible to the discretion of the user. To effect this, the compilers have had to make some rough and ready decisions on debatable points. Very likely their inspection plans work well in practice. If on the whole quality is satisfactorily maintained, with a minimum of thought-taking and complexity, it will matter little that the schemes are sometimes not quite as economical as they might be. On the other hand, an inspector armed with suitable information concerning the properties of available inspection schemes, and also having intimate knowledge of the kinds of stores he wishes to inspect, should find little difficulty in choosing schemes more exactly fitted to his requirements.

(2) The average sample sizes in Table III have been computed on the assumption that the articles are inspected one by one, so that if the required number of defectives for rejection are found the inspection terminates at once. Sometimes it is convenient to inspect the articles in small groups. If, R_1 and R_2 being integers, groups of $b+1$ articles are inspected together, or any simple fraction of this number, the operating characteristic will be unaffected, but the average sample size will be slightly increased for all values of the fraction defective except zero.

(3) The Tables have been computed on the assumption that the fraction defective p is always small, or that b is large. The effect on the operating characteristic of reducing b is illustrated in Text-table 5, which gives percentage points of $p(b+1)$ for open schemes. It will be seen that the effect is nearly linear in $1/(b+1)$.

TEXT-TABLE 5

R_1	R_2	$b+1$	Values of $p(b+1)$ when the chance of acceptance is 99%	90%	10%	1%
2	2	∞	0·3213	0·6100	1·6605	2·5565
2	2	20	0·3308	0·6186	1·6140	2·4149
2	2	10	0·3412	0·6278	1·5693	2·2835
2	2	5	0·3650	0·6481	1·4849	2·0487
3	3	∞	0·4642	0·7100	1·4159	1·9551
3	3	20	0·4766	0·7188	1·3896	1·8820
3	3	10	0·4898	0·7279	1·3639	1·8125
3	3	5	0·5190	0·7476	1·3146	1·6836

(4) It has been assumed that the sampling of the batches is truly random. In practice some sort of stratified sampling will often be used, in drawing each "sample" of $b+1$ articles. This may lead to a slight sharpening of the operating characteristic, "good" material being more certain of acceptance, and "bad" of rejection. The average sample size when quality is intermediate between the "good" and "bad" standards will tend to be increased.

(5) If some articles are misclassified during inspection, defectives being classed as non-defectives and conversely, the scheme will have a different effect from that intended. The fraction defective p referred to in the Tables must be interpreted as the apparent fraction defective, not the true one.

†(6) The operating characteristics with $R_2 = \infty$ shown in Figs. 2 and 3 were derived as follows.

Burman (1946) showed that the chance of acceptance was

$$P(p) = (1-p)^{H_1} \times \text{a function of } H_1, H_2, b, \text{and } p\,(1-p)^b.$$

Let us set $H_2 = \infty$. If $p < 1/(b+1)$, it can easily be deduced from the weak law of large numbers that

$$P(p) = 1.$$

The same result holds if $p = 1/(b+1)$, by a form of the strong law of large numbers.* If $p > 1/(b+1)$, let $p'(<1/(b+1))$ be the root of

$$p'(1-p')^b = p(1-p)^b.$$

There is just one such root. Then we must have

$$P(p) = \left(\frac{1-p}{1-p'}\right)^{H_1}.$$

In the limit as $p \to 0$, with $X = p(b+1)$ constant, we get

$$\begin{aligned} P(X) &= 1 \text{ if } X \leqslant 1, \\ &= \exp\{-R_1(X-X')\} \text{ if } X > 1, \end{aligned}$$

where X' (<1) satisfies

$$X' \exp(-X') = X \exp(-X).†$$

This limit-type of non-rectifying scheme must be distinguished from the rectifying schemes considered below. It will be noticed that what there corresponds to H_1, namely αN, is regarded as being of the same order of size as the batch-size N, and we are not interested merely in the chance that the acceptance boundary will be reached, but in how many defectives will be left in the batch if the boundary is reached.

Rectifying Inspection

9. *The tables*

The rectifying inspection procedure considered here is the following:—

From a batch of N articles, a first sample of αN articles is inspected, and then further samples of βN articles each. Defective articles found are removed or replaced by good ones. Inspection ceases after the first sample if no defectives have been found, or after the second sample if altogether one defective has been found, or, generally, after the $(r+1)^{th}$ sample if altogether r defectives have been found. Inspection is continued until either this stopping rule operates or the whole batch is inspected.

The following notation will be used: Y for the initial number of defectives in the batch, Z for the number of defectives left in it after inspection, ξ for the proportion of the batch inspected at any stage of the inspection, y for the number of defectives so far found in the inspection. Thus $0 \leqslant \xi \leqslant 1$, $0 \leqslant y \leqslant Y$, and $Z = Y -$ the value of y when inspection ceases.

The guarantee given by the scheme can be expressed in terms of a *lot tolerance*, that whatever the quality of the batch before inspection there is at most a chance ε that Z_t or more defectives

* This is a particular case of the proposition that if independent random variables z_i all have the same distribution, with $E(z_i) = 0$ and $\text{var}(z_i) < \infty$, and if $a > 0$, the probability is 1 that

$$Z(m) = \sum_{i=1}^{m} z_i \geqslant a$$

for some m. The proposition is proved by showing that it is possible to find an infinite increasing sequence of integers $\{n_r\}$ such that

$$\begin{aligned} &\text{prob}\{Z(n_1) \geqslant a\} > \tfrac{1}{3} \\ &\text{prob}\{Z(n_2) \geqslant a \mid Z(n_1) < a\} > \tfrac{1}{3} \\ &\text{prob}\{Z(n_3) \geqslant a \mid Z(n_1), Z(n_2) < a\} > \tfrac{1}{3}, \text{ etc.} \end{aligned}$$

The central limit theorem is applied to the conditional distribution of

$$Z(n_r) \mid \{Z(n_1) < a, Z(n_2) < a, \ldots Z(n_{r-1}) < a\},$$

in establishing these inequalities.

† This result was shown to me by Mr Burman soon after he had completed his paper. I do not now remember his method of proof.

will be left in the batch after inspection.* This is the type of guarantee likely to be useful in final inspection, when, for one reason or another, the product is inspected in batches and not continuously. In choosing between alternative schemes with the same lot tolerance, we shall be interested in the average number of articles inspected when the initial number of defectives Y has the values indicated by the process curve as being frequent. We may also be interested in the average number of defectives Z left in the batch after inspection (i.e. in the average outgoing quality), for any given incoming quality Y, and in particular in the maximum value $\bar{Z}_m$ of this for all possible incoming qualities (the average outgoing quality limit or AOQL).

In Tables IV and V, lot tolerance conditions with $Z_t = 5, 10, 15, 20, 25, 30, 40, 50, 60, 80, 100$, and $\varepsilon = 10\%$ and 1% are considered. It is assumed that Z_t is small compared with N. Several schemes (i.e. values of α and β) are given for each lot tolerance, and values of the average sample size and AOQL are recorded for each. For any value of α greater than the first value shown, there is a corresponding value of β such that the lot tolerance condition is just satisfied, but there is a particular sequence of values of α such that the corresponding scheme has an optimum property (Anscombe, 1946). The tables give the first eight or nine of these optimum schemes. For the r^{th} scheme tabulated ($r = 1, 2, \ldots$) the risk is precisely ε that Z_t or more defectives will remain after inspection if $Y = Z_t + r - 1$ or $Z_t + r$; for other values of Y the risk is less than ε. The average sample sizes have been calculated from a simple formula (*loc. cit.*, equation 1′) valid only when $Y < (1 - \alpha)/\beta$. For higher values of Y a much more laborious formula must be used, and except in a few cases (shown in italics) these average sample sizes have not been computed. Minimum average sample sizes in each column are shown in heavy type.

Suppose, for example, that the batch numbers 1,000 articles, that whatever the value of Y the risk must not exceed 1 % that 20 or more defectives will remain after inspection, and it is expected that most batches will actually have round about 6 defectives. We turn to the section of Table V headed $Z_t = 20$, $\varepsilon = 0{\cdot}01$, and look down the column for which $Y = 6$. The minimum average sample size occurs for the third scheme. The first inspected sample will contain 241 articles, and subsequent samples 52 articles. The full 1 % risk that $Z \geq 20$ is attained when $Y = 22$ or 23. The AOQL for the scheme is $\bar{Z}_m = 5{\cdot}9$, this value of $\bar{Z}$ being attained when $Y = 12$.

As remarked in §2, when a rectifying batch inspection scheme is used in acceptance inspection, the guarantee required will probably be one of AOQL, not lot tolerance. Tables IV and V are not adapted to such a requirement, but a suitable approximate formula is given below (10·5).

10. *Approximate formulae*

If the numbers of defectives concerned, Y, Z, and $Y - Z$, are fairly large, but still nevertheless small compared with N (so that the proportion of defectives is always small), we can obtain sufficiently accurate results with the aid of simple asymptotic formulae. It is unnecessary, therefore, to extend Tables IV and V beyond their present size.

The inspection scheme can be conveniently represented on a diagram in which the proportion ξ of the whole batch that has been inspected is abscissa and the number y of defectives found is ordinate. Inspection ceases as soon as the point (ξ, y) reaches the straight-line boundary

$$\xi = \alpha + \beta y. \qquad (10.1)$$

For a given initial number of defectives Y, we may be interested in the average sample size A, in the average number of outgoing defectives Z, and in an upper percentage point of the distribution of Z such that there is a chance ε that Z will not be less than this value. It will be necessary to assume $\alpha + \beta Y < 1$. Then we have for the first two

$$\frac{A}{N} = \frac{\alpha}{1 - \beta Y} - \frac{\alpha\beta^2 Y}{(1 - \beta Y)^3} \qquad (10.2)$$

$$\bar{Z} = Y\left\{1 - \frac{\alpha}{1 - \beta Y} + \frac{\alpha\beta}{(1 - \beta Y)^3}\right\}. \qquad (10.3)$$

* In my 1946 paper Y' was used to denote what is here called Z_t.

For the third, let c be the equivalent normal deviate to $1 - \varepsilon$, so that $c = 1{\cdot}282$ if $\varepsilon = 10\%$ and $2{\cdot}326$ if $\varepsilon = 1\%$. Then the required upper percentage value of Z is $Y - y$ where y satisfies

$$\left.\begin{aligned} \xi Y - y - \tfrac{1}{2} &= c\sqrt{\{\xi(1-\xi)Y\}}, \\ \xi &= \alpha + \beta y. \end{aligned}\right\} \qquad (10.4)$$

The maximum value of $\bar{Z}$ as Y varies is the AOQL. It is

$$\bar{Z}_m = \frac{(1-\sqrt{\alpha})^2}{\beta} + \left(\frac{1}{\sqrt{\alpha}} - 1\right), \qquad (10.5)$$

and is attained when

$$Y = \frac{1-\sqrt{\alpha}}{\beta} + \left(\frac{3}{2\sqrt{\alpha}} - 1\right). \qquad (10.6)$$

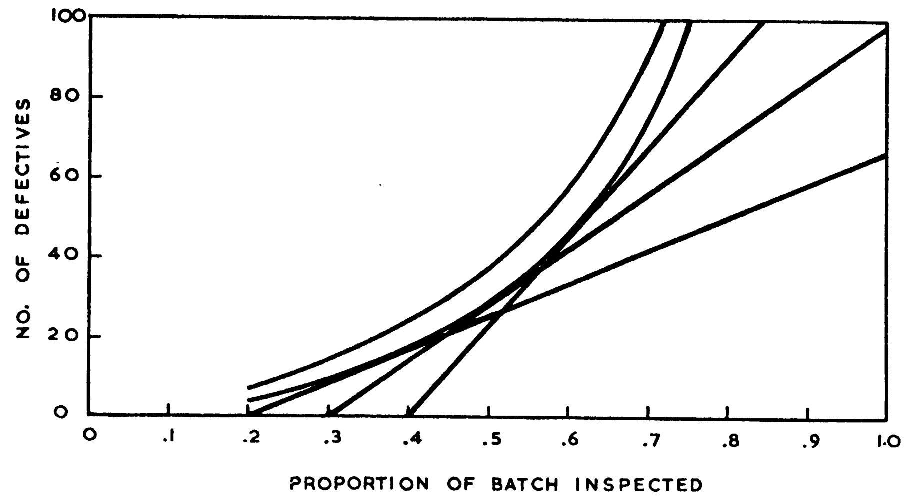

FIG. 6.—Rectifying inspection. Boundaries for sequential schemes with lot tolerance $Z_t = 50$, $\varepsilon = 0{\cdot}01$, and envelope of boundaries. Envelope of boundaries for $Z_t = 50$, $\varepsilon = 0{\cdot}10$.

The maximum value of the upper percentage point of Z, as Y varies, is the lot tolerance, Z_t. The boundary is a tangent to the curve

$$y = \frac{Z_t\xi - \frac{1}{2}}{1-\xi} - c\sqrt{\left\{\frac{\xi}{1-\xi}(Z_t + y)\right\}}. \qquad (10.7)$$

If y_t is the ordinate at the point of contact, there is precisely a risk ε that Z_t or more defectives will remain in the batch after inspection if

$$Y = Z_t + y_t. \qquad (10.8)$$

Thus if the AOQL $\bar{Z}_m$ is specified, from (10.5) we can calculate β for any not-too-small value of α. We shall probably aim at choosing α so that the average sample size A given by (10.2) is as small as possible for some expected value of Y. If, on the other hand, the lot tolerance Z_t is specified we draw the curve (10.7), which is easily done finding y by successive approximation for various values of ξ. Any tangent to this curve satisfies the specification, and if we wish to minimize A for some particular value of Y we shall take the tangent at the point where the straight line joining the origin to the point $(1, Y)$ meets the curve. For either form of specification, (10.6) or (10.8) indicates the most unfavourable value of Y.

To check the accuracy of (10.7), some boundaries have been computed having larger values of α than in Tables IV and V. They are given in Text-table 6. Those for $Z_t = 50$, $\varepsilon = 1\%$, are reproduced in Fig. 6, together with the corresponding curve (10.7). To the order of accuracy of the drawing the boundaries touch the curve. The curve for $Z_t = 50$, $\varepsilon = 10\%$, is also shown in Fig. 6 to indicate the effect of changing ε.

TEXT-TABLE 6

Lot tolerance Z_t	ε		Scheme α	β
20	1%	.	0·4	0·01664
20	1%	.	0·5	0·00952
50	1%	.	0·2	0·01208
50	1%	.	0·3	0·00713
50	1%	.	0·4	0·00442
50	10%	.	0·2	0·00871
50	10%	.	0·3	0·00550
50	10%	.	0·4	0·00352

†11. *Proof of asymptotic formulae*

The above asymptotic formulae are easily deduced from formulae given in my 1946 paper. First (10.2) is obtained, from the exact formula for A/N, supposing α and βY to be fixed and letting $\beta \to 0$. The error is $O(\beta^2)$. In fact, the second term on the right-hand side of (10.2) is a correction term, $O(\beta)$, to the first, and the first term is the abscissa of the point P where the mean path $y = \xi Y$ meets the boundary. (10.3), (10.5), and (10.6), now follow rapidly. In each case the last member on the right-hand side is a small correction to the rest. If we ignore it, (10.3) states that $\bar{Z}$ is the difference between Y and the ordinate of P. These first-order asymptotic results can easily be proved directly (*cf.* Anscombe, 1949).

To derive (10.4) we may take the exact expression for the chance that y or fewer defectives will be found before inspection ceases, apply the Euler-Maclaurin formula to replace the sum by an integral, and develop the latter asymptotically, supposing α and βy to be fixed and letting $\beta \to 0$. Setting

$$(\alpha + \beta y)Y = y(1 + \delta), \qquad (11.1)$$

we find that the first approximation, valid for $\delta = O(y^{-\frac{1}{2}})$, is a normal integral giving

$$\xi Y - y = c\sqrt{\{y(1-\xi)\}}. \qquad (11.2)$$

To the same order of approximation this is (10.4), without the $-\frac{1}{2}$ on the left-hand side. In principle, the expansion could be carried to further terms, but this is actually very laborious. We note, however, that if there were no boundary the number of defectives found when a proportion $\xi = \alpha + \beta y$ of the batch had been sampled would have a binomial distribution with mean $Y\xi$ and variance $Y\xi(1-\xi)$, so that we get our formula at once if we replace the sloping boundary by a vertical one from $(\xi, 0)$ to (ξ, y) and regard the distribution of the number of defectives as normal instead of binomial. This argument suggests that a continuity correction of $-\frac{1}{2}$ should be inserted as shown in (10.4), whereupon the formula appears to become sufficiently accurate for practical purposes. (10.7) is an immediate consequence of (10.4).

It may be noted, in passing, that the envelope of all boundaries with a given AOQL $\bar{Z}_m$ is a curve similar in general appearance to that represented by (10.7). If we ignore the last member on the right-hand side of (10.3), the curve is the rectangular hyperbola

$$(y + \bar{Z}_m)(1 - \xi) = \bar{Z}_m. \qquad (11.3)$$

Asymptotically, for large Y and $\bar{Z}_m$, this curve itself constitutes a boundary for which, whatever the (sufficiently large) value of Y, $\bar{Z} = \bar{Z}_m$; and it is the most economical boundary (giving lowest average sample sizes) of all possible boundaries satisfying the given AOQL condition. Similarly, the curve (10.7) is, asymptotically for large Y and Z_t, a boundary such that, whatever the (sufficiently large) value of Y, there is a risk equal to ε that $Z > Z_t$; and it is the most economical boundary satisfying this lot tolerance condition. In order that the aymptotic formulae should hold, however, it may be necessary to modify these curved boundaries near the origin to make negligible the chance of their being reached there when Y is large.

12. *Miscellaneous remarks*

(1) The same sorts of considerations as indicated above in §§6 and 7 for non-rectifying schemes are relevant to the choice of a rectifying scheme. In addition to sampling inspection, there is now

also the alternative possibility of 100% inspection without any preliminary sampling. Some trouble is involved in using a sequential scheme which does not occur with 100% inspection, namely keeping a score of defectives found and providing that each successive "sample" is near enough a random sample of the batch. It is not likely to be worth while to instal a sequential scheme unless the scheme and the expected quality of the batch are such that the expected sample size is considerably less than the whole of the batch; otherwise, 100% inspection will be preferable.

Little attempt has been made so far to compare sequential with single-sampling schemes. If we consider any particular lot tolerance and any expected initial quality Y, and compare the sequential and single-sampling schemes for that lot tolerance having minimum expected sample size, we find that the sequential scheme has the lower expected sample size and the higher AOQL.

(2) The effect of errors in classifying articles as defective or non-defective has been considered by Lavin (1946). The above formulae can easily be modified to allow roughly for constant chances that articles which should be classed as defective are passed as good and that good articles are classed as defective. The first sort of error inflates the true values of $\bar{Z}_m$ and Z_t, and lowers A; while the second error has the opposite effect. Actually, however, the chances of these errors may quite well not be constant, but increase as inspection proceeds. There can then be a strong case for relaxing standards sufficiently to give a low expected sample size.

Note on the computation of the tables

Preliminary work on the non-rectifying schemes was begun in 1945 by some of my colleagues and myself in the Ministry of Supply (S.R.17). Later, the Ministry of Supply placed a contract with the Scientific Computing Service (Dr H. O. Hartley) for work on non-rectifying and rectifying schemes, intended to lead to the Tables here published, but the contract came to an end before the work was finished. Thereafter the Mathematics Division of the National Physical Laboratory gave some valuable assistance, and also Rothamsted Experimental Station. Among the many persons to whom I am indebted for help in various ways, I should like to mention particularly Mr D. Newman, who in the Ministry of Supply, and later at the National Physical Laboratory, took all possible steps to bring the computing programme to fruition. Without his help and interest the work could never have been undertaken. I should also like to thank Mr C. D. Bates (National Physical Laboratory) and Mr D. East (Statistical Laboratory, Cambridge) for their very great help in the later stages of the computing. Mr P. Armitage has offered valuable criticism of the paper in draft.

Much care has been taken over Table I, and I believe all its entries are correct to the number of places shown. Table II was derived from Table I by division. The entries in Table III result from rather rough interpolation, and may occasionally be slightly in error in the last place. The values of α and β given in Tables IV and V are as supplied by the Scientific Computing Service. They were derived by an interpolation method, and are believed to be accurate to the four decimal places shown. The average sample sizes and AOQL may sometimes be slightly in error in the later parts of the Tables where β has only three significant figures.

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TABLE I.—*Operating Characteristics of Non-rectifying Sequential Schemes—Percentage Points*

Scheme				*Values of p(b + 1) when the chance of acceptance is*				
R_1	R_2	K		99%	90%	50%	10%	1%
Open								
1	1		·	0·1388	0·4371	1·1462	2·5280	4·6506
1¼	1¼		·	0·1701	0·4766	1·0869	2·1559	3·7769
1½	1½		·	0·2149	0·5247	1·0600	1·9262	3·2171
1¾	1¾		·	0·2711	0·5721	1·0463	1·7719	2·8334
2	2		·	0·3213	0·6100	1·0365	1·6605	2·5565
2¼	2¼		·	0·3600	0·6397	1·0288	1·5761	2·3484
2½	2½		·	0·3986	0·6667	1·0236	1·5108	2·1873
2¾	2¾		·	0·4337	0·6900	1·0198	1·4586	2·0592
3	3		·	0·4642	0·7100	1·0167	1·4159	1·9551
3¼	3¼		·	0·4922	0·7277	1·0144	1·3804	1·8691
3½	3½		·	0·5177	0·7435	1·0125	1·3504	1·7968
3¾	3¾		·	0·5408	0·7575	1·0109	1·3248	1·7354
4	4		·	0·5619	0·7700	1·0096	1·3027	1·6826
¾	1½		·	0·2272	0·6002	1·4132	3·2150	6·1535
1	2		·	0·3345	0·6729	1·2950	2·5550	4·6517
1¼	2½		·	0·4103	0·7194	1·2270	2·1859	3·7790
1½	3		·	0·4751	0·7557	1·1845	1·9539	3·2195
2	4		·	0·5710	0·8059	1·1339	1·6816	2·5584
2½	5		·	0·6383	0·8392	1·1050	1·5285	2·1890
¾	2¼		·	0·3894	0·7561	1·4825	3·2191	6·1535
1	3		·	0·4907	0·8029	1·3478	2·5581	4·6517
1¼	3¾		·	0·5643	0·8346	1·2714	2·1887	3·7791
1½	4½		·	0·6199	0·8575	1·2223	1·9563	3·2196
2	6		·	0·6978	0·8886	1·1631	1·6835	2·5584
1	4		·	0·5983	0·8774	1·3689	2·5584	4·6517
1	7		·	0·7692	0·9762	1·3843	2·5584	4·6517
2	3		·	0·4686	0·7318	1·1011	1·6768	2·5582
Closed								
1	1	2		0·1399	0·4573	1·2880	2·8014	4·8102
1	2	2	·	0·2285	0·5797	1·4231	2·9168	4·8748
1	2	3	·	0·2816	0·6197	1·3590	2·6811	4·6721
1	2	4	·	0·3100	0·6425	1·3308	2·6002	4·6532
2	2	4	·	0·2900	0·5846	1·1550	2·0091	2·9634
2	2	5	·	0·3049	0·5929	1·1185	1·8847	2·7596
2	2	6	·	0·3129	0·5987	1·0950	1·8079	2·6540
2	2	8	·	0·3192	0·6048	1·0678	1·7259	2·5761
2	3	4	·	0·3217	0·6133	1·1813	2·0332	2·9846
2	3	5	·	0·3586	0·6379	1·1564	1·9165	2·7831
2	3	6	·	0·3873	0·6575	1·1416	1·8436	2·6746
2	3	8	·	0·4258	0·6851	1·1252	1·7617	2·5865
2	4	6	·	0·3965	0·6651	1·1480	1·8487	2·6777
2	4	8	·	0·4499	0·7027	1·1383	1·7704	2·5891
2	4	10	·	0·4884	0·7301	1·1353	1·7310	2·5657
3	3	6	·	0·3804	0·6433	1·1084	1·7566	2·4382
3	3	8	·	0·4170	0·6656	1·0782	1·6323	2·2120
3	3	10	·	0·4382	0·6798	1·0601	1·5592	2·0909
3	3	12	·	0·4503	0·6892	1·0482	1·5128	2·0248

TABLE II.—*Operating Characteristics of Non-rectifying Sequential Schemes—Ratios of Percentage Points*

Scheme R_1	Scheme R_2	Scheme K	Ratios of percentage-point values of *p*: 99%/90%	99%/50%	99%/10%	99%/1%	90%/10%
Open							
1	1		·3175	·1211	·0549	·0298	·1729
1¼	1¼		·3569	·1565	·0789	·0450	·2211
1½	1½		·4096	·2027	·1116	·0668	·2724
1¾	1¾		·4739	·2591	·1530	·0957	·3229
2	2		·5267	·3100	·1935	·1257	·3674
2¼	2¼		·5628	·3499	·2284	·1533	·4059
2½	2½		·5979	·3894	·2638	·1822	·4413
2¾	2¾		·6286	·4253	·2973	·2106	·4731
3	3		·6538	·4566	·3278	·2374	·5014
3¼	3¼		·6764	·4852	·3566	·2633	·5272
3½	3½		·6963	·5113	·3834	·2881	·5506
3¾	3¾		·7139	·5350	·4082	·3116	·5718
4	4		·7297	·5566	·4313	·3339	·5911
¾	1½		·3785	·1608	·0707	·0369	·1867
1	2		·4971	·2583	·1309	·0719	·2634
1¼	2½		·5703	·3344	·1877	·1086	·3291
1½	3		·6287	·4011	·2432	·1476	3868
2	4		·7085	·5036	·3396	·2232	·4792
2½	5		·7606	·5776	·4176	·2916	·5490
¾	2¼		·5150	·2627	·1210	·0633	·2349
1	3		·6112	·3641	·1918	·1055	·3139
1¼	3¾		·6761	·4438	·2578	·1493	·3813
1½	4½		·7229	·5072	·3169	·1925	·4383
2	6		·7853	·5999	.4145	·2727	·5278
1	4		·6819	·4371	·2339	·1286	·3429
1	7		·7880	·5557	·3007	·1654	·3816
2	3		·6403	·4256	·2795	·1832	4364
Closed							
1	1	2	·3059	·1086	·0499	·0291	·1632
1	2	2	·3942	·1606	·0783	·0469	·1987
1	2	3	·4544	·2072	·1050	·0603	·2311
1	2	4	·4825	·2329	·1192	·0666	·2471
2	2	4	·4961	·2511	·1443	·0979	·2910
2	2	5	·5143	·2726	·1618	·1105	·3146
2	2	6	·5226	·2858	·1731	·1179	·3312
2	2	8	·5278	·2989	·1849	·1239	·3504
2	3	4	·5245	·2723	·1582	·1078	·3016
2	3	5	·5622	·3101	·1871	·1288	.3328
2	3	6	·5890	·3393	·2101	·1448	.3566
2	3	8	·6215	·3784	·2417	·1646	·3889
2	4	6	·5962	·3454	·2145	·1481	·3598
2	4	8	·6402	·3952	·2541	·1738	·3969
2	4	10	·6689	·4302	·2821	·1904	·4218
3	3	6	·5913	·3432	·2166	·1560	·3662
3	3	8	·6265	·3868	·2555	·1885	·4078
3	3	10	6447	·4135	·2811	·2096	·4360
3	3	12	·6534	·4296	·2977	·2224	·4556

TABLE III.—*Average Sample Sizes of Non-rectifying Sequential Schemes*

Scheme R_1	Scheme R_2	Scheme K	Values of $A/(b+1)$ when the chance of acceptance is 99%	90%	50%	10%	1%	Maximum value of $A/(b+1)$
Open								
1	1		1·13	1·36	1·37	0·81	0·44	1·45
1¼	1¼		1·47	1·85	1·96	1·16	0·59	2·04
1½	1½		1·87	2·47	2·72	1·64	0·82	2·80
1¾	1¾		2·35	3·20	3·63	2·24	1·15	3·71
2	2		2·88	4·02	4·66	2·93	1·52	4·74
2¼	2¼		3·44	4·91	5·79	3·68	1·91	5·87
2½	2½		4·07	5·91	7·06	4·54	2·38	7·14
2¾	2¾		4·75	7·00	8·46	5·50	2·90	8·54
3	3		5·48	8·17	9·98	6·53	3·46	10·06
3¼	3¼		6·27	9·43	11·63	7·66	4·08	11·70
3½	3½		7·11	10·79	13·40	8·89	4·76	13·47
3¾	3¾		8·00	12·24	15·29	10·20	5·49	15·36
4	4		8·94	13·78	17·31	11·61	6·27	17·39
¾	1½		0·94	1·24	1·32	0·72	0·28	1·38
1	2		1·45	2·04	2·30	1·33	0·67	2·37
1¼	2½		2·05	3·01	3·51	2·08	1·03	3·59
1½	3		2·77	4·17	4·99	3·02	1·51	5·07
2	4		4·52	7·05	8·73	5·47	2·77	8·82
2½	5		6·70	10·67	13·52	8·65	4·46	13·59
¾	2¼		1·17	1·72	1·92	1·03	0·48	2·00
1	3		1·88	2·88	3·38	1·89	0·93	3·47
1¼	3¾		2·75	4·34	5·25	3·02	1·48	5·35
1½	4½		3·78	6·09	7·52	4·43	2·18	7·65
2	6		6·35	10·48	13·31	8·09	4·04	13·47
1	4		2·36	3·81	4·54	2·47	1·19	4·68
1	7		3·97	7·01	8·26	4·20	2·01	8·70
2	3		3·66	5·48	6·62	4·18	2·14	6·70
Closed								
1	1	2	1·12	1·25	1·15	0·71	0·42	1·27
1	2	2	1·20	1·38	1·36	0·95	0·61	1·42
1	2	3	1·32	1·63	1·73	1·18	0·67	1·77
1	2	4	1·39	1·79	1·95	1·27	0·67	1·99
2	2	4	2·62	3·02	2·87	1·91	1·20	3·09
2	2	5	2·74	3·32	3·29	2·20	1·34	3·47
2	2	6	2·81	3·54	3·61	2·41	1·43	3·77
2	2	8	2·87	3·79	4·05	2·67	1·50	4·17
2	3	4	2·70	3·12	3·10	2·27	1·55	3·23
2	3	5	2·91	3·53	3·68	2·72	1·80	3·77
2	3	6	3·09	3·87	4·17	3·07	1·95	4·24
2	3	8	3·34	4·39	4·91	3·55	2·09	4·97
2	4	6	3·13	3·94	4·37	3·46	2·38	4·40
2	4	8	3·47	4·59	5·36	4·23	2·66	5·36
2	4	10	3·75	5·12	6·13	4·72	2·74	6·14
3	3	6	4·37	4·99	4·78	3·37	2·23	5·09
3	3	8	4·83	5·87	5·92	4·17	2·69	6·17
3	3	10	5·11	6·51	6·81	4·79	3·01	7·02
3	3	12	5·28	6·98	7·51	5·26	3·22	7·69

TABLE IV.—*Rectifying Sequential Schemes for Given Lot Tolerance Z_t, with Risk* $\varepsilon = 0{\cdot}10$

$Z_t = 5,\ \varepsilon = 0{\cdot}10$

Scheme		*Average sample size (A/N) for Y equal to*											
α	β	0	1	2	3	4	5	6	8	10	12	AOQL	*Y**
·3690	·1900	**·369**	**·439**	·536	·674							1·4	3
·4238	·0982	·424	·465	**·515**	**·576**	·650	·743					1·8	5
·4773	·0639	·477	·508	·542	·581	**·626**	·677	·737	·890			1·9	6
·5241	·0459	·524	·548	·574	·603	·635	**·670**	·709	·800	·916		2·0	7
·5642	·0348	·564	·584	·605	·627	·652	·678	**·706**	·769	·844	·935	2·1	8
·5986	·0275	·599	·615	·632	·651	·670	·691	·713	**·760**	·815	·877	2·2	9
·6283	·0223	·628	·642	·657	·672	·688	·705	·723	·761	·802	·849	2·2	10
·6540	·0185	·654	·666	·679	·692	·705	·719	·734	·765	·799	·835	2·2	11
·6767	·0155	·677	·687	·698	·709	·721	·733	·745	·771	**·798**	·828	2·2	12

$Z_t = 10,\ \varepsilon = 0{\cdot}10$

α	β	0	1	2	3	4	5	6	8	10	12	AOQL	*Y*
·2057	·1694	**·206**	**·241**	·287	·352	·444	*·569*	*·677*	*·820*	*·897*	*·940*	2·8	5
·2337	·0967	·234	·256	**·283**	**·316**	**·356**	·406	·469				3·7	7
·2669	·0686	·267	·285	·306	·330	·357	**·389**	**·427**	·526	·673		4·3	9
·3002	·0530	·300	·316	·334	·353	·375	·400	**·427**	·495	·584	·708	4·6	10
·3323	·0429	·332	·347	·362	·379	·397	·418	·440	**·492**	·556	·639	4·9	11
·3625	·0357	·363	·375	·389	·404	·420	·438	·456	·499	**·549**	·610	5·1	13
·3908	·0304	·391	·403	·415	·429	·443	·458	·474	·510	·552	·601	5·2	14
·4170	·0262	·417	·428	·439	·452	·464	·478	·492	·523	·559	**·599**	5·3	15
·4414	·0229	·441	·452	·462	·473	·485	·497	·510	·537	·568	·602	5·4	16

$Z_t = 15,\ \varepsilon = 0{\cdot}10$

α	β	0	2	4	6	8	10	12	14	16	18	AOQL	*Y*
·1423	·1448	**·142**	**·189**	·272	*·436*	*·617*						4·0	6
·1592	·0857	·159	**·189**	**·230**	·292	·388						5·5	9
·1808	·0629	·181	·205	·236	**·277**	·333	·413	·533				6·5	11
·2038	·0502	·204	·225	·252	·284	**·326**	·380	·453	·555			7·1	13
·2270	·0418	·227	·247	·270	·298	·332	**·374**	·427	·496	·587	·711	7·6	14
·2499	·0358	·250	·268	·290	·315	·344	·379	**·421**	·473	·539	·622	7·9	16
·2722	·0312	·272	·290	·310	·332	·358	·389	·424	**·466**	·517	·579	8·2	17
·2937	·0276	·294	·310	·329	·350	·373	·400	·431	·467	·509	·558	8·4	18
·3143	·0246	·314	·330	·348	·367	·389	·413	·440	·471	**·506**	·547	8·6	19

$Z_t = 20,\ \varepsilon = 0{\cdot}10$

α	β	0	2	4	6	8	10	12	14	16	18	AOQL	*Y*
·1088	·1258	**·109**	**·140**	·190	·281	*·441*						5·1	8
·1202	·0759	·120	**·140**	**·166**	·204	·259	·346					7·2	11
·1354	·0566	·135	·152	·172	**·198**	**·232**	·279	·345	·444			8·5	14
·1521	·0460	·152	·167	·184	·206	**·232**	**·266**	·310	·369	·452	·570	9·5	16
·1695	·0390	·169	·183	·199	·218	·241	·269	**·303**	·346	·402	·475	10·2	17
·1871	·0338	·187	·200	·215	·232	·253	·276	·305	**·339**	·381	·434	10·8	19
·2045	·0300	·204	·217	·231	·248	·266	·287	·312	·342	**·376**	·419	11·1	20
·2218	·0269	·222	·234	·248	·263	·280	·300	·322	·348	·378	·413	11·5	22
·2389	·0243	·239	·251	·264	·278	·295	·313	·333	·356	·382	**·412**	11·8	23

$Z_t = 25,\ \varepsilon = 0{\cdot}10$

α	β	0	3	6	9	12	15	18	21	24	27	AOQL	*Y*
·0880	·1114	**·088**	·125	·200	*·379*							6·2	9
·0964	·0679	·096	**·119**	·153	·212	·321						8·8	13
·1078	·0512	·108	·126	**·151**	·188	·245	·340					10·6	16
·1205	·0420	·120	·137	·158	**·187**	·228	·287	·381				11·8	18
·1339	·0360	·134	·149	·169	·194	**·227**	·272	·336	·433	·591		12·7	20
·1477	·0316	·148	·163	·181	·203	·232	**·269**	·318	·388	·490		13·5	22
·1617	·0283	·162	·176	·194	·214	·240	·272	**·314**	·369	·445	·553	14·0	24
·1761	·0255	·176	·190	·207	·226	·250	·279	·315	·360	·420	·500	14·5	25
·1907	·0232	·191	·205	·221	·239	·261	·288	·319	**·359**	·408	·472	14·9	27

* The value of *Y* shown in the last column is that for which the AOQL is attained.

TABLE IV—*continued.*

$Z_t = 30$, $\varepsilon = 0{\cdot}10$

α	β	0	3	6	9	12	15	18	21	24	27	AOQL	Y
·0739	·1002	**·074**	·101	·153	·271	*·457*						7·3	11
·0803	·0614	·080	**·097**	·122	·161	·229	*·362*					10·4	15
·0892	·0467	·089	·103	**·121**	**·147**	·185	·244	·345				12·5	18
·0993	·0386	·099	·112	·128	·148	**·176**	·216	·274	·368			14·0	21
·1100	·0333	·110	·122	·136	·154	·178	**·208**	·251	·312	·405		15·2	23
·1212	·0294	·121	·133	·146	·163	·183	·209	**·243**	·289	·354	·449	16·2	25
·1331	·0264	·133	·144	·157	·173	·192	·215	·244	·282	·332	·400	16·9	27
·1454	·0238	·145	·156	·169	·184	·201	·222	·248	**·279**	·320	·372	17·6	28
·1573	·0219	·157	·168	·180	·195	·211	·231	·254	·283	**·318**	·362	18·1	30

$Z_t = 40$, $\varepsilon = 0{\cdot}10$

α	β	0	4	8	12	16	20	24	28	32	36	AOQL	Y
·0559	·0837	**·056**	·080	·133	*·278*							9·4	13
·0601	·0518	·060	**·075**	·098	·138	·220	*·402*					13·4	18
·0662	·0397	·066	·078	**·095**	·120	·160	·233					16·3	22
·0730	·0332	·073	·084	·098	**·118**	·146	·191	·267				18·3	26
·0805	·0288	·081	·091	·104	·121	**·144**	·177	·227	·311			20·0	28
·0887	·0255	·089	·098	·111	·126	·146	**·173**	·211	·268	·360		21·5	31
·0970	·0231	·097	·107	·118	·133	·151	·175	**·207**	·251	·317	·422	22·5	33
·1050	·0213	·105	·115	·126	·140	·157	·179	**·207**	·244	·297	·376	23·4	35

$Z_t = 50$, $\varepsilon = 0{\cdot}10$

α	β	0	5	10	15	20	25	30	35	40	45	AOQL	Y
·0450	·0722	**·045**	·067	·121	*·288*							11·4	16
·0480	·0449	·048	**·061**	·083	·124	·222						16·3	21
·0524	·0346	·052	·063	**·078**	·103	·146	·234					19·9	26
·0575	·0290	·057	·067	·080	**·098**	·127	·177	·272				22·6	30
·0628	·0255	·063	·072	·083	·099	**·123**	·158	·218	·333			24·6	33
·0685	·0230	·068	·077	·088	·103	**·123**	·152	·197	·273	·418		26·2	36
·0746	·0209	·075	·083	·094	·107	·125	**·150**	·186	·241	·335		27·7	39
·0815	·0191	·081	·090	·100	·113	·130	·151	·182	·225	·293	·407	29·0	41
·0892	·0174	·089	·098	·108	·120	·135	·155	**·180**	·216	·266	·344	30·4	44

$Z_t = 60$, $\varepsilon = 0{\cdot}10$

α	β	0	6	12	18	24	30	36	42	48	54	AOQL	Y
·0376	·0637	**·038**	·058	·113	*·298*							13·3	19
·0399	·0397	·040	**·052**	·072	·115	*·232*						19·2	25
·0433	·0307	·043	·053	**·067**	·091	·136	·243					23·5	30
·0471	·0260	·047	·056	**·067**	**·085**	·115	·171	·296				26·6	35
·0512	·0229	·051	·059	·070	**·085**	·108	·146	·218				29·0	38
·0560	·0205	·056	·064	·074	·087	**·107**	·136	·185	·277			31·2	42
·0613	·0185	·061	·069	·078	·091	·108	·132	·169	·230	·346		33·3	45
·0671	·0168	·067	·074	·084	·095	·111	**·131**	·161	·206	·281	·422	35·2	48
·0728	·0155	·073	·080	·089	·100	·114	·133	**·159**	·195	·251	·346	36·9	51

$Z_t = 80$, $\varepsilon = 0{\cdot}10$

α	β	0	8	16	24	32	40	48	56	64	72	AOQL	Y
·0284	·0519	**·028**	·046	·104	*·316*							17·1	24
·0299	·0323	·030	**·040**	·058	·103	*·257*						24·8	31
·0320	·0254	·032	**·040**	·053	·076	·129						30·2	38
·0346	·0214	·035	·042	**·052**	·068	·099	·167					34·5	43
·0377	·0187	·038	·044	·053	**·067**	·089	·129	·221				38·2	48
0407	·0169	·041	·047	·055	**·067**	**·085**	·116	·173	·310			41·1	52
·0436	·0156	·044	·050	·058	·069	**·085**	·110	·153	·240			43·4	56
·0463	·0148	·046	·052	·060	·071	·086	·109	·146	·215	·369		44·8	58
·0489	·0141	·049	·055	·063	·073	·087	**·108**	·141	·198	·314		46·1	60

TABLE IV—*continued.*

$Z_t = 100$, $\varepsilon = 0{\cdot}10$

α	β	0	10	20	30	40	50	60	70	80	90	AOQL	Y
·0228	·0441	**·023**	·039	·101	*·330*							20·8	29
·0238	·0276	·024	**·032**	·050	·099	*·288*						30·0	37
·0254	·0214	·025	**·032**	·043	·065	·122						37·1	45
·0273	·0183	·027	·033	**·042**	·058	·090	·174					42·1	51
·0290	·0165	·029	·035	·043	**·056**	·079	·130					45·8	56
·0306	·0154	·031	·036	·044	**·056**	·076	·115	·214				48·1	59
·0322	·0146	·032	·038	·045	**·056**	·074	·107	·181				50·0	62
·0339	·0138	·034	·039	·047	·057	**·073**	·101	·158	·309			52·0	65
·0358	·0132	·036	·041	·048	·058	·074	·099	·147	·261			53·5	67

TABLE V.—*Rectifying Sequential Schemes for Given Lot Tolerance Z_t, with Risk* $\varepsilon = 0{\cdot}01$

$Z_t = 5$, $\varepsilon = 0{\cdot}01$

Scheme		*Average sample size (A/N) for Y equal to*											
α	β	0	1	2	3	4	5	6	8	10	12	AOQL	Y*
·6019	·1199	**·602**	**·674**	·764	·877							0·7	3
·6473	·0619	·647	·687	**·732**	·783	·841	·908	·984				0·9	4
·6872	·0396	·687	·714	·744	**·776**	·810	·848	·889	·983			1·0	5
·7199	·0279	·720	·740	·761	·784	**·807**	·833	·859	·918	·985		1·0	6
·7468	·0209	·747	·762	·779	·796	·813	**·832**	**·851**	·892	·938	·988	1·0	7
·7693	·0162	·769	·782	·795	·808	·822	·836	**·851**	·882	·915	·951	1·1	8
·7884	·0130	·788	·799	·809	·820	·831	·842	·854	**·878**	·904	·932	1·1	9
·8044	·0107	·804	·813	·822	·831	·840	·849	·859	·879	**·900**	·921	1·1	10

$Z_t = 10$, $\varepsilon = 0{\cdot}01$

α	β	0	1	2	3	4	5	6	8	10	12	AOQL	Y
·3690	·1345	**·369**	**·419**	482	·563	·672	*·803*	*·890*	*·966*	*·989*	*·996*	1·7	4
·4010	·0784	·401	·433	**·469**	**·511**	·561	·621	·693	*·878*	*·969*	*·992*	2·3	6
·4354	·0553	·435	·459	·486	·516	**·550**	·587	·630	·736	·879	*·973*	2·6	7
·4679	·0423	·468	·488	·509	·532	·558	**·586**	**·617**	·688	·777	·889	2·8	9
·4977	·0338	·498	·515	·532	·552	·572	·594	·618	·672	·735	·810	3·0	10
·5248	·0278	·525	·539	·555	·571	·588	·607	·626	**·669**	·717	·773	3·1	11
·5495	·0234	·550	·562	·576	·590	·605	·620	·637	·672	**·711**	·755	3·2	12
·5719	·0200	·572	·583	·595	·608	·621	·634	·648	·678	**·711**	·747	3·3	13
·5922	·0174	·592	·603	·613	·624	·636	·648	·660	·686	·714	**·744**	3·3	14

$Z_t = 15$, $\varepsilon = 0{\cdot}01$

α	β	0	2	4	6	8	10	12	14	16	18	AOQL	Y
·2644	·1242	**·264**	·338	·458								2·7	5
·2861	·0756	·286	**·333**	·395	·484	·614						3·7	8
·3115	·0556	·311	·348	**·394**	·452	·528	·632	·777				4·2	9
·3370	·0441	·337	·368	·405	**·450**	·505	·574	·663	·780			4·7	11
·3617	·0364	·362	·389	·421	·457	**·501**	·553	·616	·695	·793		5·0	12
·3852	·0309	·385	·410	·438	·469	·506	**·548**	·597	·656	·726	·812	5·2	14
·4073	·0267	·407	·430	·454	·482	·514	·549	·589	·636	·690	·754	5·5	15
·4282	·0234	·428	·449	·471	·496	·524	·554	**·588**	·627	·671	·721	5·6	16
·4480	·0207	·448	·467	·488	·510	·534	·561	·591	**·624**	·661	·702	5·8	17

$Z_t = 20$, ε $0{\cdot}01$

α	β	0	2	4	6	8	10	12	14	16	18	AOQL	Y
·2057	·1122	**·206**	·257	·337	·471	*·680*	*·826*	*·905*				3·7	7
·2213	·0698	·221	**·254**	**·298**	·358	·442	·570	*·748*				5·0	9
·2406	0524	·241	·267	·300	**·341**	·394	·464	·560	·697			5·9	12
·2605	·0424	·261	·284	·311	·344	**·383**	·433	·496	·577	·686		6·6	13
·2804	·0358	·280	·301	·325	·353	·386	**·425**	·473	·531	·604	·698	7·0	15
·2998	·0309	·300	·319	·341	·365	·393	·426	·465	·510	·565	·632	7·4	16
·3187	·0271	·319	·336	·356	·378	·403	·432	**·464**	·502	·545	·597	7·7	18
·3370	·0241	·337	·354	·372	·392	·415	·440	·468	**·500**	·537	·579	8·0	19
·3545	·0217	·354	·370	·387	·406	·427	·450	·475	·503	**·534**	·570	8·2	20

* The value of Y shown in the last column is that for which the AOQL is attained.

TABLE V—*continued.*

$Z_t = 25$, $\varepsilon = 0{\cdot}01$

α	β	0	3	6	9	12	15	18	21	24	27	AOQL	Y
·1682	·1016	**·168**	·231	·352								4·7	8
·1801	·0640	·180	**·219**	·278	·374	·545						6·4	11
·1952	·0487	·195	·227	**·269**	·330	·421	·568					7·5	14
·2112	·0400	·211	·239	·274	**·320**	·384	·476	·616				8·4	15
·2274	·0341	·227	·252	·283	·322	·372	·440	·533	·670			9·1	17
·2435	·0298	·243	·267	·294	·328	**·371**	·425	·495	·591	·727		9·6	19
·2592	·0265	·259	·281	·307	·337	·374	**·419**	·477	·551	·649	·784	10·0	20
·2752	·0238	·275	·296	·320	·348	·381	·420	·469	·529	·605	·705	10·4	22
·2907	·0215	·291	·310	·333	·358	·388	·423	**·465**	·515	·577	·654	10·7	23

$Z_t = 30$, $\varepsilon = 0{\cdot}01$

α	β	0	3	6	9	12	15	18	21	24	27	AOQL	Y
·1423	·0928	**·142**	·190	·277	·461							5·6	9
·1517	·0589	·152	**·182**	·226	·293	·407						7·7	13
·1639	·0452	·164	·188	**·220**	·265	·329	·428	·593				9·2	16
·1770	·0374	·177	·198	·225	**·260**	·307	·372	·467	·615			10·2	18
·1905	·0322	·190	·210	·234	·264	**·302**	·351	·419	·514	·655		11·1	20
·2040	·0284	·204	·222	·244	·271	·303	**·344**	·397	·467	·564	·703	11·7	22
·2179	·0253	·218	·235	·256	·280	·308	**·344**	·387	·442	·514	·611	12·3	23
·2320	·0228	·232	·249	·268	·290	·316	·347	**·384**	·430	·487	·561	12·8	25
·2448	·0209	·245	·261	·279	·300	·324	·352	·385	**·425**	·474	·534	13·2	26

$Z_t = 40$, $\varepsilon = 0{\cdot}01$

α	β	0	4	8	12	16	20	24	28	32	36	AOQL	Y
·1088	·0790	**·109**	·153	·244								7·3	11
·1151	·0506	·115	**·142**	·185	·258	·403						10·3	16
·1237	·0393	·124	·146	**·176**	·222	·296	·427					12·3	19
·1330	·0329	·133	·152	·178	**·213**	·264	·344	·478				13·9	22
·1430	·0285	·143	·161	·183	**·213**	·253	·311	·397	·539			15·1	25
·1535	·0252	·153	·170	·191	·217	**·251**	·296	·360	·455	·606		16·1	27
·1639	·0227	·164	·180	·199	·223	·253	**·291**	·343	·414	·519	·681	17·0	29
·1734	·0209	·173	·189	·207	·229	·257	**·291**	·336	·395	·477	·596	17·6	30

$Z_t = 50$, $\varepsilon = 0{\cdot}01$

α	β	0	5	10	15	20	25	30	35	40	45	AOQL	Y
·0880	·0690	**·088**	·129	·223								9·1	13
·0927	·0443	·093	**·117**	·158	·236	·415						12·9	19
·0991	·0347	·099	·119	**·148**	·194	·276	·446					15·4	23
·1062	·0291	·106	·124	**·148**	·182	·236	·328	·508				17·5	26
·1137	·0256	·114	·130	·151	**·180**	·223	·288	·399				19·0	29
·1211	·0230	·121	·136	·156	·182	**·217**	·269	·348	·483			20·2	31
·1289	·0210	·129	·144	·162	·186	**·217**	·260	·323	·421	·587		21·2	33
·1374	·0191	·137	·152	·169	·191	·219	**·255**	·306	·380	·494	·686	22·3	36
·1469	·0175	·147	·161	·177	·198	·223	·256	·299	·358	·443	·575	23·3	38

$Z_t = 60$, $\varepsilon = 0{\cdot}01$

α	β	0	6	12	18	24	30	36	42	48	54	AOQL	Y
·0739	·0613	**·074**	·112	·210								10·8	15
·0775	·0395	·078	**·100**	·140	·222							15·4	22
·0826	·0310	·083	·101	·128	·175	·264						18·5	26
·0881	·0263	·088	·104	**·127**	·161	·218	·327					20·9	30
·0938	·0232	·094	·109	·129	**·157**	·201	·274	·413				22·8	33
·1000	·0209	·100	·114	·132	**·157**	·194	·249	·344	·526			24·3	36
·1072	·0188	·107	·121	·138	·160	**·191**	·235	·303	·418			26·0	39
·1150	·0170	·115	·128	·144	·164	**·191**	·227	·280	·361	·498		27·5	42
·1228	·0155	·123	·135	·150	·169	·193	**·224**	·267	·329	·424	·583	29·0	45

TABLE V—*continued.*

$Z_t = 80,\ \varepsilon = 0{\cdot}01$

α	β		0	8	16	24	32	40	48	56	64	72		AOQL	*Y*
·0559	·0505	·	**·056**	·090	·195								·	14·1	19
·0583	·0325	·	·058	·078	·115	·204							·	20·3	27
·0616	·0259	·	·062	**·077**	·102	·150	·260						·	24·4	33
·0655	·0220	·	·065	·079	**·099**	·133	·196	·343					·	27·7	38
·0699	·0192	·	·070	·082	·100	·126	·170	·254	·453				·	30·7	42
·0746	·0171	·	·075	·086	·102	**·124**	·158	·216	·328				·	33·3	46
·0790	·0156	·	·079	·090	·105	·125	**·153**	·199	·277	·435			·	35.4	50
·0830	·0147	·	·083	·094	·108	·127	**·153**	·193	·258	·377			·	36·8	52
·0866	·0139	·	·087	·097	·111	·129	**·153**	·188	·243	·337	·524		·	38.1	54

$Z_t = 100,\ \varepsilon = 0{\cdot}01$

α	β		0	10	20	30	40	50	60	70	80	90		AOQL	*Y*
·0450	·0431	·	**·045**	·076	·189								·	17·3	23
·0467	·0279	·	·047	·064	·099	·200							·	25·0	32
·0492	·0221	·	·049	**·063**	·086	·133	·262						·	30·4	40
·0522	·0186	·	·052	·064	·082	·113	·177	·355					·	35·0	46
·0550	·0166	·	·055	·066	**·081**	·106	·152	·251					·	38·2	51
·0575	·0154	·	·057	·068	·082	·105	·142	·216	·401				·	40·3	54
·0598	·0145	·	·060	·070	·084	**·104**	·137	·196	·329				·	42·1	57
·0621	·0138	·	·062	·072	·085	**·104**	·134	·185	·290				·	43·6	59
·0645	·0132	·	·065	·074	·087	·105	**·133**	·178	·265	·470			·	44·8	61

The Sources and Nature of Statistical Information in Special Fields of Statistics

The Oversea Trade Statistics of the United Kingdom

By A. Maizels

CONTENTS

I. Origins

1. The records of our oversea trade are by far the oldest that have been regularly collected. Records of the quantities and values of goods loaded and discharged at our ports, and of the duties levied on them exist from the 13th century; they were first entered on parchment rolls and then—from 1428—*in Port Books*. These records, being compiled purely for Customs purposes, were unsuitable for the production of national statistics of oversea trade and were never, in fact, officially used for that purpose. Indeed, it is doubtful whether the values recorded in the *Port Books* were at all reliable.* The growth of commerce at the close of the Elizabethan era and the emergence of mercantilist ideas quickly drew attention to the need for accurate national statistics on the "Balance of Trade" with foreign countries.

2. Various estimates of the total value of imports and of exports, and of the Balance of Trade, had been made from time to time during the 17th century,† and unsuccessful attempts were also made by Official Committees to keep regular records of the value of our oversea trade. These estimates were, however, hardly more than rough approximations, especially as the duty rates on which the calculations were based were either specific or conventional ones, and were the same, in most cases, for both cheap and expensive varieties of the same commodity.

3. This situation was drastically changed in 1696 by the creation of the office of the Inspector-General of Imports and Exports to keep accurate records of our oversea trade. The Inspector-General's figures were subject to many errors and omissions, but it may be of some interest to note that for the year 1700 the Inspector-General's books show the following totals of England's trade with other countries (including Scotland) : Exports, £7,302,716 8*s*. 7*d*., and Imports, £5,970,175 1*s*. 10*d*., the excess of exports being over £1 million.‡ The defects in the earlier statistics were serious in many respects, and are touched on below, where they throw some light on the character and purpose of the present-day returns.

4. The present legal authority for the collection of statistics derives from the Customs Consolidation Act, 1876, as subsequently amended. This Act requires goods to be entered at H.M. Customs and to be described in the entry form in terms and in units in accordance with "Official Import and Export Lists," which "shall mean any lists which are now, or shall from time to time, be issued under the authority of the Commissioners of the Treasury or Customs, prescribing the denominations, descriptions and quantity by tale, weight, measure, value or otherwise by which articles of merchandise shall be required to be entered on their importation into or exportation from the United Kingdom."

5. The "Official Import and Export Lists" are revised annually on the recommendations of an

* The *Port Books* were discontinued in 1799.

† Estimates were made for 1605–11, 1612–14, 1621–2, 1662–3 (Port of London only), 1674 (trade with France only), and probably for other years also, for which records now no longer exist. (Prof. G. N. Clark, *see* Bibliography, III, 8.)

‡ See Prof. G. N. Clark's book for an extremely interesting discussion of the early trade statistics.

inter-Departmental Revising Committee, and entries made in accordance with these Lists form the basis of the official trade statistics. In 1948 there were 2,778 separate headings in the Import List and 2,514 in the Export List.

6. The statistics are published in monthly and in annual form: the monthly *Accounts relating to the Trade and Navigation of the United Kingdom* and the *Annual Statement of the Trade of the United Kingdom.* A brief description of the contents of each of these publications is given in the bibliography.

II. DEFINITIONS

7. Imports and Exports can be broadly defined as the flow of physical goods across national frontiers, and it is in this way that we normally think about them. This definition is not, however, quite accurate as it stands, since certain kinds of goods crossing national frontiers are excluded from the official statistics of imports and exports, while in certain countries—though not in the United Kingdom—the measurement of the flow of trade is made, not at the frontier, but at warehouses under Customs control.

8. For the United Kingdom we may group the flow of goods excluded from the official trade returns into two. First, and most important, are the goods which are not considered to be imports or exports proper; second, those which, though recognized as genuine imports or exports, are nevertheless excluded from the official statistics for one reason or another. The Table on the next page shows the principal categories of goods so excluded from the trade returns, and figures have been inserted wherever they are available or can readily be estimated.

9. *Goods not considered as imports or exports proper.*—Imports and exports of gold and silver bullion and coin are really a special case, such transactions being regarded as of a capital nature rather than as part of the flow of current merchandise trading. The movement of such gold and silver is therefore excluded from the trade returns, a separate account being kept by H.M. Customs.

10. A separate record is also kept of goods transhipped under bond, either within the same U.K. port or from one U.K. port to another. The goods concerned only pass through the United Kingdom as an incidental part of their voyage from one foreign country to another, so that it would be unreal to add their particulars to the record of our import and export trade. Before the outbreak of World War II, *free* goods in transit through the United Kingdom on through bills of lading were included in the import and re-export accounts, though this trade was essentially similar, in principle, to transhipments under bond, which were excluded. The arrangements for through bills of lading (which were begun in 1899) have not been recommenced by H.M. Customs, and goods which would previously have been sent in transit on through bills of lading are now transhipped under bond and are therefore excluded from the trade figures. The total value of goods in transit on through bills of lading amounted to £9 million in 1938, and this amount should be deducted from the import and re-export figures for that year before a precise comparison with the trade of war or post-war years is made.

11. Ship's stores and bunkers are also clearly a special category, being consumed on trading vessels themselves, and are also excluded from the oversea trade figures. Quantitative accounts of the coal and petroleum bunker fuel taken on board at U.K. ports by ships engaged in the foreign trade are, however, published in the official *Trade Accounts*, and an approximate valuation is given in Table 1 for 1938 and 1947. It should, of course, be remembered that a substantial proportion of the ship's stores and bunkers are taken on board by British vessels, and so are not debits in our Balance of Payments accounts. The distinction between British and foreign vessels has been made for the coal bunkers in the monthly *Accounts* as from January, 1949, but not for the petroleum bunkers, though the same point applies to them also.

12. Ships arriving at or departing from U.K. ports (other than newly-built ships departing from this country for transfer or delivery to overseas buyers, or old ships sold for breaking up) are not required to be entered as imports or exports, as they are not considered as "goods" for the purposes of the Customs Consolidation Act, 1876. Vessels engaged in the foreign trade which are sold to foreigners may be either engaged in cross-trading between foreign ports or trading direct from a U.K. port. Sales or purchases of vessels engaged in oversea trade direct from U.K. ports, as well as of those engaged wholly in trading abroad, can take place just as easily in foreign ports as when the vessels are in British ports. Sales of all old ships could be

regarded as genuine exports of part of the country's existing capital assets, but it is more convenient, for practical purposes, to treat all transactions in second-hand ships as more akin to transactions in goods held abroad rather than as U.K. merchandise trade proper. Rather different considerations apply to *new* ships and these are discussed below (para. 14). The logical position as regards aircraft would seem to be like that of ships, but the legal position differs in so far as aircraft are deemed to come within the scope of the Customs Consolidation Act and purchases and sales of aircraft, both new and old, by U.K. residents are recorded by the Customs for *Trade Account* purposes, when the aircraft arrive in, or depart from, this country. Sales when the aircraft are abroad escape record.

13. The remaining categories of excluded goods which are not considered proper to the Accounts can perhaps best be referred to as "diplomatic or extra-territorial." Diplomatic privilege is granted for goods directly imported by Ambassadors and Ministers accredited to the United Kingdom, no import entries being required by the Customs, and goods imported by U.N.R.R.A. for its official use also came in this category; the volume of such trade is, of course, relatively quite small. The principle of extra-territoriality is very much more important. Fresh fish and shell fish of British taking, landed from British ships arriving direct from the fishing grounds, are regarded as (extra-territorial) U.K. production and not as imports, and are specifically exempted from Customs entry by the Customs Consolidation Act, 1876. (No similar exemption extends, however, to whale oil landed from British whalers operating in the Antarctic, this being recorded as an import from "British Whale Fisheries"; the alternative procedure would have been to treat these British whalers as extra-territorial, but this would have resulted in only a part of our whale oil imports being recorded, i.e. landings from foreign whalers.) The treatment of British fishing vessels as "extra-territorial" implies also that fish caught by such vessels and sold directly in foreign ports should be treated as exports, but such sales are not, in fact, recorded by the Customs. Other examples of the principle of extra-territoriality are the export of British Government-owned stores exported for British Government use abroad, the import of stores for the naval, military and air forces carried on Government vessels, and during the recent war the import by Government Departments of captured enemy military equipment and the import of military stores by the U.S. Government for the use of their own forces stationed in the United Kingdom, all these movements being excluded altogether from the *Trade Accounts*.

TABLE 1.—*Goods excluded from the United Kingdom Trade Returns*

£ million

Class of goods	1938		1947	
	Inflow	*Outflow*	*Inflow*	*Outflow*
A. *Goods not considered as imports or exports proper:*				
1. Gold and silver bullion or coin	258	332	130	16
2. Transhipments under bond	33*		55	
3. Ships' stores and bunkers loaded at U.K. ports by vessels in the foreign trade	—	15†	—	33†
4. Old ships	0·5	2·4	45§	3·2
5. "Diplomatic" or "Extra territorial" goods:				
a. Imports of fish of British taking‡	17	—	43	—
b. Other	..	..	..	..
B. *Goods considered as genuine imports or exports:*				
1. New ships	3·0	—	..	—
2. Diamonds‖	3	..	12	..
3. Fissionable materials; smuggled goods; personal effects	..	..	..	..

* Excluding £9 million of goods in transit on through bills of lading.
† Approximate estimate for bunkers only.
‡ Landings in Great Britain only.
§ As given in Cmd. 7648; the figure relates to purchases in the year and may include some new ships. The figure for 1947 was abnormally high.
‖ These figures relate to exports of rough diamonds to the United Kingdom, as published in the trade returns of the main diamond-exporting countries.
NOTE.—The symbol .. is used to denote that figures are not published.

14. *Goods excluded, though considered as genuine imports or exports.*—There remains those goods which, though considered proper to the *Accounts*, are nevertheless excluded in practice for one reason or another. One of the important exclusions consists of the purchase by U.K. firms of newly-built ships from abroad. These are excluded from the import statistics because of the difficulties of Customs control, such ships not being considered as "goods" for the purposes of the Customs Consolidation Act, 1876. Newly-built ships (and aircraft) transferred from this country to owners abroad are, however, recorded as exports. Commercial aircraft flown to this country for repair constitute rather a problem, since clearly only when major repair jobs are done can these properly be regarded as "imports" on arrival and as "re-exports" on departure; the basis of valuation is arbitrary since no sale is involved, while there is the further difficulty that the "country of consignment" has no meaning in this case.

15. Imports and re-exports of unset precious and semi-precious stones and pearls (except unmounted diamonds drilled for dies) have been excluded from the statistics since the beginning of 1949. Previously they were included in as far as they were declared on Customs entries. The main trade here is in diamonds, and the essential difficulty has always been one of obtaining a complete record of the trade. Diamonds are normally sent by registered letter post, but are also frequently carried by private persons, so that the Customs entries before 1949 inevitably represented only a small proportion of the total trade, and the figures as recorded were most misleading. Table 1 gives an estimate of our imports of diamonds in 1938 and 1947.

16. A recent important exclusion has been that for fissionable materials, imports of which, though recorded, are not included in the published figures for security reasons. Personal and household effects purchased abroad and brought into the country by passengers are also excluded, though they are as much genuine imports as similar goods imported on a commercial scale. Before May 1st, 1947, however, such personal effects as were dutiable were included in the import statistics under the appropriate headings; the total so included during January-April, 1947, was £407,000*, but the figure for the summer months would have been substantially higher in view of the much larger passenger and tourist travel. Similarly, personal effects purchased in this country and taken out by foreigners are not included in the export statistics. Finally, there are goods smuggled past the Customs. While smuggling into the United Kingdom is not on any significant scale (as it still is in China, for example, or was during the prohibition period in the U.S.A.), smuggled goods are continually being discovered by Customs officers, any smuggled goods not discovered thereby escaping record.

17. With these exclusions, then, the official trade statistics represent the total inflow and outflow of the merchandise trade of the United Kingdom. The movement is not, however, a simple one, since not all the goods imported are consumed in this country—an important proportion is *re-exported.* It is most desirable for many purposes to distinguish between re-exports of imported goods and exports of home-produced goods. The distinction is, broadly, that re-exports are goods exported in the same (or essentially the same) condition in which they were imported, while exports are goods which have been transformed by processing, or manufactured in this country, or which have been grown or produced here, including fish of British taking. Our most important re-export is raw wool, followed (in 1938) by hides and skins, non-ferrous metals and tea. But the dividing line between exports and re-exports is not an easy one to make. Tea, for example, is normally blended in this country before re-export, so that, while tea blending is now considered as "productive" for Census of Production purposes, the product of the Tea Blending Trade is regarded, not as an export of British Industry, but as a re-export of a commodity the nature of which has remained essentially unchanged. These definitions must be borne in mind before comparing the trade returns with Census of Production aggregates.

18. *"General" and "Special" Trade.*—*Retained Imports* are defined in the official statistics as Total Imports in a given period *less* Re-exports in the same period. The concept of retained imports as used in the United Kingdom statistics must be distinguished from the concept of *imports for domestic consumption*—used by the U.S.A. and many continental countries—which relates to that part of the total imports in a given period intended, *at the time of importation*, for use or consumption in the importing country.

19. The differences between figures of retained imports and those of imports for home consumption are due to several factors. Goods re-exported in a given year may not have been all

* *Board of Trade Journal*, July 19th, 1947.

imported in that year; a substantial proportion may have been re-exported after having been imported in preceding years. In addition to this difficulty of time-lag, which sometimes results in negative figures for retained imports for particular commodities, there is the further complication that the same commodity normally has a higher value on re-exportation than it had at the time of its importation, the difference being merchanting, warehousing, insurance and transport charges arising in this country. A further cause of differences between the two methods of recording imports is that "imports for home consumption" are normally recorded ex-Customs warehouse, whereas Total Imports into the United Kingdom are recorded at the port whether the goods enter bonded warehouses or not. The following table illustrates the distinction between "retained imports" and "imports for home consumption"; items 3, 4 and 5 are taken from the official *Trade Accounts*, but the division of item 3 between items 1 and 2 has been estimated.

TABLE 2.—*Imports into the United Kingdom in* 1938 *and* 1948

	£ *million*	
Category	1938	1948
1. Imports for home consumption	870	2,025
2. Imports for re-export	50	50
3. Total imports	920	2,075
4. Re-exports	60	63
5. Retained imports	860	2,012

20. This distinction between retained imports and imports for home consumption is only one side of the more general distinction between the two methods of trade classification, commonly known as "General" and "Special" trade. The "General" trade system is a record of total imports on the one hand, and re-exports and domestic exports on the other; this system is in general use in the British Commonwealth. The "Special" trade system is essentially a record of imports for domestic consumption on the one hand and domestic exports on the other, but the latter also includes—in the returns of many countries using this system—"nationalized" goods which have previously been imported and on which duty has been paid (these would be included in "re-exports" in the "General" trade classification). The "Special" trade system is in use in most continental countries, since their general tariff systems readily allowed of the separate declaration of imports of goods destined for on-shipment to other countries, these being excluded altogether from the statistics (they would be included in "total imports" and in "re-exports" in the "General" trade classification). Of 83 important commercial countries in 1939, 43 used the "Special" basis, 35 the "General" basis, 4 used a mixed system, while 1 used both.*

III. PROBLEMS OF MEASUREMENT

A. *Value Measurements*

21. The basis of valuation of commodities recorded in the import and export statistics has presented some rather difficult problems ever since the figures were first compiled in 1696. The first statistics were built up from a "price list" based on information furnished by merchants engaged in the foreign trade, and these "official" prices were applied to the actual quantities recorded. The "official" prices were at first carefully revised as and when necessary, so that a "true balance of trade" should be recorded, but after the retirement of the first Inspector-General in 1703 very few amendments to this list were made, and much the same set of prices were used for valuing our export trade right through the eighteenth century, until the imposition of an *ad valorem* "Convoy Tax" on exports in 1798, under which exporters were required to declare the actual value of the goods they were exporting.

* Petruzzelli (*see* Bibliography, III, 18).

22. For imports, the "declared" value basis was introduced only in 1871; valuation on the old "price list" basis was continued as the substantive basis until 1854, when a system of "computed real values" was introduced, based on lists of current prices which were amended as frequently as was necessary. "Official" valuations based on the old "price list" were also published until the "declared" basis was introduced. Detailed revaluations of the import and export trades of both Great Britain and of Ireland were called for while the proposal for the Union of the two countries was under consideration in 1799. The Inspector-General accordingly produced Accounts showing the value, at the prices then current, of all the principal commodities imported and exported, the figures relating to the annual averages of the three years preceding January 5th, 1799. The Inspector-General for Ireland produced very similar Accounts.* A recent study by Mr. A. H. Imlah gives estimates of values of imports and exports at current prices for each year from 1795 to 1853†; according to these estimates, Great Britain and Ireland had a deficit on current merchandise trading with the rest of the world for the whole period studied.

23. The declared values now recorded represent essentially the open market value of the goods at U.K. port at the time of importation or exportation. For imports, the legal basis of valuation is laid down in the Finance Act, 1935; briefly, this value is the price which the goods would fetch on a sale in the open market at the time of importation, the goods being delivered to the buyer at the place of importation, all costs and charges (except Customs Duties and Purchase Tax) incurred in the purchase and delivery of the goods having been paid by the seller. The value of exports represents the cost of the goods to the purchaser abroad, all costs and charges accruing up to placing the goods on board the exporting vessel or aircraft, or at the land boundary of Northern Ireland, having been paid by the seller. These values are known as c.i.f. ("cost, insurance and freight") for imports (though differing from the true definition by the small addition for landing charges), and f.o.b. (i.e. "free on board") for exports.

24. Some difficulty still arises in certain cases, notably for goods transferred between branches of the same company operating in different countries and for parcel post, though during the recent war other difficulties also had to be met, especially in connection with Government imports. The difficulty in the case of intra-company transfers is, of course, that an internal costing valuation would normally be made, especially if—for taxation purposes—the company preferred to earn all its profits nominally in one country rather than in another. In such cases, however, H.M. Customs insist on a valuation on the "open market value" basis for revenue (and statistical) purposes. Equally, in the case of goods consigned for sale, the value is arrived at by making deductions for transport and profit from the value which the goods fetched when sold. For non-dutiable imports, and for exports, the values recorded may be less precise than where the revenue is involved.

25. The Parcel Post figures exclude goods liable to duty on importation, or exported from bond or claiming drawback on exportation. Since separate particulars are available for such dutiable goods, they are classified to their appropriate commodity headings in the *Trade Accounts*. The contents of the parcels included in the figures for Parcel Post are not normally known to the Customs. The basis of valuation is therefore the arbitrary one of £*x* per parcel, the estimated average values, which are different for imported and exported parcels, and may vary according to the country concerned, being subject to change as indicated by the relatively scanty data available. In 1948, Parcel Post exports amounted to £39 million on this arbitrary basis (or 2½ per cent. of total U.K. exports), while imports of Parcel Post were recorded as £16 million (under 1 per cent. of the total).

26. The valuation of imports and exports is a responsibility of the importer or exporter concerned, and he is liable to prosecution if he makes an incorrect declaration. This "declared" system has not been a universal one, however. Many countries have used "official" values based on tariff lists drawn up for the levying of Customs duties. Argentina, for example, recorded all imports on the basis of such "official" values up to July 1st, 1941, since when the "declared" system has been in use.

27. Even those countries using the declarations made by importers do not always require the c.i.f. value to be stated for imports. The U.S.A., for example, requires imports to be entered

* These Accounts are reproduced in Macpherson, *Annals of Commerce*, vol. iv, pp. 503–7.

† See Bibliography, III, 11.

at the foreign market value or the export value, whichever is the higher; in either case the value required does not include cost of carriage from place of purchase to port of shipment, so that it differs to this extent from the f.o.b. value. New Zealand, on the other hand, aiming at an estimated c.i.f. basis, records imports on the basis of f.o.b. values plus 10 per cent. There are many variations and some composite methods also in force in certain countries. An analysis of the trade returns of 83 countries during 1928–1939 showed that (while the majority used the c.i.f. basis) imports were valued on an f.o.b. basis in 25 countries and on an estimated c.i.f. or mixed basis in a further 9 countries.* For exports, nearly all countries use the f.o.b. basis.

28. The standardization of valuation practices is clearly a desirable pre-requisite for any international trade comparisons. The International Convention relating to Economic Statistics (1928) laid down that the c.i.f. basis for imports and the f.o.b. basis for exports should be followed, but allowed countries to use other methods if these were necessary also for revenue purposes, provided that in such cases an annual estimate of the c.i.f. or f.o.b. values were also given.

29. *Valuation and the Balance of Payments.*—The official statistics of our Balance of Payments† include figures of actual payments for imports and receipts for exports and re-exports. The figures for payments have been largely derived from exchange control data ever since 1939, when exchange control was imposed. Before the war the *Trade Accounts* figures were used by the Board of Trade in the annual Balance of Payments estimates published in the *Board of Trade Journal.* The following table gives the figures for 1938 and 1948 taken from the two sources.

TABLE 3.—*Valuation of U.K. Oversea Trade for Balance of Payments Purposes*

"*White Paper*" *basis*	1938 (£ *million*)	1948 (£ *million*)	Trade Accounts *basis*	1938 (£ *million*)	1948 (£ *million*)
1. Payments for Imports, f.o.b.	835	1,768	1. Value of Imports, c.i.f.	920	2,075
2. Receipts for U.K. Exports and Re-exports, f.o.b.	533	1,550	2. Value of U.K. Exports and Re-exports, f.o.b.	533	1,633
3. "Visible" trade balance	−302	−218	3. "Visible" trade balance	−387	−442
4. "Invisible" balance	+232	+98	4. "Invisible" balance (residual)	+317	+322
5. Net Balance	−70	−120	5. Net Balance	−70	−120

30. The different figures for the "visible" balance arise from differing methods of treating particular items, but this does not, of course, affect our total Balance of Payments, since compensating adjustments are made in the "invisible" account. However, the different sets of figures for our import and export trade have caused no little confusion among users of the figures, so that it seems appropriate here to explain the main reasons for the differences.

31. First, the pre-war basis related to credits and debits, whereas the present figures are based on payments made and received. The relevant magnitudes in a current account are clearly the actual payments and receipts in the period taken, rather than the value of the goods recorded as having been imported and exported in that period. Since there is a time-lag between the shipment of the goods and the payment for them, there will be a significant difference in the two sets of figures, for this reason alone, in periods when trade is moving rapidly upwards or downwards. The time-lag for exports is a main reason for the differing figures for 1948 given above.

32. Second, it is more convenient when considering payments to value imports as purchased in countries of origin (i.e. on an f.o.b. basis) rather than to use the recorded c.i.f. values in the *Trade Accounts*; the difference (apart from time-lag) represents freight and marine insurance, and the proportion of this due to firms overseas is estimated separately. A separate account for shipping services can then be included in the Balance of Payments estimates.

* Petruzzelli (see Bibliography, III, 18).
† See Bibliography, I, 6*b*.

33. Third, the Balance of Payments figures of merchandise trade should make a clear distinction between goods which are paid for (either in monetary form or by barter) and goods for which no current payment arises. The *Balance of Payments Manual*, issued in 1948 by the International Monetary Fund for use by member countries, discusses the different categories of real and money flows and contains detailed instructions on the method of treatment to be used for each item. The U.K. official Balance of Payments estimates, however, which—as mentioned above—relate to money flows (i.e. payments and receipts), also include *all* imports and exports, even in cases where no payments arise. The most important of these "non-payment" imports and exports are *gifts*, both government and private, and *transfers*, both by Governmental agencies and between branches of the same company. "Gifts" were extremely important during the war (Lease-Lend, Mutual Aid, etc.) and in the post-war period (relief and rehabilitation supplies); non-cash imports under E.R.P. are, however, treated as part of our merchandise imports in the Balance of Payments White Paper. "Transfer" would include oil equipment exported by U.K. oil companies to their refineries abroad, though in this case the White Paper includes such equipment in the figure for exports, and makes corresponding deduction in the "invisible" account. Another example is the import of U.K. stocks held abroad which were paid for during the war; once again, these are included in the White Paper as "imports" in 1946, 1947 and 1948, offsetting items being included in the "invisibles." There seems to be a good case for amending the presentation of these statistics to distinguish between the *value* of goods imported and exported and the *money flows* arising either from payments in foreign currency or from the liquidation of accumulated sterling balances.

34. Fourth, the items mentioned in the previous section as being excluded from the *Trade Accounts*, though properly classifiable as imports and exports, should be added back for Balance of Payments purposes. However, the only items which are mentioned in the White Paper as having been added back are old ships sold to customers abroad and ships purchased from abroad; i.e. the figures for merchandise trade in the official Balance of Payments estimates include the sale and purchase of old as well as of newly-built ships (see the argument in para. 12 above). Diamonds are mentioned as being excluded from the figures of merchandise trade, while the treatment of bunkers, personal effects of passengers and fissionable materials is not discussed at all; these would all be taken into account in the final item of Table 3.

35. Finally, a deduction has to be made from the *Trade Accounts* figures in respect of trade with the Channel Islands, which are treated as part of the United Kingdom in the Balance of Payments Accounts.

B. *Quantity Measurements*

36. The recording of quantitative information is clearly essential for the proper interpretation of the oversea trade statistics, since changes in the value figures may be due to price changes as well as to changes in the volume of trade.

37. It is not, of course, practicable to record all commodities by quantity, because there are bound to be headings in any finite statistical classification containing a heterogeneous collection of items, for each of which the appropriate unit of quantity would be different. Moreover, there has been a long-term trend towards more fabricated products, especially on the export side, and these are generally more difficult to measure quantitatively. These difficulties have, however, been more than counterbalanced by successive improvements in the statistical classification, which is now very much more detailed than ever before, thus allowing for a greater proportion of items to be specified by one or other unit of quantity. As a result of the overhaul of the classification after the first World War and subsequent annual revisions, the proportion of trade recorded by value only was reduced from 11 per cent. for imports and 17 per cent. for U.K. exports in 1912* to 2 and 8½ per cent., respectively, in 1938. The corresponding proportions for 1946—the latest year for which the full detail has been published—were 2 and 10½ per cent.

38. Apart from the obvious difficulties in comparing the quantity figures published by different countries (due to the use of different units of quantity), there are various traps awaiting those who wish to use the U.K. figures themselves over a long period of years, in particular as a result

* Cd. 7432 (1914).

of the incessant smuggling of dutiable goods in the eighteenth and early nineteenth centuries, as well as the "ostentation" over-entries of the London merchants in the earlier period. There are also various difficulties of interpretation when the quantity units were changed, especially in cases where the units themselves were not very precise (e.g. "pieces" of cloth).

39. A rather more subtle difficulty also exists even where the unit of quantity has remained unchanged. This arises in cases where the unit adopted is not fully suitable for the measurement of the quantum of trade. A good case in point is machinery, which is measured by tons. Since there is a definite trend towards the use of light alloys and more automatic and complex machinery, the average "ton" of machinery now exported embodies a greater amount of productive resources than the average "ton" of 10 or 20 years ago. This trend must always be borne in mind in interpreting the quantitative information available. This type of difficulty could probably be largely overcome by the use of appropriate multiple quantity units. Cotton piece goods, for example, are recorded by square yards, linear yards and cwt., though it is doubtful whether even these units taken together provide sufficient information on changes in average quality. The units in use for the hull and fittings of motor vessels exceeding 50 tons gross (number, gross tonnage and net tons*) are even more dubious from this point of view, the average "quality" in this case depending on a variety of materials and of workmanship quite independent of the gross or net tonnage.

40. For many purposes (allocation of shipping space, comparison between national statistics, etc.) it is important to obtain a global quantitative measure of oversea trade. One method of doing this is to convert all quantity units into weight terms. *The International Convention relating to Economic Statistics* (1928) recognized the importance of a common measure of quantity, and it included the following provision:

> "When the quantity of goods of any kind is expressed in any unit or units of measure other than weight, an estimate of the average weight of each unit, or multiple of units, shall be shown in the annual returns."

In accordance with this provision, the annual U.K. returns have been prefaced, since 1932, with a table of suitable factors for the conversion of all the important quantity units other than weight into tonnage terms. It should always be remembered that the weights recorded in the trade statistics are *net*, i.e. exclusive of all packing.

41. For estimating the *total* weight of imports or exports it is, of course, also necessary to convert the "value only" headings into weight on the basis of whatever relevant information is available. These figures are of particular interest to the shipping community, the total net weight of our imports and exports being computed each year by the Liverpool Steam Ship Owners' Association and published in their annual reports. Official estimates were published for 1913 and for a few years after the first World War. The weight of dry cargo imports was given for the war years in *Statistics relating to the War Effort of the United Kingdom* (Cmd. 6564), while the annual review of oversea trade in the *Board of Trade Journal* has given figures for both dry cargo and tanker imports for 1938 and for 1946 and each subsequent year.

c. Volume Measurements

42. The measurement of trade movements by global weight figures, though indeed useful for many purposes, has the serious disadvantage for the economic statistician that it takes no account of changes in the relative prices of different commodities. An expansion in the number of, say, motor cars exported may well be balanced in terms of weight by a reduction for, say, lead ingots, but clearly the net result has been an increase in terms of the amount of productive resources devoted to exports.

43. This disadvantage is met by the use of the concept of *volume*. There has frequently been misunderstanding as to the meaning of the term. It is best thought of as the *value* of imports or exports *at constant prices*. This raises a further problem of the best set of prices to use, since different results will be obtainable if price relationships are very different from one year to another. To meet this, it is necessary to change the base year of the calculation as soon as the base year prices are seriously at variance with the current price relationships.

* These tonnages not being weights but representing calculated cubic capacities (see M. G. Kendall, "United Kingdom Merchant Shipping Statistics," *J.R.S.S.*, Pt. II, 1948).

44. The first official attempt to eliminate the effect of price changes on the recorded values of imports and exports was made by Mr. (later Sir Robert) Giffen in 1878, in a report to the Secretary of the Board of Trade, in which he gave index numbers of prices to show the contribution made by price fluctuations to the changes in the recorded values.* From here it was but a short step to the use of index numbers of volume, which were introduced by Mr. Stephen Bourne at the 1885 meeting of the British Association, and later elaborated by him at the British Association meeting of 1888 and in a Paper read before the Royal Statistical Society in 1889. The method used by Bourne was also adopted in the next official series, covering the years 1900–1913, trade in each of these years being revalued in detail at the average values of the year 1900.† After World War I a detailed revaluation of trade was made quarterly, and this practice has been continued ever since, the results being published in the *Board of Trade Journal*. The present series of volume index numbers is based on average unit values of trade in 1938; each item recorded in the monthly *Trade Accounts* for the current period is revalued at the average value of the corresponding item in 1938, and the total revalued figure is then related to recorded trade in 1938 to derive the volume index. A full description of the method of calculation can be found in the *Board of Trade Journal* for May 4th, 1946.

45. The contrast between the *volume* and *weight* measurements is clearly brought out in the following comparisons for imports and exports:

TABLE 4.—*Index Numbers of the Volume and Weight of U.K. Imports and Exports*

(1938 = 100)

Year	*Total Imports*		*U.K. Exports*	
	Volume Index	*Weight Index*	*Volume Index*	*Weight Index*
1943 .	77	48 .	29	*
1946 .	67	73 .	99	31
1947 .	76	85 .	109	22
1948 .	78	98 .	136	47

* No estimate available.

46. The discrepancy between the two series is much more marked for exports than for imports. The low weight index for exports is due entirely to the reduced export of coal, which is very heavy but also very low-valued, so that, while it dominates the weight index, it has a much smaller influence on the volume index. The increase in the export volume over the pre-war figure is due essentially to the increased export of manufactured goods, which have high values per ton of weight.

47. The discrepancy in the case of imports is perhaps rather more illuminating. The low war-time figure for 1943 illustrates how much the tonnage of imports was reduced to suit the limited shipping capacity then available, while at the same time maintaining the volume at a relatively high level. This was effected by (1) a drastic cut in imports of low-valued, bulky goods (e.g. animal feeding-stuffs and timber); (2) the substitution of more finished goods for raw materials or semi-manufactures (e.g. semi-finished steel for iron ore or flour for wheat); and (3) the elimination of water and other unnecessary parts of normal imports (e.g. dried eggs instead of fresh eggs).‡ This policy cut down the weight of imports drastically, but reduced the value (and therefore the volume index) by only a relatively small amount. The process was reversed after the end of the war, the weight of imports in 1948 being almost equal to that in 1938, whereas the volume was lower by one-fifth, in an endeavour to reduce the adverse Balance of Payments by importing less finished products.

* See Bibliography, I, 6*a* (1).
† See Bibliography, I, 6*a* (3).
‡ See *Board of Trade Journal*, August 11th, 1945.

D. *Price Measurements*

48. Since the volume index is defined as representing changes in values at *constant* prices (see para. 43), a "price" index can readily be obtained as a result of the volume calculation. This is the *index of average values*, which measures the relationship between the total value of goods imported or exported in the current period and the value which the same bundle of goods would have at base year prices. This index is published each quarter with the results of the detailed quarterly revaluation. Since the weighting used in the official average value index numbers always represent the quantities imported or exported in the current period, this index cannot be used to indicate price movements between adjacent periods, since the weighting would be different in each and the resultant index is affected by this as well as by any genuine price changes. This is particularly important for imports, where seasonal movements are of greater importance than for exports.

49. To overcome this difficulty a separate series of *price index numbers* is published each month. This index measures the change in the aggregate value of a representative selection of imports and exports. There are two essential differences between this monthly price index and the quarterly average value index:

(a) *Coverage*.—The average value index covers *every* commodity specified in the monthly *Trade Accounts*, the "value only" headings being revalued on the basis of the price movement shown by a related commodity or group of commodities. The price index, on the other hand, includes only items where the unit value behaves like a true price, and the movement in these is taken as representative of price changes in other imports or exports.

(b) *Weighting*.—Whereas the weighting of the average value index changes each quarter, the weights used for the price index are kept constant throughout the year, so that changes in the composition of trade from month to month have no effect on the price index. As from this year, 1949, the weighting procedure for the price index has been revised; previously the weights were based on forward estimates of trade in the ensuing year, but now the actual pattern of trade in the preceding year is used as a basis. The present method is adequate when the composition of trade shows little change from year to year, but may well give some misleading results if the pattern alters radically, as it was doing in the years immediately succeeding the war.

50. The weighting for the price index is revised at the beginning of each year, the new weights also being used to recalculate the monthly index for the preceding year. Thus, both the 1948 and 1949 index numbers now being published are based on 1948 weights; next year the 1949 figures will presumably be recalculated on 1949 weights. The price index and the average value index will therefore give different results when a comparison is made between the current year as a whole and the base year, to the extent that the current weighting (used by the average value index) differs from that for the preceding year (used by the price index).

51. *Terms of trade*.—In current discussions of our Balance of Payments position the terms of trade play a vital role. For most practical purposes they can be defined as *the cost of a given quantum of imports in terms of exports*, though, of course, other definitions are probably more useful for special purposes. Changes in the terms of trade from one year to another are best measured by the ratio of import and export *average value* index numbers, since these allow for the changing composition of trade in different years as well as for price changes. When the quarterly movement is being studied, however, it is better to use the *price index* numbers, since variations in the composition of trade due to seasonal and irregular factors are thereby eliminated. The following Table shows the changes in our terms of trade with the rest of the world between pre-war and post-war years, and the quarterly movement since the beginning of 1948. The figures for the years up to 1947 are based on the average value index numbers, while for the quarterly movement since the beginning of 1948 the price index numbers have been used for the reason given above. The index numbers shown for the whole year 1948 are the averages of the quarterly price index numbers; had the average value index numbers been used to show the price movement between 1947 and 1948 the results would have been slightly different. The Table shows that our terms of trade have deteriorated by about 20 per cent. compared with 1938—when import prices were falling—but by only 12 per cent. when compared with 1937.

TABLE 5.—*Terms of Trade*

(1947 = 100)

Period	*Import Prices (c.i.f.)*	*Export Prices (f.o.b.)*	*Terms of Trade**
1937	41	44	93
1938	39	45	87
1947	100	100	100
1948	114	110	104
1948: 1st quarter	108	107	101
2nd ,,	114	109	104
3rd ,,	115	111	104
4th ,,	117	112	104
1949: 1st ,,	118	113	105
2nd ,,	117	113	104

* A rise indicates an adverse movement.

IV. CLASSIFICATION OF COMMODITIES

52. The basis of commodity classification is clearly most important, not only for obtaining the most useful interpretation of the movement in the figures, but for international comparisons also. The problem of a satisfactory logical system of commodity classification is one which has for long been the concern of various international bodies and conferences. The subject was discussed at the St. Petersburg International Statistical Congress, 1872, at which a list of 103 commodities was adopted for recommendation to Governments. This list was, however, not detailed enough to have any significant influence on the existing national classifications. The need for an international classification was becoming more necessary, and the International Commercial Conference at Brussels just before the first World War adopted a more detailed list (185 headings) for use by national Governments. The *Brussels List*, published in 1913, was adopted by a number of countries as their main or subsidiary method of classification.

53. This subject was also considered by the International Conference relating to Economic Statistics (1928), and one of the Conference recommendations was that a commodity classification for international trade statistics should be prepared, so as to allow the trade statistics of different countries to be readily compared. This was done by the League of Nations Committee of Statistical Experts, and was circulated to Governments in 1935 as the *Minimum List for International Trade Statistics*, a revised version being circulated in 1938. The major principles on which the *Minimum List* is based are:

(*a*) Nature and origin of the commodity or its basic material.

(*b*) Stage of production—crude, semi-manufactured, fully manufactured.

(*c*) Use to which the product is put, as for food, clothing, transport, etc.

54. The Committee of Statistical Experts stressed that no one principle should be the sole, or dominant, basis, but that several different principles should be taken into account. Clearly, all three of the principles mentioned above are useful, each for its own purpose. But each has limitations in practice. For example, there are difficulties in classifying composite manufactured goods (containing, say, metal, wood and plastic) under the "basic material" principle; again, it is not always possible to say when a particular commodity is "semi" or "fully" manufactured; while even greater difficulties may arise, in some cases, in deciding the future use to which a particular commodity may be put.

55. The League's "*Minimum List*" gave prominence to the "basic material" principle, but the other principles were also taken into account. In addition, Governments were urged to re-classify commodities according to "stage of production and use," but this latter classification had

not, by the beginning of the war, found such a wide application as had the *Minimum List* itself, this being used by 22 countries as a supplementary classification of their oversea trade, and by a further 3 countries as the basis of their main classification. The *Minimum List* is now under process of revision by the United Nations Statistical Commission. A new draft has recently been circulated to member Governments, and this differs considerably in many respects from the League *Minimum List*. In the new draft, nearly half the items of the League List have been either sub-divided, amalgamated or transposed, and all optional items abolished. The grouping gives prominence to end-use, except where the "basic material" is obviously identifiable, e.g. in the case of raw materials.

56. *The U.K. Classification.*—Before 1920 the United Kingdom figures for individual commodities were published alphabetically, the first three headings in 1913, for example, being "Aerated and Mineral Waters," "Aeroplanes, Airships, Balloons, and Parts thereof," and "Animals Living," though there were also summaries published bringing all the food items together, etc. This was an unsatisfactory method of presentation, and in 1920 the whole classification was revised, the system then introduced being, substantially, still in operation. This system takes into account all three major principles enunciated by the Committee of Statistical Experts. The main classes distinguished, and the value of trade in each class in 1938 and 1948, are shown below. The segregation of Classes I and IV follows the principle of "end use," whereas the distinction between Classes II and III depends on the principle of "stage of production." Within each of the three main classes the individual groups distinguished largely follow the principle of "origin or basic material" (e.g. "Iron and Steel and Manufactures thereof"), though "end use" is also taken into account (e.g. "Apparel").

TABLE 6.—*U.K. Oversea Trade in each of the Classes distinguished in the U.K. Classification*

£ million

Class	*Total Imports*		*U.K. Exports*		*Re-exports*	
	1938	1948	1938	1948	1938	1948
I. Food, Drink and Tobacco	430	888	36	94	12	10
II. Raw Materials and articles mainly unmanufactured	248	683	57	68	30	43
III. Articles wholly or mainly manufactured	234	485	365	1,378	18	11
IV. Animals, not for food	3	8	1	4	1	1
V. Parcel Post	4	16	12	40	—	—
Total	920	2,080	471	1,583	62	65

57. In addition to the 5 main classes shown in the Table above, a special account for "Munitions" was kept from 1942 to 1945 inclusive. For this purpose, "Munitions" were defined as aircraft and other vehicles and parts (except tyres and tubes for road vehicles), and arms, ammunition and military and naval stores imported or exported by Government Departments, excluding, of course, extra-territorial trade (see para. 13).

58. As from 1936, the United Kingdom statistics were re-classified on the *Minimum List* basis, as a subsidiary classification, the figures for 1936, 1937 and 1938 having been published before the war; the figures were value totals only, no quantities or country detail being given. The first post-war publication was for 1946, when country detail, and as much quantitative information as possible, was also given (see *Annual Statement of Trade for* 1946, Vol. I).

59. *Influence of the Tariff.*—The oversea trade figures are compiled by H.M. Customs, which Department is also responsible for the collection of the duty on imported goods and the payment of drawbacks and rebates on goods shipped out of the country. For administrative reasons it is essential to dovetail the statistical and the revenue accounts as closely as possible, and the headings in the statistics are therefore dependent to a very large extent on the heads of duty specified in the Tariff. As a result, the individual commodity headings are in many cases really tariff headings and, especially before 1946, were not always ideal for economic analysis. As from 1946 separate

Import and Export Lists have been issued, and there has been almost a complete overhaul of the individual commodity headings in the light of the developments in industry since 1939 and the need for more precise information about our oversea trade. The Import List has a number of headings needed for tariff purposes, but any of these not of economic significance have been eliminated from the Export List.

V. CLASSIFICATION OF COUNTRIES

60. There are various methods which could be used to record the countries involved in international trade, and a clear distinction needs to be made between these before a correct interpretation of the country figures is possible. This is, of course, especially important when examining the figures of the Balance of Trade between individual countries. The problem is complicated by the fact that different methods are used by different countries, while the practice in the United Kingdom itself has also varied.

61. The three major systems of classification of countries for international trade statistics were defined in the 1928 International Convention. They are:

(a) *Countries of origin (for imports) and consumption (for exports).*—"Country of origin" means the country where the goods being imported were actually produced, or in the case of manufactured goods, the country where they were transformed into their existing condition. Similarly, "country of consumption" means the country in which the goods will be put to the use for which they were produced, or in which they will undergo a process of transformation.

(b) *Countries of consignment.*—This means the country from which (or to which) the goods were originally despatched, with or without breaking bulk in the course of transport, but without any commercial transaction in any intermediate country.

(c) *Countries of purchase (for imports) and sale (for exports).*—This means the country in which the seller (or the buyer) of the goods carries on his business.

62. *Country of origin (or consumption)* particulars are the most important for studies of the economic inter-relationships between various trading countries, and are essential where trade preferences are involved. *Country of consignment* is useful when considering the immediate trade relations between different countries, and this basis is of particular interest to countries such as the United Kingdom where a considerable transit trade is carried on. *Country of purchase (or sale)* is clearly the most significant basis for Balance of Payments purposes, but the primary purpose of the trade statistics is to record the movement of goods and not the movement of money. The payments basis would, in any case, make international comparisons of trade statistics virtually impossible because of the importance of trading by agents, brokers, etc., resident in one country and buying and selling in other countries. For a great many of the commodities entering international trade, the figures on each of these bases would probably be the same. Nevertheless, a considerable proportion of world commerce goes through intermediaries in third countries, this being especially true of the trade in primary commodities, and in these cases, the statistics would be very different according to which method of classification was in use. An analysis of the classification methods for recording country detail in use by 75 important countries before World War II showed that for imports, 38 countries used the "consignment" basis, 28 the "origin" basis, and 2 the "purchase" basis; a further 4 used both "purchase" and "origin," while 3 used both "consignment" and "origin."*

63. The methods used in classifying countries for oversea trade statistics was one of the subjects discussed at the St. Petersburg meeting of the International Statistical Congress in 1872, when it was recommended that countries of origin and final destination be used (i.e. in effect the first method described above). At that time, however, as Sir R. Giffen reported to the next meeting of the International Statistical Congress in 1876, the United Kingdom used this basis so far as the information was available on the Customs entries, though the "country of shipment" was the substantive system of classification, i.e. goods were recorded according to the country from which (or to which) they were directly shipped in trade with the United Kingdom. The "shipment" basis could usually be qualified in the case of exports, but this could not be done to

* Petruzzelli (see Bibliography, III, 18).

any substantial extent for imports, and the country figures as published can easily lead to most misleading conclusions unless this is borne in mind. Switzerland, for example, did not appear at all in the statistics until the "shipment" basis was superseded, while trade with countries such as Jamaica, doing a considerable transit and entrepôt trade, was greatly overstated. In 1904 the "consignment" basis was adopted by the United Kingdom as a supplementary system of classification, and as the substantive system from 1909. The great advantage of the "consignment" basis is its simplicity; the country of consignment is always known to the importer or exporter, whereas it is not always possible to ascertain the country of origin or of final consumption. Indeed, for imports of some manufactured goods more than one country may be involved, while exports may be shipped on consignment and the country of final destination not known.

64. This subject was also discussed at the first British Empire Statistical Conference, 1920, where it was considered "that it would be of advantage if, in addition to statistics on any other basis, statistics of imports classified under countries of origin were available throughout the Empire." The major step towards international comparability, however, was taken by the 1928 International Conference, the resulting Convention providing that each country should conduct an experiment showing the results on each of the three systems of classification. This experiment was undertaken by 17 countries, that for the United Kingdom being carried out for trade in 1931.* The results were communicated to the League of Nations Committee of Statistical Experts, who reported that the "country of origin (or consumption)" basis appeared to be the most suitable for purposes of international comparisons,† and that countries using the "consignment" basis should supplement their statistics by a further record on the "origin" basis for a selected list of basic commodities. As a result of this recommendation, the United Kingdom has, since 1936, required country of origin particulars in respect of imports of a more restricted list of commodities, for which the earlier experiment had shown that countries of origin and consignment might be significantly different.

VI. Trade at Individual Ports

65. Since Customs entries have to be lodged at the ports at which goods are imported and exported, the statistics of trade at each port are readily available. In the earlier period copies of the individual Bills of Entry for the Port of London were available for public inspection on payment of a fee, and it was in order to deceive their rivals that the London merchants made over-entries (see para. 38). The Bills of Entry were also used as a basis for various private publications of London's oversea trade in the late seventeenth and eighteenth centuries.

66. For recent years separate figures for each of the principal commodities imported and exported at each port and at the Land Boundary with Eire were published each year in the *Annual Statement of Trade* up to the issue for 1938. Separate figures were also given for Transhipments under Bond. No figures for trade at individual ports have been published for any year since 1938.

VII. Nationality of Carrying Vessel

67. An important innovation was made in the recording of the U.K. oversea trade statistics in 1936; as from the beginning of that year importers and exporters were required to state on Customs entries the nationality of the vessel in which their goods were imported or exported. It was now possible, for the first time, to show accurately the part played in our oversea trade by British shipping and by shipping of each of the other trading nations. The 1937 results showed, for example, that 67 per cent. by value of our total imports were carried in British ships, the corresponding proportions for United Kingdom exports and re-exports being 79 and 54 per cent. The detailed results, relating to classes by value and main groups of commodity by quantity, are clearly most valuable for consideration of shipping programmes and policy, while the figures can also be used (for Balance of Payments purposes) as a basis for estimating freights earned by British and foreign shipping. For a comprehensive discussion of the work done by British shipping in the carriage of goods see the paper read by H. Leak before the Royal Statistical Society (1939).

* See *Board of Trade Journal*, January 12th, 1933.
† League of Nations, Economic and Financial, 1933, IIA, 31.

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2. *Annual Statement of the Trade of the United Kingdom.*—Published in 4 volumes and Supplement, the principal contents being as follows:

   Volume I: Summary volume containing all the aggregates in full detail, but without cross-classification.

   Volume II: Particulars of *imports* and *re-exports*, showing principal countries under commodities.

   Volume II, *Supplement:* Particulars of the amounts of *duty* collected under each Act, distinguishing the principal dutiable commodities, the quantities and values of imports also being shown.

   Volume III: Particulars of *exports*, showing principal countries under commodities.

   Volume IV: Particulars, for each *country* with which we trade, of the principal commodities imported, exported, and re-exported. Trade at individual ports and at the Land Boundary with Eire was shown up to 1938 in similar commodity detail, but no comparable figures have yet been published for any later year.

3. *Import and Export Lists* (annual).

4. *Oversea Trade of the United Kingdom*: *Nationality of Carrying Vessels*, 1936, 1937.

5. *Board of Trade Journal.*—Regular series of articles are published on the following aspects of the trade statistics:

   (*a*) *Reviews of U.K. Trade.*—Monthly, quarterly and annual reviews of U.K. Trade in the main commodities and with the principal countries.

   (*b*) *Volume and Value.*—Quarterly articles reviewing the trend of U.K. trade by volume. Index numbers of volume and of average values are given for each group distinguished in the classification. See *Board of Trade Journal* of May 4th, 1946, for a full description of the method of calculation of these index numbers.

   (*c*) *Price Index Numbers.*—Monthly reviews of the trend of import and export prices. See *Board of Trade Journal*, issues of April 13th, 1946, and March 12th, 1949, for a full description of the method of calculation.

   Before the war an annual article in the *Board of Trade Journal* gave estimates of the U.K. Balance of Payments, using the *Trade Accounts* figures of merchandise trade, together with estimates of each main item in our "invisible" trade with the rest of the world. The last of these articles was published in the issue for February 23rd, 1939.

6. *White Papers.*

   a. *Volume and Value.*

      (1) "*Prices of Imports and Exports.*"—This series covers the years from 1840 (for exports) and 1854 (for imports) down to 1886. Separate particulars are given of the price movements of each main commodity, as well as totals for imports and exports. The Command numbers are: C.2247 (1879), C.2484 (1880), C.3079 (1881), C.4456 (1885), and C.5386 (1888).

      (2) "*Statistical Tables and Charts relating to British and Foreign Trade and Industry* (1854–1908)," Cd. 4954 (1909).—This contains a table giving, for Total Imports and for U.K. Exports, values at the prices of 1900, for each year from 1880 to 1908 (computed from data published annually in the *Economist*).

      (3) "*Imports and Exports at Prices of* 1900."—This series covers the years 1900 to 1913, and gives index numbers of average value and volume based on 1900 for each main class distinguished in the statistics, as well as for Total Imports, U.K. Exports and Re-exports. The Command numbers are: Cd. 2894 (1906), Cd. 3446 (1907), Cd. 4115 (1908), Cd. 4867 (1909), Cd. 5160 (1910), Cd. 6314 (1912), Cd. 6782 (1913), and Cd. 7432 (1914).

   b. *Balance of Payments.*—This series was started after the end of World War II, and contains particulars of the main categories of imports on an f.o.b. basis, as well as useful notes. The Command numbers are Cmd. 7324 (1947), Cmd. 7520 (1948) and Cmd. 7648 (1949).

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PROCEEDINGS OF SECTIONS.

PROCEEDINGS OF THE STUDY SECTION, 1948–49.

OCTOBER 13th, 1948. Subject: "Financial Sources." Principal Speaker, Miss M. S. Rix; Attendance, 34.

The difficulty of keeping oneself informed on sources of data is appreciated by all engaged in statistical work. Miss Rix's paper touched on data from a field which was not generally well known. Her appraisal of the Stock Exchange Year Book, the Bush House records and similar financial sources was valuable in helping to fill what many present must have recognized as a serious gap in their knowledge of sources.

In the discussion which followed, several speakers expressed the need for a comprehensive source book. The source book of the Ministry of Labour's statistics was quoted as a good example of what is to be done for other official data and what remains to be done for semi-official sources.

NOVEMBER 13th, 1948. Joint Meeting with the London Group of the Industrial Applications Section.

A report of this meeting was published in the *Journal, Series A*, 1948, Part III, p. 238.

DECEMBER 8th, 1948. Subject: "Installation and Operation of Punched Card Accounting Machine Equipment for Routine Statistical Analysis." Principal Speaker, Mr. H. Gearing; Attendance, 47.

Mr. Gearing described the organization of a large Powers-Samas system designed for centralised accounting and records. He explained the problems of getting the best out of the installation by means of "spreading the load" and laid considerable emphasis on the need for planning in advance all stages of the operations, particularly sorting and tabulating.

The nature of the staff required for various types of operation was dealt with, as well as other associated staff problems, including that of maintaining the morale of workers engaged on purely routine processes.

The discussion which followed was, as might be expected, concerned to some extent with comparisions between the Hollerith and Powers equipments and their respective suitability for different kinds of work.

JANUARY 12th, 1949. Subject: "Pictorial Presentation." Principal Speaker, Mr. P. Redmayne; Attendance, 66.

An appreciative meeting listened to Mr. Redmayne's exposition of the art of presentation which he illustrated with some excellent lantern slides.

The bar chart, graph and "pie" chart were the basic forms used for many attractive isotype presentations. These aimed at helping the eye and mind, by showing, as symbols, a picture of the subject matter underlying the figures charted by this means.

Nine members joined in the discussion. While some doubted the value of the more pictorial forms of presentation, it was generally agreed that to plot a problem, or to illustrate it in some way at the outset frequently helped towards its solution.

FEBRUARY 9th, 1949. Joint Meeting with the Market Research Society. Subject: "Teaching of Statistics—(*b*) Teaching for Market Research." Principal Speakers: Mr. A. H. Elliott, Mr. A. P. McAnally, Mr. W. F. F. Kemsley, Mr. L. T. Wilkins; Attendance, 49.

The first two speakers, representing the Market Research Society, outlined the essential features of market research in relation to the training of statisticians. They stressed the point that advanced techniques were seldom likely to be used in that field. The need to convince the manufacturer or distributor of the research work involved frequently obliged that statistician to use methods less exact and rigorous but more capable of explanation in non-technical terms.

Mr. Kemsley thought that a case could be made out for training computing assistants with emphasis on methods rather than on the theory. Mr. Wilkins pointed out that teaching was difficult because students came from schools with little or no idea of functional analysis. He

wondered whether the basic ideas of distributions and of variation could not be introduced at an earlier age. Teaching was closely allied to methodological research and he stressed the need for such an examination into the techniques of market research.

The discussion was continued by many members and visitors and some widely differing views were expressed.

MARCH 9th, 1949. Subject: "The Organization of a Statistics Department." Principal Speakers: Mr. J. Stafford, Mr. B. Benjamin, Mr. G. S. Browne, Mr. A. W. Swan; Attendance, 27.

The collective experience of the four principal speakers at this Open Forum may be said to have covered the widely varying problems of organizing a statistics department in Central Government, industry, and an independent advisory service.

Mr. Stafford outlined the organization of the Board of Trade's Statistical Division and paid particular attention to the Census of Production Office and the nucleus staff already assembled to deal with the forthcoming Census of Distribution. Mr. Benjamin described his section at the London County Council, which was primarily allied to the Health Department. He showed how the duties of qualified statisticians were integrated with those of the medical staff and also with the work of other Departments of the Council.

The widely different organizations described by Mr. Browne and Mr. Swan opened up a field very different from that usually associated with statistical departments in the industrial and commercial field. Mr. Browne, as head of the "Economist" Intelligence Unit, was responsible for the organisation of statistical material covering an extremely wide range of subjects in both the national and international field. Much of this information had to be in such a form that it could be produced at short notice and cover current questions. Mr. Swan's position was very different in that he was providing an advisory service to a group of steel works and dealing mainly with the application of modern techniques of statistical analysis to problems of productive industry.

An enthusiastic discussion followed and the "guillotine" had to be applied, leaving much still to be said.

APRIL 6th, 1949. Subject: "Some Applications of Discriminant Functions in Biology." Principal Speaker, Mr. P. Armitage; Attendance, 27.

Mr. Armitage gave a valuable account of the discriminant function and showed its relationship to the work of Professor R. A. Fisher. He showed how the discriminant function ensured the minimum of variance within a class and the maximum between classes. Using some measurements of certain germs, he discussed the problems of identifying and classifying them according to the classes to which they would "most likely" belong. In one case it was demonstrated that two measurements, length and breadth, provided sufficient data for classification.

In the subsequent discussion it was made clear that this particular analytical technique should be capable of a much wider use than was being made of it at present, possibly because its processes were as yet less familiar than those of, say, analysis of variance.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Fundamentals of Statistics.* By J. B. Scarborough and R. W. Wagner. Boston and London: Ginn, 1948. 9″. vii + 145 pp. 13*s.*

This book has a more unusual raison d'être than can be claimed for most statistical texts. It was originally prepared to form the basis of a course of about twenty lectures to be introduced into the post-war curriculum of the United States Naval Academy. Unfortunately, the study of statistics gave way before the more pressing demands of electronics and aeronautics, and the authors now offer the work for the use of scientific and technical students in general. It should not be confused with a larger book of the same title by T. L. Kelley.

The aim has been to present fundamental principles and techniques to students whose mathematical repertoire includes a knowledge of elementary calculus, and emphasis is directed more to the proofs of propositions than to lengthy discussion.

In the first chapter the reader is introduced to various methods of representing data and to frequency distributions. The next two chapters deal with measures of location and dispersion, but include no reference to range or mean deviation. The widespread use of range in connection with quality-control charts might have justified its inclusion. Chapter IV is concerned with the comparison of distributions and the determination of shape coefficients. An illustration containing twelve histograms of various shapes is particularly instructive. The following chapter deals with correlation and regression. The computation of the correlation coefficient and the correlation ratio is explained in some detail. Chapter VI introduces probability density functions with particular reference to the Binomial distribution. It also includes a discussion of Tchebycheff's Inequality and the law of large numbers. The next chapter examines the Normal Curve and its use as an approximation to the Binomial distribution. This is followed by several pages on the Gram-Charlier series and its application to the graduation of frequency distributions. Sampling is the subject of Chapter VIII, which includes the sampling distributions of the mean and standard deviation and a description of the testing of the difference between two means. No reference is made to the possibility of unequal variances, nor is the t distribution introduced. In fact, throughout the book, any detailed discussions of small sample techniques have been specifically avoided. An appendix, which includes a large proportion of problems and examples, is devoted to *a priori* probability. The only tables which are given, or indeed required, in the book, are the Normal probability function, its integral, and the second, third and fourth derivatives.

The main criticism of the work lies not so much with the method of presentation, which is both clear and concise, but with the choice of subject matter. For example, the omission of Chi-squared tests and contingency tables is unfortunate, and their inclusion might well have been effected at the expense of Tchebycheff's Inequality, Bernoulli's theorem and the Gram-Charlier series. Despite the serious limitations imposed by the brevity of the course, more might have been accomplished in providing statistical techniques which could be widely applied to problems likely to be encountered by the technical student. D. R. W.

2.—*A Statistical Analysis of Advertising Expenditure and the Revenue of the Press.* By Nicholas Kaldor and Rodney Silverman. Cambridge: University Press, 1948. xiv + 200 pp. 9½″.

This book presents the results of far the most extensive and scientific enquiry into (pre-war) advertising statistics which has ever been made in this country. It attempts to answer almost all the questions which a statistical investigation could reasonably explore, and to assess the margin of error of many of the results.

The field is inherently a difficult one. The proper definition of advertising is far from obvious —indeed for different purposes one clearly wants different definitions (e.g. announcements of births, deaths and marriages undoubtedly contribute to the advertising revenue of the newspapers, but are not included in what the economist has in mind when discussing advertising); the definition used in this book is an uneasy compromise based mainly on expediency, and it might have been wiser to stress this point by giving several different figures for the total, including and excluding various doubtful items.

The main difficulty, however, is the lack of any systematic or comprehensive statistics even on parts of the field. The authors have shown remarkable ingenuity and persistence in finding ways of preparing estimates for each part of the total, and consider that there is only one chance in 20 of their final figure of £89·4 million being out by more than 5 per cent. This may be an over-optimistic view, which *inter alia* allows far too little for the danger of a few major errors all pulling the same way. In a pioneering venture of this kind there are almost bound to be some "errors of principle," and one has already come to light—the omission of any allowance for the considerable expense of having blocks made for press advertisements. I would, however, prefer to raise the risk factor to (say) one chance in four, rather than to increase the margin of 5 per cent.

The total is, however, less important than some of the "bits," especially in view of its arbitrary definition. The authors analyse it in three distinct ways: by the commodities, etc., advertised, by the "medium" used, and by the industry which is paid for the work. Perhaps the most interesting is the first, which is further subdivided to show separately advertising directed to the final consumer as opposed to trade press advertising, advertising of raw materials, etc. Figures are given for a very large number of commodities or groups, and the advertising expenditure is related to the value of manufacturers' sales.

The authors try to explain the very varying percentages which arise from this last analysis. They do not find any common factor in the *nature* of the highly advertised goods, which include luxuries and necessities, goods bought frequently and others bought at long intervals, articles of mass consumption and others sold only to a limited class. Their explanation runs in terms of the structure of the industry, high advertising being associated with an oligopolistic situation.

This book is one which should be in the hands of everyone concerned with advertising. The post-war picture, however, would doubtless be very different, and it is satisfactory that the Advertising Association is intending to collect information about it. In some respects it should be able to improve on the rather devious methods which had to be used in an enquiry conducted during the war, but it will clearly find this pioneering work of great value. W. B. R.

3.—*Comparative Productivity in British and American Industry.* By L. Rostas (National Institute of Economic and Social Research. Occasional Papers, XIII). Cambridge University Press, 1948, xxiii + 263 pp. 9½″. 18*s*.

To read this book gives one somewhat the same feeling that I imagine one would have on now visiting Hiroshima. It was in 1943 that Dr. Rostas first startled industrialists and the public with the suggestion that the output per man is very much lower in British than in American industry; but only in December, 1948, was the present book, which gives a full account of his researches, published, although it was completed in 1946. Nevertheless, it will for some time be an essential book for every investigator of industrial productivity to read.

The main conclusion of the work, that in the immediate pre-war years the output per worker in American manufacturing industry was on the average about twice that in British manufacturing industry, is irrefutable. The conclusion, as is appropriate for a scientific discussion, is stated in precise terms; more light would have been shed and less heat generated had the public controversies been in equally precise terms and not in terms of such vague concepts as the efficiency of industry. Dr. Rostas has, through giving his conclusions, contributed powerfully to the awakening of this country to an awareness of the problem of industrial productivity, but the most valuable features of his book for the present and future are the discussion of methodology and of the factors that influence productivity, and the almost encyclopaedic references to other investigations in the field.

The investigation was inevitably based largely on data obtained from censuses. Dr. Rostas's study is very thorough and he has ably exploited his method to the utmost, but its inherent limitations leave much information to be desired. The investigation gives little indication of the

causes of differences in productivity or of what to do in order to improve matters. The discussions of factors affecting productivity, both general and specific to various industries, are exhaustive, well-informed and realistic; but they are successful only at the qualitative level, and such quantitative information as is brought into the discussions advances knowledge very little. Dr. Rostas is aware of this, and admirably sums up the position in the following passage, taken from his Preface:

"I feel that we are still at the beginning of enquiry into the fascinating subject of relative productivity, and I hope that this study will stimulate further research. It shows the limitations on how far we can get *on the basis of available data and information* [author's italics] towards knowing what our relative productivity is *vis-à-vis* other countries; and it shows as well the limitations on our attempts to account for the differences. It is to be hoped that, once more detailed and more appropriate data are collected and the methods of productivity measurement and of the study of productivity problems have developed, we shall be able to enlarge our knowledge on all questions of productivity." L. H. C. T.

4.—*Demographic Survey of the British Colonial Empire.* By R. R. Kuczynski. Vol. I. West Africa. London: Geoffrey Cumberlege, 1948. 9¾". xiii + 821 pp. 75*s.*

This is the first of four volumes which the late Dr. Kuczynski prepared under the auspices of the Royal Institute of International Affairs. It was a bold undertaking to base a survey on the defective data which are available for the various areas of the Colonial Empire. However, although the material dealing with population and vital statistics is scanty and often inconclusive, this first volume is most impressive and much more than a reference book or a "series of monographs" as the author modestly described the work.

Much of the book is devoted to the interpretation of the demographic figures given in, or conclusions based on, colonial reports which are no longer defensible when they are scrutinized, and therefore need re-interpretation. The available material is such that not infrequently it offers an insoluble task to the demographer. The author resignedly remarks: "To appraise fertility, morbidity, mortality or migration is about as difficult in most African Dependencies as to appraise the frequency of adultery in this country."

The book deals with and presents a demographic survey of the following territories:

Sierra Leone; Gambia, consisting of the Island of St. Mary, the rest of the Colony and the Protectorate; Gold Coast and Togoland; the Gold Coast Colony, comprising the Dependencies Ashanti and the Northern Territories; Nigeria (including Lagos) and the Cameroons.

The section on Sierra Leone, by no means the largest territory dealt with, is the largest chapter of the book. This is the result of the material available which more often than not is disproportionate to the size of the area. Much information is derived from reports of medical officers, pamphlets, letters and diaries and local studies which contain either demographic data or demographic history of the Dependencies. These documents are especially valuable for the periods for which no censuses have been taken.

Apart from statistics dealing with fertility and mortality of Africans and Non-Africans, the book contains innumerable data relating to the prevalence of the chief diseases which were or are still rife in the various districts of the West-African Colonies. The book will be indispensable to demographers, historians and sociologists alike. It is extremely well produced, though one wonders how many scholars can in fact afford to purchase it. M. J. E.

5.—*Current Financial Problems and the City of London.* A series of lectures delivered at the Institute of Bankers' International Summer School, Christ Church, Oxford, September, 1948. London : Europa Publications, 1949. [4] 219 pp. 8½". 15*s.*

Since the war the Institute of Bankers has arranged two very useful series of lectures on current economic problems. The first, arranged in conjunction with the University of London, was published under the title "The Industrial Future of Great Britain" and dealt with various British industries. The second, which is the subject of this review, covers a variety of topics of more direct interest to students of banking.

The lectures which are reprinted in this volume must have proved fascinating material for the students of various nationalities to whom they were first delivered at the bankers' International Summer School. They constitute an equally valuable survey of the contemporary British financial scene for the general reader who has been unable to keep abreast of the many changes which have taken place since 1939. The book is, however, open to one major criticism. A collection of lectures necessarily tends to lack continuity, since each lecturer tries to make his paper as self-contained as possible. In this volume the lack of continuity is accentuated by the illogical order in which the contributions have been arranged.

Of the thirteen lectures, seven deal specifically with banks and the City. Here the "Survey of Financial Institutions of the City of London," by Mr. David Sachs, forms a useful introduction to papers on the discount market, the Stock Exchange, merchant banking, bank organization and accounting, and the work of trustee departments and foreign branches. On international aspects there is an admirable commentary on international banking organizations by Mr. Paul Bareau, and short papers on central banking and the Sterling Area by Professor Sayers and Sir Henry Clay. Miss Irene Shrigley contributes a classified bibliography of banking literature. Finally there are two papers of a more general nature, one by Mr. Harrod on the balance of payments, and the other a general survey of British monetary policy by Mr. W. F. Crick.

Current Financial Problems and the City of London does not set out to break new ground. It is a straightforward account of the contemporary situation by experts in their various fields. It will provide a handy statement for the general reader and, in addition, may help to bridge the gap which sometimes exists in the minds of students between policy and institutions, by showing some of the implications of recent developments in international trade and financial policy for the bankers' day to day work.

P. A. D.

6.—*Other New Publications*

Essays on Local Government. ed. C. H. Wilson. London: Blackwell, 1948. vii + 246 pp. 8½". 18*s*.

[The five essays which make up the body of this book are the fruits of research by workers at Nuffield College. Some of them cover a period of a century or more, some deal with the effects of recent legislation. All are factual and should prove useful sources for students. A digest of the proposals made by Local Authorities Associations and others for reform is added as an appendix. The editor himself provides a sixth essay as introduction, examining in it the principles underlying local government as we know it. He concludes that, as these are political, viz., the provision of the fullest possible education in and exercise of political responsibility by the individual, reforms are to be judged by their contribution to this provision rather than by any gain in administrative convenience they may offer.]

Rose, T. G. Business Charts. 4th ed. London: Pitman, 1949. xv + 126 pp. 8½". 18*s*.

[Too often charts and graphs are produced for business men who do not know what they want by clerks who do not know what they are doing. The standard of charting could hardly fail to benefit if managers would not only keep this book on their shelves for reference, but encourage their statistical clerks to become fully acquainted with its contents. Mr. Rose writes primarily for managers and executives, and is admirably clear as to the object of business charts and the advantages of various types for specific purposes. The directions he gives are marked by practical common sense, and even an untrained junior clerk should be able to put them into practice.

Statistical students, whether intending to enter business or not, will find this 4th edition a useful introduction to elementary graphic methods. It does not claim to deal with the presentation of advanced mathematical data.]

STATISTICAL NOTES

(1) British Official Statistics

The interim index of retail prices compiled by the Ministry of Labour and National Service, which had risen by 2 points in May as a result of some of the increases in food prices announced in the Budget speech, remained unchanged at 111 up to the middle of August. There was, however, a further slight rise in the food index in June due to increases in the price of butter, margarine and cheese which completed the Budget series. There was also a slight rise in the fuel index due to an increase in coal prices in some areas. The figures for other groups of items showed little change in the period. The detailed figures for May to August were as follows :

(Prices at June 17th, 1947 = 100)

Date	*Food*	*Rent and Rates*	*Clothing*	*Fuel and Light*	*Household Durable Goods*	*Miscellaneous Goods*	*Services*	*Drink and Tobacco*	*Total*
Weights	348	88	97	65	71	35	79	217	1,000
May, 17th, 1949	114·3	100·1	118·1	111·5	108·4	113·1	105·2	107·5	111
June 14th, „	115·5	100·1	118·3	112·5	108·4	113·1	105·3	107·5	111
July 12th, „	115·6	100·1	118·6	112·9	108·5	113·1	105·3	107·5	111
August 16th, „	116·0	100·1	118·7	112·9	108·3	113·1	105·4	107·5	111

In publishing the figures the Ministry of Labour states that they are in the form in which they are used in the procedure adopted for calculating the index for all the groups combined, i.e. to the nearest first place of decimals. The decimals are shown only in order that, if desired, calculations can be made of the effect of combining particular groups and excluding others. The information available as to price changes, however, is such that no precise significance can be attributed to the decimals, and for any other purposes, therefore, the figures should be used to the nearest whole number.

The Ministry of Labour index of weekly wage rates, which rose to 108 in January, remained at that figure in February and March. The following is a summary of the figures since June, 1947, when the present series was instituted. The rise in June was only fractional, and resulted from the fact that the index is quoted to the nearest whole number.

(Wage Rates at end of June, 1947 = 100)

Date (end of month)	*Men*	*Women*	*Juveniles*	*All Workers*
June, 1947	100	100	100	100
September, ,,	101	101	102	101
December, ,,	103	103	106	103
March, 1948	105	106	107	105
June, ,,	105	107	108	106
September, ,,	106	108	109	106
December, ,,	107	109	110	107
January, 1949	107	109	110	108
February, ,,	107	109	110	108
March, ,,	108	110	111	108
April, ,,	108	110	111	108
May, ,,	108	111	111	108
June, ,,	108	111	111	109
July, ,,	108	111	112	109
August, ,,	108	111	112	109

The total working population and the numbers in civil employment shown by the new series of manpower figures, which became available early in 1949, are as follows:

Thousands

	Total Working Population			*Numbers in Civil Employment.*		
	Males	*Females*	*Total*	*Males*	*Females*	*Total*
Mid-1948	16,057	7,089	23,146	14,945	6,981	21,926
End 1948	16,069	7,116	23,185	15,019	6,992	22,011
April, 1949	16,058	7,151	23,209	15,061	7,038	22,099
May, 1949	16,039	7,170	23,209	15,070	7,066	22,136
June, 1949	16,023	7,171	23,194	15,079	7,078	22,157
July, 1949	16,036	7,169	23,205	15,095	7,075	22,170

It will be seen that between April and July, 1949, the total working population showed little change, a decrease of 22,000 in the number of males being offset by an increase of 18,000 females. The number in civil employment was 71,000 higher in July than in April, both males and females showing a rise.

The level of unemployment fell by 40,522 in June and by a further 20,602 in July. In August there was a rise of 18,348, more than half of this number being school-leavers seeking employment.

Number of Unemployed Persons on the Registers of the Employment Exchanges of the Ministry of Labour and National Service

Date	*Men and Boys*	*Women and Girls*	*Total*
May 9th 1949	227,943	76,222	304,165
June 13th, ,,	199,212	64,431	263,643
July 11th, ,,	185,655	57,386	243,041
August 15th, ,,	194,216	67,173	261,389

The total for August, 1949, includes 27,812 married women. The totals do not include registered disabled persons who were classified as suitable only for employment under sheltered conditions.

(2) Other Statistics

A pamphlet entitled *The Rent of Agricultural Land in England and Wales*, 1870–1946, has recently been published by the Central Landowners' Association (Price 6s. post free). This pamphlet contains two Reports prepared by the University of Oxford Agricultural Economics Research Institute; one summarizing and analysing information obtained in enquiries made by the Association and the Ministry of Agriculture in 1938 and 1946 and in other recent years; and a second bringing together and examining the information available on the trends of agricultural rents over the last 75 years. An exhaustive list of publications and references is also given.

In the Report relating to the years 1938 and 1946, the difficulties in the way of obtaining a representative sample of agricultural rents are discussed, especially in view of the wide variation in the rents paid for farms of different sizes, but it is considered that the comparability of the figures as between the two years is not materially affected. On this basis it appears that the average gross rents rose during the war period from 25s. 6d. per acre in 1938 to 27s. 6d. in 1946, or by about 8 per cent. This small increase is in striking contrast to the movement in farm prices generally which, according to the official index, rose by 102 per cent. in the same period while minimum wage rates increased by 115 per cent. The gross rents were, moreover, subject to charges for maintenance, repairs, insurance and management, all of which tended to rise more rapidly than the rents themselves; these expenses absorbed 62 per cent. of the gross rents in 1934–38 and 79·7 per cent. in 1946 so that the balance of the gross rent remaining in owners' hands had shrunk from 38 per cent. in the former years to just over 20 per cent. in the latter.

In the second Report by the Agricultural Economics Research Institute, a longer view is taken and comparison is carried back into the last century. Here the available evidence shows that the rents of agricultural land, though fluctuating, have moved definitely downwards, those for the period 1872–78 being higher than in any subsequent year.

It is perhaps of interest to mention that in this review, use is made of an enquiry published in this Journal over forty years ago. (*R.S.S. Journal*, 1907, Part IV.)

CURRENT NOTES

Statistics in Cambridge.—The Statistical Laboratory of the University of Cambridge was formally opened as a separate field of activity, and in its own premises, on March 2nd, 1949. In charge of the Reader in Statistics, the venture is under the auspices of the Faculty of Mathematics. The present building is a temporary one, but the laboratory hopes to become a permanent part of a Mathematical Institute which is planned. The staff consists of the Reader, Dr. John Wishart; Henry E. Daniels and Francis J. Anscombe, University Lecturers; and Dennis V. Lindley, University Demonstrator, and provision is made for clerical and computing assistance. The students to be accommodated will be graduates who are either research students or enrolled for the course leading to a Diploma in Mathematical Statistics.

Mr. G. Udny Yule, appointed in 1912, was the first lecturer in statistics in the University, his duties being divided between the School of Agriculture and the Faculty of Economics. He became the first Reader in Statistics, and on his retirement on September 30th, 1931, was succeeded by Dr. Wishart. From this point onwards statistical teaching was catered for separately in the Faculty of Economics, and has considerably expanded under Mr. Charles F. Carter; in addition the Department of Applied Economics under the directorship of Mr. Richard Stone has made rapid progress since the war ended. Dr. Wishart continued the association of statistics with the School of Agriculture, but also introduced the subject into the Mathematical.Tripos, and a school of mathematical statisticians was the result. This work will be continued in the Statistical Laboratory, and the applications of statistical methods will be catered for in various ways—by lectures to non-mathematicians, by a consultative statistical service to University and other Research Institutes, and by the requirement that Diploma students should study in a field of application of their choice. Among other statistical developments at Cambridge, it is of interest to mention that the Medical School has appointed a statistician, Mr. Norman T. J. Bailey.

The Secretary of State for the Colonies desires to make known the particulars of Research Fellowships and Grants which it is proposed to offer for the encouragement of fundamental research into colonial economic problems. The purpose is to assist the Colonial Office in the development and welfare of Colonial peoples.

The fields of research in which further investigation is considered especially desirable include: Economic problems of migrant labour; Internal trade and marketing; National income and the balance of payments; Capital investment; The relation of land tenure systems to economic development; Comparative efficiency of different methods of production in agriculture and industry; Conditions affecting growth of local enterprise; Incentives to production; Supply of and demand for skilled manpower; Patterns of cash expenditure; Problem of peasant agriculture in Colonial conditions; Problems of large-scale production; Economic surveys of selected areas; Economic studies of selected industries; Price movements and production; Transport economics.

Some of the subjects would relate to Colonial territories generally; others to conditions in particular territories. Some would involve work by a team of investigators. In some cases the research would necessitate visits to, or temporary residence in, the selected territory.

The Secretary of State is prepared to make grants to suitable workers as follows:

Colonial Research Fellowships.—A limited number of Research Fellowships will be available for British graduates of not more than 35 years of age, with some research experience, for investigations to be undertaken largely or entirely in Colonial territories. Fellowships will normally be awarded for periods not exceeding 3 years and the salary will not normally exceed £575.

Special Research Grants, carrying salaries comparable to United Kingdom standards, may also be made to suitably qualified persons attached to a university or research institution in the United Kingdom for whom the terms of the Fellowships are not appropriate. These awards will normally be made for periods not exceeding 3 years.

Grants for Development of Special Researches, of especial interest at the present time. These may be awarded to research workers of acknowledged standing who have already worked at the subject; they may be applied to investigations to be carried out here or in the specified territory.

The amount will vary according to the qualifications of the worker and the expense involved in the research. Grants to research institutions may also be included.

Research workers and heads of institutions who are interested in the fields of work specified above should write for application forms (stating the kind of grant for which they wish to apply) to the Under Secretary of State, Colonial Office (Research Department), Sanctuary Buildings, Great Smith Street, London, S.W.1.

The Trustees of the Houblon-Norman Fund, on the recommendation of the Advisory Committee, have made the following awards for 1949/50:

Fellowship.—J. A. S. L. Leighton-Boyce, Research Student, University of Oxford, "Country Banking in the 18th and early 19th centuries."

Research Grants.—W. C. E. Hartley, Bank Official, "History of Banks, Bankers and Banking in Yorkshire"; J. M. Holden (Fellow, 1948/49), Assistant Lecturer, London School of Economics, "Historical Study of Negotiable Instruments"; F. R. J. Jervis, Assistant Lecturer, Technical College, Derby, "Economic Development of the Large-Scale Retail Unit"; F. A. Judd, Bank Official, "Migration of Industry from the United Kingdom to British Territories Overseas since 1945"; R. O. Roberts (renewed for a second year), Lecturer, University College of Swansea, "Economic History of Non-Ferrous Metal Smelting in Wales"; P. E. Smart (renewed for a second year), Bank Official, "Banking Case Law"; J. A. P. Treasure, Assistant Lecturer, University College of South Wales and Monmouthshire, Cardiff, "Economic and Financial Problems of the British Film Industry"; J. S. G. Wilson, Lecturer, London School of Economics, "Development of Indian Banking since 1930."

An offer of awards for 1950/51 will be made early in 1950. Further information may be obtained from the Secretary, Houblon-Norman Fund, c/o The Bank of England, London, E.C.2.

The President has appointed as his Vice-Presidents for the Session 1949–50, Professor R. G. D. Allen, Mr. H. Campion, Professor A. Bradford Hill, and Sir George Maddex.

Fellows will be interested to hear of the appointment of Dr. M. G. Kendall to a newly created chair of statistics at the London School of Economics, University of London. Dr. Kendall was recently awarded the degree of Sc.D. of the University of Cambridge.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Accounting Research—

July 1949—The effects of the Local Government Act, 1948, and other recent legislation on the finances of local authorities: *A Research Working Party.* Plant, depreciation, and 1949 price levels: *T. H. Sanders.* Depreciation allocations in relation to financial capital, real capital and productive capacity: *H. Norris.* Standardized accountancy considered internationally: *S. J. Lengyel.* Measuring industry's output: *G. L. S. Shackle.* The classification of post-war refunds of excess profits tax: *R. W. Moon.* Worsted spinning costs: *F. Sewell Bray, C. Smith* and *D. R. Bedford Smith.* Some aspects of university finance and accounts: *R. Lewin.*

The Advancement of Science—

April 1949—Problems of old age—report of a symposium. The contribution of instruments to industrial progress: *Sir Ewart Smith.* Some post-war economic developments in Egypt: *A. McK. Frood.*

Annals of Eugenics—

June 1949—A non-existence theorem for an infinite family of symmetrical block designs: *M. P. Schützenberger.* Parental and fraternal correlations for fitness: *J. B. S. Haldane.* The incidence of parental consanguinity in diabetes mellitus: *H. Harris.* The meaning of "fitness" in human populations: *L. S. Penrose.* Note on the elimination of insignificant variates in discriminatory analysis: *M. H. Quenouille.* The sib-sib age of onset correlation among individuals suffering from a hereditary syndrome produced by more than one gene: *H. Harris* and *C. A. B. Smith.* The truncated binomial distribution: *D. J. Finney.* A quantitative survey of the fingerprints of a small sample of the British population: *S. B. Holt.* A test for homogeneity of records of familial abnormalities: *J. B. S. Haldane.*

The Banker—

July 1949—America—recession or slump? Germany's capital needs: has credit policy been too restrictive? The balance of payments since the war: *G. A. Duncan.* Ireland's investment programme: *A. Fitzgerald.*

Biometrika—

June 1949—The infectiousness of measles: *M. Greenwood.* A note on the analysis of grouped probit data: *K. D. Tocher.* A generalization of Poisson's binomial limit for use in ecology: *M. Thomas.* The estimation and comparison of residual regressions where there are two or more related sets of observations: *A. H. Carter.* Cumulants of multivariate multinomial distributions: *J. Wishart.* On the Wishart distribution in statistics: *A. C. Aitken.* The spectral theory of discrete stochastic processes: *P. A. P. Moran.* On a property of distributions admitting sufficient statistics: *V. S. Huzurbazar.* On a method of trend elimination: *M. H. Quenouille.* On the estimation of dispersion by linear systematic statistics: *H. J. Godwin.* On the reconciliation of theories of probability: *M. G. Kendall.* The derivation and partition of χ^2 in certain discrete distributions: *H. O. Lancaster.* A note on the subdivision of χ^2 into components: *J. O. Irwin.* The first and second moments of some probability distributions arising from points on a lattice and their application: *P. V. K. Iyer.* Probability tables for the range: *E. J. Gumbel.* Systems of frequency curves generated by methods of translation: *N. L. Johnson.* Rank and product-moment correlation: *M. G. Kendall.* Tests of significance in harmonic analysis: *H. O. Hartley.* The non-central χ^2- and F-distributions and their applications: *P. B. Patnaik.* On a method of estimating frequencies: *D. J. Finney.* A further note on the mean deviation from the median: *K. R. Nair.*

British Journal of Psychology (*Statistical Section*)—

March 1949—On estimating oblique factors: *G. Thomson.* Evidence of a space factor at 11 and earlier: *W. G. Emmet.* Multivariate analysis applied to differences between neurotic groups: *C. R. Rao* and *P. Slater.* The concept of equivalent scores in similar tests: *P. D. Greenall.* Subdivided factors: *C. Burt.*

British Journal of Social Medicine—

January 1949—Biological factors affecting family size: *F. A. E. Crew.* Studies of the diet of students at Edinburgh University: *A. H. Kitchin, R. Passmore, M. Pyke* and *G. M. Warnock.* Mortality from rheumatic heart disease in children and young adults in England and Wales: *J. Knowelden.*

Economica—

May 1949—The economist in the twentieth century: *L. Robbins.* Mr. Harrod's dynamic theory: *J. R. Hicks.* The German currency reform and the revival of the German economy: *F. A. Lutz.*

Economic Journal—

June 1949—Wholesale prices, 1938–48: *R. G. D. Allen.* The problem of French recovery: *R. V. Rosa.* Mill and the wages fund: *A. C. Pigou.* International factor-price equalisation once again: *P. A. Samuelson.* The value of the pound: *Colin Clark.* The future of banking in Australia: *J. S. G. Wilson.*

Eugenics Review—

July 1949—The population problems of India and Pakistan: *S. Chandrasekhar.* Race mixture: a social or biological problem? *A. Dickinson.*

Institute of Actuaries, Journal—

Vol. LXXIV, Part II, No. 339—The Centenary of the Institute of Actuaries. The effect of changed economic conditions and State insurance on private pension fund benefits, contributions and valuations: *W. F. Marples.* Punched-card equipment: *N. E. Coe, K. J. Hedley* and *L. H. Longley-Cook.* The measurement of reproductivity: *A. H. Pollard.* The rationale of the use of the geometric average as an investment index: *C. D. Rich.* The balance of the sexes in Great Britain: *W. S. Hocking.* The recent trend of mortality in England and Wales: *W. S. Hocking.*

Journal of Political Economy—

June 1949—Food and agriculture in the Soviet Union, 1917–48: *M. K. Bennett.*

Manchester School of Economic and Social Studies—

May 1949—The Economic Survey for 1949: *E. Devons.* The development areas: *J. Sykes.* An agricultural production function for the United Kingdom, 1924 to 1947: *K. S. Lomax.* Federalism and finance: *A. H. Birch.* The concept of a dollar shortage: *T. Balogh.* The economic recovery of Belgium: *R. Pâquet.* The British Monopolies Act: *W. A. Lewis.*

Oxford Economic Papers—

June 1949—The function of exchange rates: *R. G. Hawtrey;* A comment: *Sir Hubert Henderson.* Profits in British industry from 1924 to 1935: *R. Hope.* Some thoughts on the nature of interest: *E. V. Morgan.* Static models and current problems in international economics: *T. Balogh.* The theory of comparative costs reconsidered: *G. M. Meier.* Oligopoly and imperfect competition: *E. G. Dowdell.* The foundations of welfare economics: *I. M. D. Little.* The origins of industrial peace: the case of the British boot and shoe industry: *E. Brunner.* A note on current Marxist definitions of the national income: *D. Seers.* A note on price and quality: *N. H. Leyland.*

Oxford University Institute of Statistics, Bulletin—

May 1949—The movement of labour in 1948 (Part I): *C. A. R. Crossland.* The cost of living, 1938–1948: *D. Seers.*

June 1949—A "human needs" diet: *T. Schulz.* Changes in real national income: *D. Seers* and *P. F. D. Wallis.*

Royal Meteorological Society, Quarterly Journal—

April 1949—Post-glacial climatic change: *F. Hoyle, H. Godwin, G. Manley* and *C. E. P. Brooks.*

INDIA—

Calcutta Statistical Association, Bulletin—

May 1949—Teaching of statistics. Improvement of official crop-forecasts in India: *A. N. A.* Tests for topographic randomness: *B. Ghosh.* Symposia held at the Indian Science Congress.

Sankhyā—

March 1949—Anthropometric survey of the United Provinces, 1941: a statistical study. Part I: The field survey: *D. M. Majumdar;* Part II: Statistical analysis: *P. C. Mahalanobis* and *C. R. Rao;* Part III: Anthropological observations: *P. C. Mahalanobis.*

UNION OF SOUTH AFRICA—

South African Journal of Economics—

June 1949—Wool in the South African economy: *F. J. C. Cronje.* Some thoughts on the Union's economic outlook: *C. S. Richards.* Report of the Coal Commission, 1946/7: *B. H. Davies.* Some new emphases in population analysis: *R. Ross.* Returns to capital invested in the gold mining industry in South Africa: *S. R. Hellig.*

UNITED STATES—

American Academy of Political and Social Science, Annals—

May 1949—The Soviet Union since World War II. (Whole number.).
July 1949.—World Government. (Whole number.)

American Economic Review

May 1949—Papers and proceedings of the sixty-first annual meeting of the American Economic Association. (Whole number.)
June 1949—International disequilibrium: *R. F. Mikesell.* Prices, money and the distribution of goods in postwar Germany: *H. Menderhausen.* Analysis of dissaving: *G. Katona.*

American Statistical Association, Journal—

June 1949—The current status of State and local population estimates in the Census Bureau: *H. S. Shryock, Jr.*, and *N. Lawrence.* The uses and usefulness of binomial probability paper: *F. Mosteller* and *J. W. Tukey.* Teaching statistical quality control for town and gown: *E. G. Olds* and *L. A. Knowler.* The use of sampling in Great Britain: *C. A. Moser.* Unemployment and migration in the Depression (1930–1935): *R. Freedman* and *A. H. Hawley.* Mininmum χ^2 and maximum likelihood solution in terms of a linear transform with particular reference to bioassay: *J. Berkson.* Some inadequacies of the federal censuses of agriculture: *R. J. Jessen.* The edge marking of statistical cards: *A. M. Lester.* Conrad Alexander Verrijn Stuart (1865–1948): *W. F. Willcox.*

Annals of Mathematical Staztistics—

June 1949—Statistical decision functions: *A. Wald.* The multiplicative process: *R. Otter.* Application of the Radon-Nikodym theorem to the theory of sufficient statistics: *P. R. Halmos* and *L. J. Savage.* On designing single sampling inspection plans: *F. E. Grubbs.* On the range-midrange test and some tests with bounded significance levels: *J. E. Walsh.* Asymptotic studentization in testing of hypotheses: *H. Chernoff.* Some low moments of order statistics: *H. J. Goldwin.* On a theorem of Hsu and Robbins: *P. Erdös.* Brownian motion on the surface of the 3-sphere: *K. Yosida.* On the strong stability of a sequence of events: *A. Dvoretzky.* A note on weighing design: *K. S. Banerjee.* Control chart for largest and smallest values: *J. M. Howell.* Sufficiency, truncation and selection: *J. W. Tukey.* On a probability distribution: *M. A. Woodbury.* A graphical determination of sample size for Wilks' tolerance: *Z. W. Birnbaum* and *H. S. Zuckerman.*

Biometrics—

March 1949—Problems of the optimum catch in small whitefish lakes: *R. B. Miller.* Statistics of a lake trout fishery: *F. E. J. Fry.*
June 1949—Comparing individual means in the analysis of variance.: *J. W. Tukey.* Methods of estimating total runs and escapements of salmon: *G. A. Rounsefell.* The analysis of

extinction time data in bioassay: *K. Mather*. The general theory of prime-power lattice designs: *W. T. Federer*. A relation between the logarithmic, Poisson and negative binomial series: *M. H. Quenouille*. The statistical analysis of insect counts based on the negative binomial distribution: *F. J. Anscombe*.

Econometrica—

April 1949—A system of equations explaining the United States trade cycle, 1921 to 1941: *C. Clark*. Identification problems in economic model construction: *T. C. Koopmans*. A scheme of international compensation: *L. R. Klein*. A scheme of international compensation: postscript: *M. H. Ekker*.

Estadística—

December 1948—El censo de prueba de Turrialba, Costa Rica: *R. S. Franco* (English summary). Sobre errores en las investigaciones: *W. Edwards Deming* (English summary). El papel futuro del Instituto Internacional de Estadística: *S. A. Rice* (English summary). El programa de entrenamiento de la Dirección del Censo de los Estados Unidos: *C. L. Dedrick*. El Censo Mundial de Agricultura de 1950: *C. Taeuber*. La carrera de "Estadístico Matemático" en la Universidade del Litoral, Rosario, Argentina: *C. E. Dieulefait*. 1950 Continental Census, report on minimum suggestions submitted by the Sociedade Brasileira de Estatística. Plan para un Punto Focal Nacional de Intercambio de Estadística Internacional. First session of the Co-ordinating Board of the COTA: summary and appendices. Disposiciones legales sobre el Censo de 1950: Bolivia, Ecuador, Colombia, Peru, Guatemala, Republica Dominicana.

Journal of Experimental Education—

March 1949—Projections of three types of factor pattern: *C. W. Harris*. An experimental study of the effectiveness of filmstrips in teaching geometry: *D. A. Johnson*. An experimental evaluation of the relative effectiveness of certain audio-visual aids in vocational agriculture: *R. R. Bentley*.

Milbank Memorial Fund Quarterly—

July 1949—Characteristics of stable and non-stable families in the morbidity study in the eastern health district of Baltimore: *J. Downes, S. D. Collins* and *E. Jackson*. Cultural differences and census concepts: *C. L. Dedrick*. The sixth revision of the international lists of diseases and causes of death: *J. T. Marshall*. Internationally comparable statistics of food and agriculture: *C. Taeuber*. Problems in the collection and comparability of international labor statistics: *R. M. Woodbury*. Statistics of the distribution of family incomes by size: *J. B. D. Derksen*. General assessment of international statistics and outlook for the future: *W. R. Leonard*.

Quarterly Journal of Economics—

May 1949—Vilfredo Pareto (1848-1923): *J. A. Schumpeter*. The accelerator as a generator of steady growths: *S. S. Alexander*. Central banking in the light of recent British and American experience: *R. S. Sayers*. The British payments and exchange control system: *H. A. Shannon*. The A and P case : a study in applied economic theory: *M. A. Adelman*. The multiplier in a tri-fiscal economy: *H. M. Somers*.

Review of Economics and Statistics—

May 1949—Methodological issues in quantitative economics: Koopmans on the choice of variables to be studied and of methods of measurement: *R. Vining*. A reply: *T. C. Koopmans*. A rejoinder: *R. Vining*. Effect of income changes on the rate of saving: *G. Katona*. Various views on the monopoly problem: Introduction: *E. S. Mason*. Advertising outlays under oligopoly: *R. W. Jastram*. A dynamic aspect of the monopoly problem: *C. Kaysen*. The large firm and its suppliers: *M. A. Adelman*. The development of monopolistic competition and the monopoly problem: *A. Nicols*. Some final comments: *E. H. Chamberlin*. Some limits to the income elasticity of income tax yields: *W. Vickrey*. Note on "stocks" and "flows" in monetary interest theory: *W. Fellner* and *H. M. Somers*. Timing and flexibility of a public works program: *S. J. Maisel*.

BELGIUM—

Bulletin de Recherches Economiques et Sociales—

June 1949—La Belgique en 1948. (Whole number.)

DENMARK—

Nationaløkonomisk Tidsskrift—

Vol. 87, Part 1–2—Konjunkturomslag I U.S.A.?: *S. Laursen.* Om estimeringsproblemer for makromodeller: *P. N. Rasmussen.*

FRANCE—

Journal de la Société de Statistique de Paris—

May–June 1949—Contribution à la statistique des coûts de production en agriculture: *P. Raèous.* Les fluctuations de la construction d'habitations urbaines: *L. Flaus.* Chronique des salaires: *P. Delagrange.*

July–August 1949—Le recensement général des agents des services publics effectué en 1947: *M. Brichler.* Quelques errements dans l'utilisation économique des statistiques agricoles: *H. Brousse.* Remarques sur l'évolution économique et financière de la France depuis la libération: *A. Vene.* Chronique des statistiques agricoles: *M. Augé-Laribé.* Chronique des statistiques financières: *R. Jolivot.* Fluctuations longues et oscillations cycliques: *R. la Garrigue.*

Population—

April–June 1949—Le controle des recensements: *L. Henry.* Minimum vital et niveau d'existence des familles suivant le nombre d'enfants: *G. Malignac.* La famille devant le problème du logement: *R. Colin.* Le problème sucre-alcool et la population française: *L. Tabah.* Progrès technique et répartition professionelle de la population (2e partie): *A. Sauvy.*

GERMANY—

Weltwirtschaftliches Archiv—

Vol. 62, Part I—The concept of elasticity in economics: *T. Surányi-Unger.* Theorie der Währung: *A. Lösch.* Der Zins in der kapitalistischen und sozialistischen Wirtschaft: *G. Mackenroth.* Zur Liquidstätstheorie der Zinses: *E. Schneider.*

HOLLAND—

Statistica—

Vol. 3, No. 2—De betekenis van massaal consumentenondersoek voor het bedrijfsleven: *G. van der Wal.* Het probleem der waarnemingsfouten een massaal onderzoek: *J. Sittig.* Anthropologische opmerkingen over anthropo-sociologische feiten: *A. de Froe.* Statistiek van lineaire macromoleculen: *Th. Hekker.* (English summaries.)

ITALY—

Annali dell'Istituto di Statistica—

Vol. XXIV—La produttività del lavoro: *G. Lasorsa.*

Statistica—

January–March 1949—Nuovi procedimenti di calcolo degli indici di dissomiglianza e di connessione: *T. Salvemini.* La popolazione studentesca universitaria di Palermo dalle origini ai nostri giorni: *A. di Pasquale.*

SWITZERLAND—

Schweizerische Zeitschrift für Volkswirtschaft und Statistik—

April 1949—Die Berechnung des schweizerischen Volkseinkommens und ihr Erkenntniswert: *U. Zwingli.* Statistical investigation of capital formation: *J. Judik* and *R. Nötel.*

INTERNATIONAL—

International Labour Review—

March 1949—The guaranteed weekly wage in the British metal trades: *J. B. Jeffreys.*

April 1949—The I.L.O. manpower programme.

June 1949—Post-war trends in social security: income and security.

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part IV, 1948, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom

Agriculture and Fisheries, Ministry of. The co-operative marketing of horticultural produce in England and Wales. (Economic Series, 49.) London, H.M.S.O., 1948. 68 pp. 9½″. 1*s.* 3*d.*

Central Statistical Office. Studies in official statistics, No. 1. The interim index of industrial production. London, H.M.S.O., 1949. 52 pp. 8½″. 1*s.*

Civil Aviation, Ministry of. Report of the committee of the National Civil Aviation Consultative Council on accident investigation procedure ... London, H.M.S.O., 1948. Cmd. 7564. 46 pp. 9¾″. 9*d.*

Colonial Office

African labour efficiency survey, edited by C. H. Northcott. London, H.M.S.O., 1949. 123 pp. 9½″. 2*s.*

Colonial Annual Reports, 1947: Aden. 76 pp. 8½″. 2*s.* 6*d.* Barbados. 63 pp. 8½″. 2*s.* Basutoland. 79 pp. 8½″. 4*s.* 6*d.* Bechuanaland Protectorate. 56 pp. 8½″. 4*s.* 6*d.* Bermuda. 32 pp. 8½″. 1*s.* 6*d.* British Guiana. 98 pp. 8½″. 3*s.* British Honduras. 48 pp. 8½″. 2*s.* Brunei. 56 pp. 9½″. 4*s.* 6*d.* Cayman Islands. 31 pp. 8½″. 1*s.* 3*d.* Cyprus 56 pp. 8½″. 2*s.* 6*d.* Dominica. 56 pp. 8½″. 2*s.* Falkland Islands. 30 pp. 8½″. 2*s.* 6*d.* Fiji. 62 pp. 8½″. 2*s.* Gambia. 51 pp. 8½″. 2*s.* Gibraltar. 48 pp. 8½″. 2*s.* Gold Coast. 172 pp. 9½″. 5*s.* Hong Kong. 154 pp. 9½″. 12*s.* 6*d.* Jamaica. 163 pp. 9½″. 7*s.* 6*d.* Kenya. 108 pp. 8½″. 3*s.* Leeward Islands. 44 pp. 8½″. 2*s.* Malayan Union. 132 pp. 9½″. 7*s.* 6*d.* Mauritius. 107 pp. 8½″. 3*s.* 6*d.* Nigeria. 108 pp. 8½″. 3*s.* North Borneo. 58 pp. 8½″. 2*s.* Northern Rhodesia. 50 pp. 8½″. 2*s.* Nyasaland. 67 pp. 8½″. 2*s.* 6*d.* St. Helena. 44 pp. 8½″. 2*s.* St. Vincent. 66 pp. 8½″. 2*s.* Sarawak. 102 pp. 8½″. 5*s.* Seychelles. 31 pp. 8½″. 2*s.* Sierra Leone. 71 pp. 8½″. 2*s.* 6*d.* Singapore. 137 pp. 9½″. 7*s.* 6*d.* Swaziland. 71 pp. 8½″. 4*s.* Tonga. 24 pp. 8½″. 1*s.* 3*d.* Turks and Caicos Islands. 26 pp. 8½″. 1*s.* 3*d.* Trinidad and Tobago. 103 pp. 8½″. 5*s.* Uganda. 103 pp. 8½″. 3*s.* Zanzibar. 52 pp. 8½″. 1*s.* 6*d.* London, H.M.S.O., 1949.

Colonial Primary Products Committee. Second report, January, 1949. London, H.M.S.O., 1949. 60 pp. 9½″. 1*s.*

Production of fish in the colonial empire. London, H.M.S.O., 1949. 14 pp. 9¾″. 4*d.*

Commonwealth Economic Committee

Fruit: a summary of figures of production and trade relating to apples, pears, plums, peaches, oranges, grapefruit, lemons, bananas, grapes, raisins and currants, wine and canned fruit. London, H.M.S.O., 1949. 88 pp. 9¾″. 5*s.*

Meat: a summary of production trade and consumption relating to beef, live cattle, mutton and lamb, live sheep, bacon and hams, pork, live pigs, canned meat, offals. London, H.M.S.O., 1948. 91 pp. 9¾″. 5*s.*

General Register Office. Sickness in the population of England and Wales, 1944–1947, by Percy Stocks ... (Studies on Medical and Population Subjects, 2.) London, H.M.S.O., 1949. iv, 51 pp. 9¾″. 1*s.*

Health, Ministry of

The cost of house-building: first report of the committee of inquiry appointed by the Minister of Health. London, H.M.S.O., 1948. vi, 65 pp., table. 9½″. 1*s.* 3*d.*

Neonatal mortality and morbidity: report by a joint committee of the Royal College of Obstetricians and Gynaecologists and the British Paediatric Association. (Reports on Public Health and Medical Subjects, 94.) London, H.M.S.O., 1949. 92 pp. 9½″. 1*s.* 6*d.*

Health, Ministry of. Department of Health for Scotland. Ministry of Labour. Working party on the recruitment and training of nurses. Minority report by John Cohen. London, H.M.S.O., 1948. vi, 78 pp. 9½″. 1*s.* 6*d.*

Labour and National Service, Ministry of. Time rates of wages and hours of labour. 1st September, 1948. London, H.M.S.O., 1948. iv, 177 pp. 9½″. 3*s.*

Medical Research Council

Observations on the pathology of hydrocephalus, by Dorothy S. Russell. (Special Report Series, 265.) London, H.M.S.O., 1949. vi, 138 pp. 9½″. 6*s*.

The physique of young adult males, by W. J. Martin. (Memorandum, 20.) London, H.M.S.O., 1949. 66 pp. 9½″. 1*s*. 3*d*.

The Rh blood groups and their clinical effects, by P. L. Mollison, A. E Mourant and F. R. Race. (Memorandum, 19.) London, H.M.S.O., 1948. 74 pp. 9½″. 1*s*. 6*d*.

Studies in air hygiene, by R. B. Bourdillon, O. M. Lidwell and J. E. Lovelock, with others. (Special Report Series, 262.) London, H.M.S.O., 1948. [4] 356 pp. 9½″. 7*s*. 6*d*.

Vitamin A requirement of human adults; an experimental study of vitamin A deprivation in man: a report of the vitamin A sub-committee of the Accessory Food Factors Committee, compiled by E. M. Hume and H. A. Krebs. (Special Report Series, 264.) London, H.M.S.O., 1949. 145 pp. 9½″. 3*s*.

Scotland, Department of Health for

Diphtheria: report by the Infectious Diseases Sub-Committee of the Scientific Advisory Committee. Edinburgh, H.M.S.O., 1948. 31 pp. 9½″. 6*d*.

Report of the Committee on Scottish Building Costs. Edinburgh, H.M.S.O., 1948. iv, 66 pp. 1*s*. 3*d*.

Scottish Home Department. Industry and employment in Scotland, 1948. Edinburgh, H.M.S.O., 1949. Cmd. 7676. 79 pp. 9½″. 1*s*. 6*d*.

Trade, Board of

Distribution of industry. London, H.M.S.O., 1948. Cmd. 7540. 52 pp. 9½″. 1*s*.

A forecast of the future development of the Japanese economy and the opportunities for British trade with Japan, by H. A. Macrae ... London, H.M.S.O., 1948. 36 pp. 9½″. 9*d*.

Overseas Economic Surveys: Belgium, Oct., 1947. iv, 62 pp. 1948. 1*s*. 3*d*. British East Africa, March, 1948. vii, 93 pp. 1948. 2*s*. Canada, 1947. iv, 166 pp. 1948. 3*s*. Portugese East Africa, September, 1948. iv, 48 pp. 1949. 1*s*. Spain, April, 1948. vii, 179 pp. 1949. 3*s*. Switzerland, March, 1948. vi, 101 pp. 1948. 2*s*. Iran, April, 1948. vi, 50 pp. 1948. 1*s*. United States of America, February, 1948. iv, 207 pp. 1948. 3*s*. 6*d*. London, H.M.S.O., 1948–9. 9½″.

Report to the President of the Board of Trade by the United Kingdom Engineering Mission to Canada, 1948 ... London, H.M.S.O., 1949. 47 pp., maps. 9½″. 1*s*.

Reviews of Commercial Conditions: Chile, December, 1948. 27 pp. 6*d*. Colombia, April, 1948. 39 pp. 1*s*. El Salvador, June, 1948. 19 pp. 6*d*. Greece, September. 1948. 41 pp. 1*s*. Guatemala, July, 1948, 22 pp. 6*d*. London, H.M.S.O., 1948–49. 5 vols. 8½″.

(b) Other National and International Publications

Aden

Census of Aden, 1946. Report and tables by John Goepel. Aden, 1947. 130 pp. 9¾″ × 14½″.

Australia

National Health and Medical Research Council. Report of committee of inquiry into the decline of the birth rate, including reports of special investigations. (Special Report Series, 4.) Canberra, 1948. 98 pp. 9½″.

Brazil

Instituto Brasileiro de Geografia e Estatistica

Estudos de estatistica teórica e aplicada. Estatistica demográfica. 1. O aproveitamento das apurações do censo demográfico de 1940 para a determinação das correntes de migração interior. 69 pp. 2. Os cegos no Brasil segundo o censo demográfico do 1° de setembro de 1940. 96 pp. 3. Os surdos-mudos no Brasil segundo o censo demográfico de 1° de setembro de 1940. 28 pp. 4. Estimativas da taxa de natalidade para o Brasil, as Unidades da Federação e as principais capitais. 59 pp. Rio de Janeiro, 1948. 5 vols. 11″.

Recenseamento geral do Brasil (1° de setembro de 1940) Sinopse do censo agricola. Datos gerais. xi, 47 pp. Sinopse do censo comercial. Datos gerais. xx, 73 pp. Sinopse do censo industrial e do censo dos serviços. Datos gerais. xx, 74 pp. Rio de Janeiro, 1948. 3 vols. 11″.

Canada

Department of Trade and Commerce. Investment and inflation with special reference to the immediate post-war period. Ottawa, 1949. 290 fols. 12¾″.

Columbia

Contraloria General de la Republica. Direccion General del Censo. Primer censo industrial de Colombia, 1945. Departamento del Tolima. Departmento del Valle del Cauca. Intendencias y comisarias. Bogota, 1948. 3 vols. 9¾″.

Denmark

Det statistiske Departement

Folketaellingen 1940, Bind II. Befolkningens Fordeling efter Erhverv. (Statistisk Tabelvaerk, 5, A, 23.) Copenhagen, 1949. 60, 150 pp. 11½″.

Vurderingen til Grundskyld og Ejendomsskyld pr. 1 Oktober, 1945. (Statistisk Tabelvaerk, 5, E, 21.) Copenhagen, 1948. 54, 65, 49 pp. 11½″.

Finland

Finlands Officiella Statistik VI. Befolkningsstatistik, A. 21. Dödlighets- och livslängdstabeller för åren 1941–45. Helsinki, 1948. 25 pp. 11¾″.

France

Direction Générale de l'Urbanisme et de l'Habitation. Institut National de la Statistique et des Études Économiques. Résultats statistiques d'une enquête sur la propriété bâtie dans les communes rurales. Bretagne. 81 pp. Nord. 121 pp. Normandie. 89 pp. Sud. 105 pp. Sud- Est. 93 pp. Paris, 1947–8. 5 vols. 10½″.

Institut National de la Statistique et des Études Économiques.

Dénombrement de la population, 1946. Paris, 1947. 925 pp. 10¼″.

Le marché mondial du coton (Études et Documents. C. 2). Paris, Presses Universitaires de France, 1948. 270 pp. 9½″. 600*f.*

Résultats statistiques du recensement générale de la population effectué le 10 mars, 1946. Vol. I. Population légale ... Paris, 1948. 162 pp., map. 10½″.

Germany

Deutsche Wirtschaftskommission für die sowjetische Besatzungszone. Statistisches Zentralamt. Volks- und Berufszählung vom 29 Oktober, 1946, in der sowjetischen Besatzungszone Deutschlands. Band I: Amtliches Gemeindeverzeichnis. 104 pp. Map. Band II: Gemeindestatistik. Heft 1: Land Brandenburg. 40 pp. Heft 2: Land Mecklenburg. 43 pp. Heft 3: Land Sachsen-Anhalt. 47 pp. Heft 4: Land Thüringen. 42 pp. Heft 5: Land Sachsen. 45 pp. Berlin, 1948. 6 vols. 11½″.

Statistisches Amt des Vereinigten Wirtschaftsgebietes. Volkszählung vom 29 Oktober, 1946. Amtliches Gemeindeverzeichnis für das Vereinigte Wirtschaftsgebiet ... Stuttgart, W. Kohlhammer-Verlag, 1949. 92 pp. 11½″.

International Labour Office.

Cost-of-living statistics: methods and techniques for the post-war period ... (Studies and Reports, N.S. 7, Pt. 2.) 1947. [6] 58 pp. 9¼″. 1*s.* 6*d.*

Employment, unemployment and labour force statistics: a study of methods ... (Studies and Reports, N.S. 7, Pt. 1.) 1948. [6] 130 pp. 9¼″. 3*s.* 9*d.*

Housing and employment. (Studies and Reports, N.S. 8.) 1948. iv, 148 pp. 9½″. 3*s.* 9*d.*

Methods of statistics of industrial injuries ... (Studies and Reports, N.S. 7, Pt. 3.) 1947. [6] 34 pp. 9½″. 1*s.*

The second session of the Permanent Migration Committee. (Studies and Reports, N.S. 10.) 1948. [4] 172 pp. 9½″. 5*s.* Geneva, 1947–8 (London, Staples Press).

Luxemburg (Grand Duchy)

Office de la Statistique Générale. Recensement de la population du 31 décembre, 1947. Premiers résultats et liste alphabétique des localités. (Publications de l'Office de la Statistique, 78.) Luxemburg, 1948. 90 pp. 11¼″.

Mozambique

Repartição Técnica de Estatistica. Recenseamento da população não indigena em 12 de Junho de 1945. Lourenço Marques, 1947. xxx, [8] 235 pp. 10¼″.

Netherlands

White paper regarding the measures for the currency rehabilitation in the Netherlands. The Hague, Government Publishers, 1947. (London, H.M.S.O., 1948.) 219 pp. 9½″. 10*s*.

New Zealand.

Census and Statistics Department. Population Census 1945. Vol. I. Increase and location of population. 1947. xiii, 68 pp. Vol. II. Island territories. Cook Islands and Niue. Tokelau Islands. Western Samoa. 1947, 14 pp. Appendix A. Census of poultry. 1947. 14 pp. Interim returns of ages, marital status, religious profession, birthplaces, duration of residence of overseas-born, race, war service, industries, occupations, occupational status, travelling time. [4] 26 pp. Wellington, 1947–48. 4 vols. 12″.

Poland

Central Statistical Office. Statistics of industrial and handicraft plants on the basis of applications for registration cards, 1946. [Text in Polish and English.] (Statistics of Poland D. 5.) Warsaw, 1948. xxx, 67 pp. 11¾″.

Southern Rhodesia

Central African Statistical Office. Sixth report on the census of industrial production, 1938–1946. Salisbury, 1948. [5] 39 fols. 13″.

Sweden

Statistiska Centralbyrån. Folkräkningen den 31 december, 1945, av Statistiska Centralbyrån. I. Areal och folkmängd inom särskilda fövaltningsomraden m.m.befolkningsagglomerationer. II: 1, 2, 3. III: 1, Partiella undersökningar (Tolvtedelssamplingen). Stockholm, 1947–49. 5 vols. 9½″.

Singapore

Department of Social Welfare. A social survey of Singapore: a preliminary study of some aspects of social conditions in the municipal area of Singapore, December, 1947. viii, 165 pp., table, map. 8¼″.

Switzerland

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REGISTRATION OF THE UNITED KINGDOM

No. I.—ENGLAND AND WALES

BIRTHS, DEATHS *and* MARRIAGES *registered in the Calendar years* 1944–1948 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* per* 1,000 *persons living.* (*Deaths under* 1 *year of age; rate per* 1,000 *related Live Births. Stillbirths per* 1,000 *births.*)

Years	1944†		1945		1946		1947†		1948†	
Estimated Mid-year Popln. in thousands ‡	42,449		42,636		42,700		43,050		43,502	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	744,843	*17·5*	685,273	*16·1*	819,894	*19·2*	886,820	*20·6*	777,618	*17·9*
Stillbirths	21,212	*28*	19,452	*28*	22,915	*27*	21,916	*24*	18,415	*23*
Deaths[1]	492,176	*11·6*	488,108	*11·4*	492,090	*11·5*	517,615	*12·0*	470,282	*10·8*
Marriages	302,714	*7·1*	397,626	*9·3*	385,606	*9·0*	401,210	*9·3*	393,098	*9·0*
Infant Mortality ..	33,455	*45*	31,959	*46*	33,541	*43*	36,849	*41*	26,635	*34*
Effective reproductive rate[2]	*0·996*		*0·909*		*1·103*		*1·205*		*1·070*	
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.–Mar.	183,973	*17·4*	177,946	*16·9*	181,220	*17·2*	241,530	*22·8*	202,184	*18·7*
Apr.–June	199,039	*18·9*	175,221	*16·5*	203,808	*19·1*	235,196	*21·9*	203,711	*18·8*
July–Sept.	183,594	*17·2*	167,807	*15·6*	213,051	*19·8*	216,508	*20·0*	192,073	*17·6*
Oct.–Dec.	178,237	*16·7*	164,299	*15·3*	221,815	*20·6*	193,586	*17·8*	179,680	*16·4*
	Stillbirths									
Jan.–Mar.	5,587	*29*	5,190	*28*	5,202	*28*	6,347	*26*	5,043	*24*
Apr.–June	5,496	*27*	4,853	*27*	5,767	*28*	5,831	*24*	4,733	*23*
July–Sept.	5,018	*27*	4,676	*27*	5,831	*27*	5,073	*23*	4,433	*23*
Oct.–Dec.	5,111	*28*	4,733	*28*	6,115	*27*	4,665	*24*	4,206	*23*
	Deaths[1] (excluding Stillbirths)									
Jan.–Mar.	146,039	*13·8*	157,743	*15·0*	155,227	*14·7*	181,736	*17·1*	132,705	*12·3*
Apr.–June	115,548	*10·9*	110,972	*10·4*	113,908	*10·7*	118,015	*11·0*	110,356	*10·2*
July–Sept.	107,251	*10·1*	97,157	*9·0*	100,409	*9·3*	97,099	*8·9*	101,664	*9·3*
Oct.–Dec.	123,335	*11·6*	122,236	*11·4*	122,546	*11·4*	120,765	*11·1*	125,557	*11·5*
	Marriages									
Jan.–Mar.	62,879	*5·9*	76,975	*7·3*	78,237	*7·4*	75,241	*7·1*	94,197	*8·7*
Apr.–June	82,583	*7·8*	99,709	*9·4*	101,213	*9·5*	109,146	*10·2*	91,977	*8·5*
July–Sept.	82,312	*7·7*	119,447	*11·1*	109,750	*10·2*	119,426	*11·0*	122,201	*11·2*
Oct.–Dec.	74,940	*7·0*	101,495	*9·5*	96,406	*9·0*	97,397	*9·0*	84,723	*7·8*
	Infant Mortality									
Jan.–Mar.	10,152	*58*	10,676	*60*	9,637	*56*	12,561	*55*	8,358	*41*
Apr.–June	7,983	*42*	7,227	*41*	7,657	*40*	9,195	*40*	6,336	*31*
July–Sept.	7,376	*40*	6,429	*37*	7,100	*35*	7,141	*32*	5,468	*28*
Oct.–Dec.	7,944	*43*	7,627	*45*	9,147	*43*	7,952	*38*	6,473	*35*

* All rates are based on the estimated population as at the middle of the corresponding year.

† Provisional figures.

‡ Inclusive of non-civilians of England and Wales temporarily abroad; and exclusive of non-civilians of Foreign Countries, Dominions, etc., temporarily in England and Wales.

[1] Including deaths of non-civilians registered in England and Wales.

[2] Based not upon current, but upon estimated future mortality.

No. II.—SCOTLAND

BIRTHS, DEATHS *and* MARRIAGES *registered in the Calendar years* 1944–1948, *and in the Quarters of those years. Numbers, Annual and Quarterly Rates, per* 1,000 *persons living.** (*Deaths under* 1 *year of age, rate per* 1,000 *Live Births; Stillbirths per* 1,000 *Births.*)

Years	1944		1945		1946		1947†		1948†	
Estimated Popln. in thousands‡	4,654		4,674		4,901		5,139		5,169	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	95,941	*18·5*	86,932	*16·9*	104,413	*20·3*	113,147	*22·0*	100,343	*19·4*
Stillbirths	3,221	*32*	2,949	*33*	3,483	*32*	3,563	*31*	2,960	*29*
Deaths	64,603	*13·6*	62,655	*13·2*	64,605	*13·1*	66,200	*12·9*	60,979	*11·8*
Marriages	37,017	*7·1*	48,642	*9·4*	45,851	*8·9*	44,411	*8·6*	43,741	*8·5*
Infant Mortality ..	6,237	*65*	4,889	*56*	5,621	*54*	6,309	*56*	4,486	*45*
Quarters					**Live Births in the Quarters of each Calendar Year**					
Jan.–Mar.	23,231	*18·0*	21,644	*16·9*	22,910	*18·1*	30,479	*24·1*	25,324	*19·8*
Apr.–June	25,618	*19·9*	22,579	*17·6*	25,851	*20·2*	30,366	*23·7*	26,561	*20·7*
July–Sept.	23,405	*17·9*	21,344	*16·4*	27,033	*20·9*	27,028	*20·8*	24,389	*18·7*
Oct.–Dec.	23,687	*18·2*	21,365	*16·5*	28,619	*22·1*	25,274	*19·5*	24,069	*18·5*
					Stillbirths					
Jan.–Mar.	816	*34*	746	*33*	810	*34*	1,020	*32*	758	*29*
Apr. June	874	*33*	774	*33*	864	*32*	911	*29*	774	*28*
July–Sept.	751	*31*	671	*30*	854	*31*	853	*31*	709	*28*
Oct.–Dec.	780	*32*	758	*31*	955	*32*	779	*30*	719	*29*
					Deaths (excluding Stillbirths)					
Jan.–Mar.	17,988	*15·2*	18,674	*16·0*	19,741	*16·3*	21,198	*16·8*	16,825	*13·1*
Apr. June	15,288	*12·9*	15,364	*13·0*	15,185	*12·2*	15,653	*12·2*	14,698	*11·4*
July–Sept.	14,745	*12·3*	13,082	*10·9*	13,594	*10·8*	13,447	*10·4*	13,652	*10·5*
Oct.–Dec.	16,582	*13·9*	15,535	*12·9*	16,085	*12·7*	15,902	*12·3*	15,804	*12·2*
					Marriages					
Jan.–Mar.	9,020	*7·0*	9,747	*7·6*	9,985	*7·9*	9,508	*7·5*	9,964	*7·8*
Apr.–June	9,243	*7·2*	12,362	*9·6*	11,483	*9·0*	11,254	*8·8*	10,492	*8·2*
July–Sept.	9,847	*7·5*	13,622	*10·5*	12,857	*9·9*	12,762	*9·8*	13,518	*10·4*
Oct.–Dec.	8,907	*6·8*	12,911	*10·0*	11,526	*8·9*	10,887	*8·4*	9,767	*7·5*
					Infant Mortality					
Jan.–Mar.	1,756	*76*	1,465	*68*	1,579	*69*	1,915	*63*	1,303	*51*
Apr.–June	1,462	*57*	1,234	*55*	1,221	*47*	1,609	*53*	1,144	*43*
July–Sept.	1,510	*65*	979	*46*	1,222	*45*	1,353	*50*	972	*40*
Oct.–Dec.	1,509	*64*	1,211	*57*	1,599	*56*	1,432	*57*	1,067	*44*

* Death rates from 1944 to 1946 are based on civilian deaths and civilian population; from 1947 they are based on all deaths registered in Scotland, and on total population. Birth and marriage rates are based on total population (including persons in the Services).

† Provisional figures.

‡ 1944–46 = Mean of four quarterly estimates of the civilian population.
1947–48 = Estimated mid-year total population.

No. III.—NORTHERN IRELAND

BIRTHS, DEATHS *and* MARRIAGES *registered in the Calendar years* 1944–1948 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* per* 1,000 *persons living.* (*Deaths under* 1 *year of age; rate per* 1,000 *Live Births.*)

Years	1944		1945		1946		1947		1948†	
Estimated Mid-Year Popln. in thousands‡	1,316		1,317		1,333		1,339		1,347	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	30,900	*23·5*	29,007	*22·0*	30,134	*22·6*	31,254	*23·3*	29,532	*21·9*
Stillbirths[1]					Not registered					
Deaths	16,791	*12·8*	16,264	*12·3*	16,666	*12·5*	16,913	*12·6*	15,132	*11·2*
Marriages	9,508	*7·2*	10,452	*7·9*	9,801	*7·4*	9,517	*7·1*	9,375	*7·0*
Infant Mortality	2,083	*67*	1,975	*68*	1,626	*54*	1,658	*53*	1,348	*46*
Quarters					**Live Births in the Quarters of each Calendar Year**					
Jan.-Mar.	7,612	*23·2*	7,332	*22·2*	7,278	*21·8*	8,318	*24·9*	7,412	*22·1*
Apr. June	8,312	*25·3*	7,509	*22·8*	7,924	*23·8*	8,539	*25·5*	8,030	*23·9*
July-Sept.	7,503	*22·8*	7,188	*21·8*	7,646	*22·9*	7,516	*22·4*	7,262	*21·5*
Oct.-Dec.	7,473	*22·7*	6,978	*21·1*	7,286	*21·9*	6,881	*20·5*	6,828	*20·2*
					Deaths‡ (excluding Stillbirths)					
Jan.-Mar.	5,108	*15·5*	5,249	*15·9*	5,435	*16·3*	5,902	*17·7*	4,314	*12·8*
Apr. June	4,072	*12·4*	3,962	*12·0*	4,026	*12·1*	4,011	*12·0*	3,821	*11·4*
July-Sept.	3,483	*10·6*	3,339	*10·1*	3,280	*9·8*	3,276	*9·8*	3,325	*9·9*
Oct.-Dec.	4,128	*12·6*	3,714	*11·3*	3,925	*11·8*	3,724	*11·1*	3,672	*10·9*
					Marriages					
Jan.-Mar.	2,044	*6·2*	2,140	*6·5*	2,224	*6·7*	1,820	*5·5*	2,157	*6·4*
Apr.-June	2,532	*7·7*	2,806	*8·5*	2,509	*7·5*	2,517	*7·5*	2,167	*6·4*
July-Sept.	2,646	*8·1*	2,984	*9·0*	2,840	*8·5*	2,905	*8·6*	2,962	*8·8*
Oct.-Dec.	2,286	*7·0*	2,522	*7·6*	2,228	*6·7*	2,275	*6·8*	2,089	*6·2*
					Infant Mortality[2]					
Jan. Mar.	650	*85*	700	*95*	464	*64*	497	*60*	395	*53*
Apr.-June	501	*60*	504	*67*	436	*55*	442	*52*	354	*44*
July-Sept.	420	*56*	384	*53*	331	*43*	314	*42*	301	*41*
Oct. Dec.	508	*68*	382	*55*	388	*53*	405	*59*	298	*44*

* Rates are based on Civilian population only.
† Provisional figures.
‡ Civilians only.
[1] Stillbirths are not registered in Northern Ireland. The birth of one living child and one stillborn, or one living child and two stillborn, is counted as one birth.
[2] The annual figures have been corrected, so that slight differences appear between them and the aggregates of the quarterly figures.

No. IV.—EIRE

BIRTHS, DEATHS *and* MARRIAGES *registered in the Calendar years* 1944–1948 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* per* 1,000 *persons living.* (*Deaths under* 1 *year of age, rate per* 1,000 *Live Births.*)

Years	1944		1945		1946		1947		1948†	
Estimated Mid-year Popln. in thousands	2,941		2,983		2,963		2,972		2,997	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	65,425	*22·2*	66,861	*22·4*	67,922	*22·9*	68,978	*23·2*	65,584	*21·9*
Stillbirths‡					Not registered					
Deaths	45,128	*15·3*	42,762	*14·3*	41,457	*14·0*	44,061	*14·8*	36,502	*12·2*
Marriages	16,772	*5·7*	17,301	*5·8*	17,525	*5·9*	16,290	*5·5*	16,331	*5·4*
Infant Mortality ..	5,198	*79*	4,739	*71*	4,390	*65*	4,687	*68*	3,212	*49*
					Live Births in the Quarters of each Calendar Year					
Quarters										
Jan.–Mar.	17,066	*23·2*	16,073	*21·6*	17,005	*23·0*	17,537	*23·6*	16,177	*21·6*
Apr.–June	17,663	*24·0*	17,940	*24·1*	18,122	*24·5*	18,946	*25·5*	18,006	*24·0*
July–Sept.	15,749	*21·4*	17,244	*23·1*	17,230	*23·3*	17,424	*23·5*	16,410	*21·9*
Oct.–Dec.	14,947	*20·3*	15,604	*20·9*	15,565	*21·0*	15,071	*20·3*	14,991	*20·0*
					Deaths (excluding Stillbirths)					
Jan.–Mar.	14,814	*20·1*	13,715	*18·4*	13,042	*17·6*	15,975	*21·5*	10,653	*14·2*
Apr.–June	11,074	*15·0*	10,754	*14·4*	10,515	*14·2*	11,161	*15·0*	9,419	*12·6*
July–Sept.	8,785	*11·9*	8,892	*11·9*	8,539	*11·5*	8,288	*11·2*	7,886	*10·5*
Oct.–Dec.	10,455	*14·2*	9,401	*12·6*	9,361	*12·6*	8,637	*11·6*	8,514	*11·4*
					Marriages					
Jan.–Mar.	4,222	*5·7*	3,697	*5·0*	4,478	*6·0*	3,769	*5·1*	3,673	*4·9*
Apr.–June	4,069	*5·5*	4,594	*6·2*	4,147	*5·6*	4,023	*5·4*	4,058	*5·4*
July–Sept.	4,599	*6·2*	4,928	*6·6*	5,161	*7·0*	4,899	*6·6*	4,963	*6·6*
Oct.–Dec.	3,882	*5·3*	4,082	*5·5*	3,739	*5·0*	3,599	*4·8*	3,637	*4·9*
					Infant Mortality					
Jan.–Mar.	1,719	*101*	1,486	*92*	1,408	*83*	1,639	*93*	1,027	*63*
Apr.–June	1,241	*70*	1,097	*61*	1,051	*58*	1,241	*66*	831	*46*
July–Sept.	1,021	*65*	1,032	*60*	948	*55*	832	*48*	629	*38*
Oct.–Dec.	1,217	*81*	1,124	*72*	983	*63*	975	*65*	725	*48*

* Rates are based on the total estimated population as at the middle of the corresponding year.

† Provisional figures.

‡ Stillbirths are not registered in Eire. The birth of one living child and one stillborn, or of one living child and two stillborn, is counted as one birth.

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Foundations of Probability and Statistical Inference*

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1. *The Problem*

Recent work in econometrics and economic statistics, especially some of the work done by members of the Cowles Commission,† makes it likely that in the not too distant future there may be a fresh interest in questions arising relating to the *probability of a theory*. On the basis of certain empirical evidence we may, for instance, ask about the probability of the Keynesian as compared to the "classical" economic system.‡

The following essay presents and exemplifies a new theory of probability, which forms a part of an ambitious system of inductive logic developed by the logician and philosopher Rudolf Carnap.§ We feel that his ideas may lead at least to partial and as yet tentative answers to questions like the probability of a theory or hypothesis.||

We will in this section briefly indicate the existing systems of statistical inference and their relations to foundations of probability. The remainder of the essay will then be devoted to a presentation of the solution proposed by Carnap and to the applications of the theory in statistics.

The following approaches to *foundations of probability* are in current use by statisticians and theoreticians of probability:

(*a*) Probability related to relative frequency.¶ Probability is sometimes defined as the limit of the relative frequency in a random series; or otherwise related to the relative frequency in a hypothetical infinite population.

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† L. R. Klein: "The uses of econometric models as a guide to economic policy," *Econometrica*, vol. 15, 1947, pp. 111 ff. M. A. Girshik and T. Haavelmo: "Statistical analysis of the demand for food," *ibid.*, vol. 15, 1947, pp. 79 ff. T. Haavelmo: "Quantitative research in agricultural economics. The dependence between agriculture and the national economy," *Journal of Farm Economics*, vol. 30, 1947, pp. 910 ff. G. Cooper: "The role of econometric models in economic research," *ibid*, vol. 30, 1948, pp. 101 ff. See also for a different method:—G. Tintner: "Multiple regression for systems of equations," *Econometrica*, vol. 14, 1946, pp. 5 ff.

‡ See, e.g., L. R. Klein: *The Keynesian Revolution*. New York, 1947.

§ R. Carnap: "On inductive logic," *Philosophy of Science*, vol. 12, 1945, pp. 72 ff. "The two concepts of probability," *Philosophy and Phenomenological Research*, vol. 5, 1945, pp. 513 ff. "Remarks on induction and truth," *ibid.*, vol. 6, 1946, pp. 590 ff. "On the application of inductive logic," *ibid.*, vol. 8, 1947, pp. 133 ff. "Probability as a guide in life," *Journal of Philosophy*, vol. 44, 1947, pp. 141 ff.

|| E. Nagel: "Principles of the theory of probability," *International Encyclopaedia of Unified Science*, vol. 1, no. 6, Chicago, 1939, pp. 60 ff.

¶ R. von Mises: *Wahrscheinlichkeitsrechnung*. Vienna, 1931. *Probability, Statistics and Truth*. London, 1939. H. Reichenbach: *Wahrscheinlichkeitslehre*. Leiden, 1935. *Experience and Prediction*. Chicago, 1938.

(*b*) Probability as a frequency on the basis of set-theory.* Empirical probability is (mostly rather vaguely) identified with an abstract concept defined in terms of set theory.

(*c*) "Logical" concepts. Here we have the classical concept of probability as given by Laplace and its modern adaptations,† e.g. the theories of Keynes‡ and Jeffreys.§ Probability is defined as relating to propositions.

In *statistical inference* we have different approaches, which sometimes lead to different answers to empirical statistical problems:—

(*a*) An adaptation of the classical approach proposed by Jeffreys. Statistical inference bases itself here upon a modification of Bayes' theorem.‖

(*b*) Theories which do not use Bayes' theorem or its modification. Various approaches are in use by many statisticians:

(1) Fisher's fiducial theory. Fiducial distributions are derived from sampling distributions. This approach has, perhaps, similarity to the one proposed by Carnap.¶ Barnard's theory is closely related to Fisher's fiducial approach.**

(2) The Neyman-Pearson theory†† uses only a frequency interpretation of probability. Two types of error are distinguished for testing hypotheses. This theory is often not applicable because of the appearance of nuisance parameters.‡‡

(3) Wald's theory of statistical decision functions.§§ Statistical inference is based upon the assumption of a risk function which makes it possible to evaluate the empirical consequences of decisions. This theory is derived from the Neyman-Pearson approach.

None of these methods seems to be entirely satisfactory. What is desired is a "pure" theory of induction which would permit us to evaluate ultimately the degree of confirmation of given statistical hypotheses without any pragmatic considerations, as implied in Wald's theory.

This is particularly important in economic statistics, and more generally in the statistical practice relating to social phenomena. I do not believe that there a sensible risk function can be

* A. Kolmogoroff: *Grundbegriffe der Wahrscheinlichkeitsrechnung*. Berlin, 1933. J. Neyman: "Outline of a theory of statistical estimation based on the classical theory of probability," *Philosophical Transactions*, series A, vol. 236, 1937, pp. 333 ff. T. Haavelmo: "The probability approach in econometrics," *Econometrica*, vol. 12, 1944 supplement.

† J. V. Uspensky: *Introduction to Mathematical Probability*. New York, 1937.

‡ J. M. Keynes: *A Treatise on Probability*. London, 1921. B. Russel: *Human Knowledge*, Its Scope and Limitations. London, 1948, pp. 353 ff.

§ H. Jeffreys: *Theory of Probability*. 2nd ed. Oxford, 1948.

‖ Maria-Pia Geppert: "Das Bayessche Rueckschlussproblem," *Deutsche Mathematik*, vol. 7, 1942, pp. 1 ff. C. Gini: "I pericoli della statistica," *Societa Italiana di Statistica Atti della Riunione de Pisa*, 1939. "I testi di significativita," *ibid.* H. von Schelling: "Ma non puo dopo essero caso,"? *ibid.* C. Gini: "Gedanken zum Theorem von Bernouilli" *Schweizerische Zeitschrift fuer Volkswirtschaft und Statistik*, vol. 82, 1926, pp. 27 ff. O. Anderson: "Zum Problem der Wahrscheinlichkeit a posteriori in der Statistik," *ibid.*, vol. 83, 1947, pp. 33 ff.

¶ R. A. Fisher: "Inverse probability," *Proceedings Cambridge Philosophical Society*, vol. 26, 1930, pp. 528 ff. "The logic of inductive inference," *Journal of the Royal Statistical Society*, vol. 98, 1935, pp. 39 ff. *The Design of Experiments*. 4th ed. London, 1947, pp. 179 ff. See also: -M. G. Kendall: *The Advanced Theory of Statistics*, vol. 2, London, 1946, pp. 85 ff.

** G. A. Barnard: "Statistical Inference," to be published in *J. R. Statist. Soc.*, B, XI, 2, 1949.

†† J. Neyman and E. S. Pearson: "On the use and interpretation of certain test criteria for purposes of statistical inference," *Biometrika*, vol. 20A, 1928, pp. 175 ff, 263 ff. "On the problem of the most efficient test of statistical hypotheses," *Phil. Transactions*, series A, vol. 231, 1933, pp. 289 ff. See also:— M. G. Kendall: *op. cit.*, pp. 269 ff. H. Cramer: *Mathematical Methods of Statistics*. Princeton, 1946, pp. 525 ff.

‡‡ H. Hotelling: "The selection of variates for use in prediction with some comments on the general problem of nuisance parameters," *Annals of Mathematical Statistics*, vol. 11, 1940, pp. 271 ff. M. G. Kendall, *op. cit.*, pp. 134 ff.

§§ A Wald: "On the principles of statistical inference," *Notre Dame Mathematical Lectures*, No. 1, Notre Dame Ind., 1942. "Contributions to the theory of statistical estimation and testing hypotheses," *Annals of Mathematical Statistics*, vol. 10, 1939. "Statistical decision functions which minimize the maximum risk;" *Annals of Mathematics*, vol. 46, 1945, pp. 265 ff. "An essentially complete class of admissible decision functions," *Annals of Mathematical Statistics*, vol. 18, 1947, pp. 549 ff. *Sequential Analysis*, New York, 1947, pp. 20 ff. "Foundations of a General Theory of Sequential Decision Functions," *Econometrica*, vol. 15, 1947, pp. 279 ff. C. West Churchman: *Theory of Experimental Inference*. New York, 1948.

constructed because of the multiplicity of conflicting ends pursued in society. It is by no means obvious how a single risk function can be constructed.

Apart from this particular difficulty there is a more general objection to Wald's pragmatic approach. How should we, for instance, construct a risk function which expresses the consequences of accepting a false scientific theory? This problem is relevant to statistical applications in all sciences, natural and social. Wald's theory remains still valid for industrial and similar applications, for which it was primarily constructed. Gain and loss can sometimes be expressed in monetary terms and the above objections lose their force.

We believe that Carnap's theory outlined below gives a "pure," i.e. non-pragmatic approach to these problems, and may point the way to a satisfactory solution of many philosophical and practical questions connected with statistical inference.

It should be pointed out that Carnap's theory is restricted to the theory of attributes. It has not yet been extended to a coordinate language or the quantitative language of physics and other sciences, e.g. economics. Hence it cannot yet deal with some important and puzzling statistical problems like the Fisher-Behrens problem. But it seems to offer very interesting new aspects in statistical inference in the restricted field in which it is applicable. There is also hope that the theory will eventually be generalized to deal with a greater variety of statistical problems.

The subject discussed in this essay belongs to logic or semantics, i.e. it is concerned with questions of *meaning*. From this ought to be distinguished problems of statistical methodology, like, for instance: how to design experiments, how to plan sample surveys, how to compute index numbers, etc.

Rudolf Carnap has, in some remarkable essays, made fundamental contributions to the clarification of certain problems in probability theory. He distinguishes between two concepts of probability.* Probability$_1$ is the degree of confirmation, a purely logical concept without empirical implications. This concept belongs to inductive logic. Like deductive logic, probability$_1$ is purely logical and not factual or empirical. It lays down certain logical and mathematical rules by which the degree of confirmation of a hypothesis can be evaluated on the basis of a given evidence. This will be illustrated in the subsequent sections.

The other probability concept is called probability$_2$. It is related to empirical frequency. It may, for instance, be defined in the sense of the theories of von Mises.†

It is believed that the concept of probability$_1$ or degree of confirmation may also be utilized to deal with some problems in statistical inference. This will be demonstrated in the latter parts of this essay.

2. *Carnap's Theory of Probability$_1$ (Degree of Confirmation) for Simple Attributes and Simple Properties*

Carnap gives in his essay‡ a complete theory of various types of the degree of confirmation. A positive and a comparative concept of confirmation are discussed first. These are both non-metrical. We will concern ourselves here only with the metrical concept of the degree of confirmation as applied to the theory of attributes.

(a) *Some Semantical Concepts*

Since probability$_1$ is a purely logical non-empirical idea, we have to introduce into its definition a few modern semantical concepts.§ We deal throughout with a finite language system, i.e. a system with a finite number of *individuals*. There are altogether N individuals designated by $a_1, a_2, \ldots, a_N$. Carnap's extension to an infinite language will be omitted here. In the special case of the theory of simple attributes (alternatives) there are also p *attributes* $P_1, P_2, \ldots P_p$. The logical symbols used are: $\sim$ for not, $\vee$ for disjunction (or), . for conjunction (and). Each simple attribute can assume only the two values P and $\sim P$.

* R. Carnap: "The two concepts of probability," *op. cit.*
† R. von Mises: *Probability, Statistics and Truth.* London, 1939.
‡ R. Carnap: "On inductive logic," *op. cit.*
§ R. Carnap.: *Introduction to Semantics.* Cambridge, Massachusetts, 1942.

Example 1.—Take, for instance, $N = 3$. Let, e.g., a_1, a_2, a_3 denote to the first, second and third throw of a coin. Let $p = 1$. Then P_1 denotes the property of a head appearing in the throw in question. $\sim P_1$ stands for the appearance of a tail.

P_1a_1 means that a head is obtained in the first throw of the coin. Propositions like this are called atomic sentences. $\sim P_1a_3$ means that no head (i.e. a tail) is obtained in the third throw of the coin.

$(P_1a_1) \vee (\sim P_1a_3)$ means that a head is obtained in the first throw or a tail in the third throw. $(P_1a_1) \,.\, (P_1a_2) \,.\, (\sim P_1a_3)$ means the following situation: A head is obtained in the first throw, a head in the second throw and a tail in the third throw, etc.

(b) *Q-properties*

The Q-properties are conjunctions of all the p simple attributes $P_1, P_2, \ldots P_p$ and their negations. We have $k = 2^p$ Q-properties.

Example 2.—Consider again the situation of Example 1. We have only one simple attribute P_1. Now we form:

$$Q_1 = P_1$$

$$Q_2 = \sim P_1.$$

From the previous interpretation of P_1 in Example 1 it follows that Q_1 designates heads and Q_2 tails.

It should be noted that the total number of Q-properties is here $k = 2^1 - 2$, since $p = 1$.

We have, e.g., $P_1a_1 = Q_1a_1$ (heads in the first throw). $\sim P_1a_3 = Q_2a_3$ (tails in the third throw). $(P_1a_1) \vee (\sim P_1a_2) = (Q_1a_1) \vee (Q_2a_2)$ (heads in the first throw, or tails in the second). $(P_1a_1) \,.\, (\sim P_1a_3) = (Q_1a_2) \,.\, (Q_2a_3)$ (heads in the first throw, and tails in the third, etc.).

Example 3.—Consider now the case of a language with $N = 2$ individuals and $p = 2$ simple attributes $\sim$; a_1, a_2, stand now, e.g. for *two* throws with two coins, a nickel and a dime. Let P_1 be the property of the nickel showing a head and $\sim P_1$ the property of the nickel showing a tail. Let P_2 be the property of the dime showing a head and $\sim P_2$ the property of the dime turning up a tail. The Q-properties are now 4 in number: ($p = 2$, $k = 2^2 = 4$):

$$(P_1) \,.\, (P_2) = Q_1$$

$$(P_1) \,.\, (\sim P_2) = Q_2$$

$$(\sim P_1) \,.\, (P_2) = Q_3$$

$$(\sim P_1) \,.\, (\sim P_2) = Q_4.$$

The interpretation of the Q-properties is as follows: Q_1 = nickel head, dime head. Q_2 = nickel head, dime tail. Q_3 = nickel tail, dime head. Q_4 = nickel tail, dime tail.

Hence we have, for instances: $(P_1a_1) \,.\, (P_2a_1) = Q_1a_1$. Nickel head, dime head, in the first throw. $(P_1a_2) \,.\, (\sim P_2a_2) = Q_2a_2$. Nickel head, dime tail in the second throw, etc.

(c) *State Description*

A state description is a conjunction (*And*-connection) of atomic sentences relating to all individuals in a given language. They are most conveniently expressed in terms of the Q-properties. The set of all possible state descriptions expressed in terms of the Q-properties is called the *Q-matrix*. (This term is used in a sense which is different from Carnap's.) The number of state descriptions in a language with N individuals and p simple attributes is k^N, where $k = 2^p$.

Example 4.—Take again $p = 1$, $N = 3$. The Q-matrix of all possible state descriptions is now given by (Example 1):

Q-Matrix

State descriptions		*Q-properties* a_1	a_2	a_3
i_1	.	1	1	1
i_2	.	1	1	2
i_3	.	1	2	1
i_4	.	1	2	2
i_5	.	2	1	1
i_6	.	2	1	2
i_7	.	2	2	1
i_8	.	2	2	2

Since $p = 1$, we have $k = 2^1 = 2$. Also, $N = 3$, since there are three individuals. Hence the total number of state descriptions is $2^3 = 8$.

We interpret the state descriptions with the help of the examples given in Example 1 and 2: Q_1 means heads, Q_2 tails. a_1, a_2, a_3 are three consecutive throws with a coin.

For example, i_1 means that we obtain heads in all three throws. i_2 signifies heads in the first two and tail in the third throw. i_3 denotes a situation in which we obtain heads in the first and third, and tail in the second throw, etc.

This list is evidently exhaustive for the outcomes of three throws with one coin. Hence the 8 state descriptions given in the Q-matrix exhaust all possible state descriptions.

Example 5.—Consider now the Q-matrix for $N = 2$, $p = 2$ (Example 3). We have here $2^2 = 4$ Q-properties, Q_1, Q_2, Q_3, Q_4.

State description	.	*Q-properties* a_1	a_2
i_1	.	1	1
i_2	.	1	2
i_3	.	1	3
i_4	.	1	4
i_5	.	2	1
i_6	.	2	2
i_7	.	2	3
i_8	.	2	4
i_9	.	3	1
i_{10}	.	3	2
i_{11}	.	3	3
i_{12}	.	3	4
i_{13}	.	4	1
i_{14}	.	4	2
i_{15}	.	4	3
i_{16}	.	4	4

We can interpret this situation in the same way as in Example 3.

We have, for instance, for i_1: First throw shows nickel heads, dime heads. Second throw has nickel heads, dime heads. i_7: first throw nickel heads, dime tail, second throw nickel tail, nead. i_{16}: first throw and second throw, nickel tail, dime tail.

.t is again evident that the 16 state descriptions in the Q-matrix exhaust all the possibilities or the outcome of two throws with 2 coins.

(d) *Isomorphic State Descriptions. Structure*

Two state descriptions are called isomorphic if they can be transformed into each other by exchanging the individuals which have specified Q-properties. Let there by n_i state descriptions

which are isomorphic to a given state description *i*. The disjunction (*Or*-relationship) of the n_i state description which are isomorphic is called a ***structure description.*** The number of structure descriptions in a language with N individuals and p properties is $m = \binom{N+k-1}{N}$.

Example 6.—Consider the totality of state descriptions given in the Q-matrix in Example 4. We have $p = 1$, $k = 2$, $N = 3$. Hence there are $m = \binom{3+2-1}{3} = \binom{4}{3} = 4$ distinct structure descriptions:

Structure	*Structure descriptions* / *State descriptions*	*Q-properties* a_1	a_2	a_3
S_1	i_1	1	1	1
S_2	i_2	1	1	2
	i_3	1	2	1
	i_5	2	1	1
S_3	i_4	1	2	2
	i_6	2	1	2
	i_7	2	2	1
S_4	i_8	2	2	2

It appears from this example that for the structure description the important thing is the *number* of the individuals with various Q-properties in a state description, regardless with which individual a_1, a_2, a_3 they are associated. All the individuals in the given language are treated on a par. This is in agreement with the general practice in probability theory and statistics.

There is evidently only one single state description i_1 in which the Q-property 1 appears for all three individuals. Hence i_1 forms the structure description S_1.

Consider now the state description i_2. The Q-property 1 appears twice, for individuals a_1 and a_2. The Q-property 2 appears once, for individual a_3. Hence the state description i_3 is isomorphic to i_2. It becomes i_2 if the individual a_2 takes the place of a_3, and *vice versa*. State description i_5 is also isomorphic to i_2. It becomes i_2 if individuals a_1 and a_3 change places. The disjunction of the three isomorphic state descriptions i_2, i_3, i_5 is the second structure description S_2, etc.

It appears that no state description within a given structure description is isomorphic to any state description in another structure description. For instance, i_1 is not isomorphic to any other state description. The state description i_2 is not isomorphic to i_1, i_4, i_6, i_7 or i_8, etc. The four structure descriptions indicated above exhaust the number of all possible structures in our language.

Example 7.—Consider now the data presented in Example 6. The structure descriptions are as follows:

In the structure descriptions S_1, S_5, S_8, S_{10} the same Q-properties appear for both individuals a_1 and a_2. Hence there is only one unique state description within each structure description.

In all other structure descriptions there are two state descriptions. It can be easily seen that they are isomorphic to each other. Within each structure description one state description can be derived from the other by having a_1 and a_2 exchange places.

The total number of structure descriptions is here:

$$m = \binom{2+4-1}{2} = \binom{5}{2} = 10.$$

Structure Descriptions

Structure		State description		Q-properties a_1	Q-properties a_2
S_1	.	i_1	.	1	1
S_2	.	i_2	.	1	2
		i_5	.	2	1
S_3	.	i_3	.	1	3
		i_9	.	3	1
S_4	.	i_4	.	1	4
		i_{13}	.	4	1
S_5	.	i_6	.	2	2
S_6	.	i_7	.	2	3
		i_{10}	.	3	2
S_7	.	i_8	.	2	4
		i_{14}	.	4	2
S_8	.	i_{11}	.	3	3
S_9	.	i_{12}	.	3	4
		i_{15}	.	4	3
S_{10}	.	i_{16}	.	4	4

(e) *Q-distributions. Number of Isomorphic State Descriptions*

We consider state descriptions which are isomorphic to each other. Then it is only important *how many* of the individuals with various Q-properties appear in each state description and not which individual is associated with a given Q-property. Hence all the isomorphic state descriptions can be characterized by the following Q-distribution:

Q-property		*Q-distribution* Number
1	.	N_1
2	.	N_2
. . . .	. . . .	. . . .
k	.	N_k

Here N_1 is the number of individuals associated with the Q-property Q_1. N_2 is the number of individuals associated with the Q-property Q_2, . . . N_k the number of individuals associated with the last Q-property Q_k in the given state description and all state descriptions isomorphic to it. $N_1 + N_2 + \ldots N_k = N$. Here N is the number of individuals in the language considered. The number of isomorphic state descriptions is given by the combinatorial formula:

$$n_i = \frac{N!}{N_1!\,N_2!\ldots N_k!}$$

Example 8.—Consider the situation represented in example 6. ($N = 3$.) To the state description i_1 in S_1 corresponds the following Q-distribution:

Q-property		*Q-distribution* — *Number*
1	.	3
2	.	0

Hence the number of state descriptions isomorphic to i_1 is: $n_i = \frac{3!}{3!\ \ 0!} = 1$. There is one single state description in S_1.

Example 9.—Consider the structure description S_8 in Example 7. To the state description i_{11} corresponds the following Q-distribution:

Q-property		*Q-distribution* — *Number*
1	.	0
2	.	0
3	.	2
4	.	0

The number of isomorphic state descriptions corresponding to this distribution is:

$$n_i = \frac{2!}{0!\ 0!\ 2!\ 0!} = 1.$$

There is just one single state description, i_{11}, in this structure description S_8.

(f) *Measure of a State Description*

We now associate with each structure description a measure $m(S_j)$. The sum $\Sigma_j m(S_j) = 1$, where the summation is extended over all structure descriptions.

Let the state description i belong to the structure description S_j. There are n_i state descriptions which are isomorphic to i. We decide to give equal measure to isomorphic state descriptions. Hence the measure of the state description i is defined as:

$$m(i) = \frac{m(S_j)}{n_i}. \tag{2.1}$$

Carnap associates with each of the m structure descriptions the *same* measure $1/m$. There are n_i isomorphic state descriptions in each structure description. The measure of each state description i is defined as

$$m(i) = \frac{1}{mn_i}. \tag{2.2}$$

Example 10.—Consider again the situation described in Example 6. We obtain by the above rule the following table of measures:

Measure of State Descriptions

Structure description		*State description*		*Measure*
S_1	.	i_1	.	1/4
S_2	.	i_2		1/12
		i_3	.	1/12
		i_5	.	1/12
S_3	.	i_4	.	1/12
		i_6	.	1/12
		i_7	.	1/12
S_4	.	i_8	.	1/4

Note that the sum of all the measure of isomorphic state descriptions adds up to $1/m$ for each structure description. The sum of the measures of all state descriptions within a given structure description is here 1/4. Since there are altogether 4 structure descriptions, the sum of their measures is one.

Let us compare these measures with the *classical theory of probability*. We use the interpretation given in Example 1. Then Q_1 stands for obtaining a head, Q_2 for obtaining a tail. In classical probability theory we use the principle of indifference. Hence the probability of Q_1 and of Q_2 is 1/2. The probabilities of all state descriptions computed by the classical formula are 1/8. It should be noted that the probabilities of the structure descriptions are *not* equal, according to the classical theory. Indeed, the probability of S_1 and of S_4 is 1/8. The probability of S_2 and of S_3 is 3/8.

It is interesting to note that we may obtain prior probability equal to Carnap's measure if we do not use the principle of indifference, but instead assume a rectangular *a priori* distribution of the probability in the binomial distribution. In our example the classical formulae for the probabilities of x successes in n trials (corresponding to our structures S) are:

$$\binom{n}{x} p^x(1-p)^{n-x}, \tag{2.3}$$

where p is the probability of a success in a single trial.

Assume now that p has a rectangular distribution in the interval 0, 1; i.e. not knowing anything about the probability p, we assume that it is equally probable to be anywhere between zero and one. This is analogous to the classical procedure of Bayes' theorem and its modification by Jeffreys.*

Integrating the above expression with respect to p we obtain:

$$\int_0^1 \binom{n}{x} p^x (1-p)^{n-x}\, dp = 1/(n+1). \tag{2.4}$$

Example 11.—From the data in Example 7 we derive the following measure of state descriptions:

Measure of State Descriptions

Structure description	*State description*	*Measure*
S_1	i_1	1/10
S_2	i_2	1/20
	i_5	1/20
S_3	i_3	1/20
	i_9	1/20
S_4	i_4	1/20
	i_{13}	1/20
S_5	i_6	1/10
S_6	i_7	1/20
	i_{10}	1/20
S_7	i_8	1/20
	i_{14}	1/20
S_8	i_{11}	1/10
S_9	i_{12}	1/20
	i_{15}	1/20
S_{10}	i_{16}	1/10

* H. Jeffreys: *Theory of Probability*. *Op. cit.*, pp. 29 ff.

Here again within any given structure description the sum of the measures of all isomorphic state descriptions is $1/m$. The sum of the measures of all state descriptions within a given structure description is 1/10. The sum of the measures of all structure descriptions is one.

(g) *Measure of Sentences*

In a language with N individuals assume a sentence j. The measure of this sentence $m(j)$ is defined as the sum of all the measures of the state descriptions which correspond to j.

Example 12.—The situation is as in Example 1. Let the sentence be: $j = (P_1a_1) \, . \, (\sim P_1a_2)$. In terms of Q-properties this can be written:

$$j = (Q_1a_1) \, . \, (Q_2a_2).$$

Now consider the Q-matrix given in Example 4. For our sentence we have to select the state descriptions which have one in the first column (for a_1) and two in the second (for a_2). These state descriptions together with their measures (from Example 10) are given in the following table:

Measure of $(Q_1a_1) \, . \, (Q_2a_2)$

State description		a_1	a_2	a_3		*Measure*
i_3	.	1	2	1	.	1/12
i_4	.	1	2	2	.	1/12

Hence the measure of the sentence $j = (P_1a_1) \, . \, (\sim P_1a_2)$ is $m(j) = 1/12 + 1/12 = 1/6$.

Example 13.—The situation is the same as in Example 3. Consider the sentence $j = (P_1a_1) \vee (\sim P_2a_2)$. From the table of Q-properties given in the example we see that this is equivalent to the following Q-properties: $(Q_1a_1) \vee (Q_2a_1) \vee (Q_2a_2) \vee (Q_4a_2)$.

Hence we have to select from the Q-matrix in Example 5 all the state descriptions which have either 1 or 2 in the first (a_1) column *or* have either 2 or 4 in the second (a_2) column. These are all state descriptions except the following: $i_9, i_{11}, i_{13}, i_{15}$.

The measure of the state descriptions are taken from Example 11. The sum of the measures of the state descriptions and hence the measure of the sentence j is: $m(j) = 15/20 = 3/4$.

(h) *Degree of Confirmation*

In a given language assume two sentences, h and e. The sentence h is the hypothesis and the sentence e is the evidence. Form the conjunction (*And*-connection) $e \, . \, h$. The degree of confirmation of h on e is defined as:

$$c(h, e) = m(e \, . \, h)/m(e). \tag{2.5}$$

Example 14.—Consider the situation described in Example 4. Let the sentence e be: $(P_1a_1) \vee (\sim P_1a_2)$. The sentence h is: $(\sim P_1a_3)$.

Expressed in Q-properties, the sentence e is: $(Q_1a_1) \vee (Q_2a_2)$. The sentence h becomes: (Q_2a_3). Their conjunction $e \, . \, h \, .$ is:

$$[(Q_1a_1) \vee (Q_2a_1)] \, . \, (Q_2a_3).$$

Consider first the sentence e (evidence). The state descriptions corresponding to it must have either one in the first column (for a_1) *or* 2 in the second column (for a_2) in the Q-matrix in Example 4. This gives the following state descriptions: $i_1, i_2, i_3, i_4, i_7, i_8$.

The measures of the various state descriptions are taken from Example 10. The sum of the measures of the state descriptions corresponding to e is: $m(e) = 5/6$.

Now consider the sentence $e \, . \, h$ (conjunction of evidence and hypothesis). The state descriptions corresponding to this sentence must have either one in the first column of the Q-matrix or 2 in the second column, but all must have 2 in the third column. They are: i_2, i_4, i_8.

Hence the measure associated with the sentence $e\ .\ h$ is $m(e\ .\ h) = 5/12$. The degree of confirmation is:

$$c(h, e) = \frac{(5/12)}{(5/6)} = 1/2.$$

Example 15.—Consider now the situation in Example 5. Let the sentence e (evidence) be: $(\sim P_1 a_1)$. The sentence h is: $P_2 a_2$. The sentence $e\ .\ h$ is $(\sim P_1 a_1)\ .\ (P_2 a_2)$.

In terms of the Q-properties the sentence e is: $(Q_3 a_1) \vee (Q_4 a_1)$. The sentence h is: $(Q_1 a_2) \vee (Q_3 a_2)$. The conjunction $e\ .\ h$ is in terms of the Q-properties: $[(Q_3 a_1) \vee (Q_4 a_1)]\ .\ [(Q_1 a_2) \vee (Q_3 a_2)]$.

To find the measure of sentence e we take from the Q-matrix in Example 5 all the state descriptions which have either 3 or 4 in the first column: These are the state descriptions i_9, i_{10}, i_{11}, i_{12}, i_{13}, i_{14}, i_{15}, i_{16}.

The measures of these state descriptions are taken from Example 11. The sum of the measures of all these state descriptions which correspond to the sentence e is: $m(e) = 1/2$.

Now consider the conjunction $e\ .\ h$ (conjunction of evidence and hypothesis). The state descriptions in the Q-matrix must have either 3 or 4 in the first column *and* either 1 or 3 in the second column. Hence these state descriptions are: i_9, i_{11}, i_{13}, i_{15}.

The sum of these measures and hence the measure of $e\ .\ h$ is: $m(e\ .\ h) = 1/4$. The degree of confirmation of h on e is:

$$c(h, e) = \frac{(1/4)}{(1/2)} = 1/2.$$

(i) *Simple Properties. Logical Width*

With applications of the theory of attributes we are especially interested in properties, without caring which individuals in the language in question have these properties. All the individuals in the language are treated on a par. A property M can be expressed as a disjunction (*or*-connection) of various Q-properties. In a language with p attributes we denote by w_1 the number of Q-properties which correspond to the property M. This is the logical width of M. And we denote by $w_2 = k - w_1$ the number of Q-properties which correspond to the negation of the property M, $\sim M$; w_2 is the logical width of $\sim M$.

Example 16.—Consider now the situation in Example 2. Let M be the simple property P_1. There is only one single Q-property corresponding to M: Q_1. Hence $w_1 = 1$. From this follows that the logical width of the property M is 1. But $k = 2$. Hence the logical width of the property $\sim M$ is: $w_2 = 2 - 1 = 1$.

Example 17.—Consider now the situation in Example 3. Take the simple property $M = (P_1) \vee (\sim P_2)$. From the table given in the example it follows that the three Q-properties Q_1, Q_2 and Q_4 correspond to this property M. Hence the logical width of M is $w_1 = 3$. The logical width of $\sim M$ is: $w_2 = 4 - 3 = 1$, since $k = 4$.

(j) *Measure of a Sentence Regarding Simple Properties*

Let M be a simple property in a language of p simple attributes and N individuals. Assume that M is a disjunction of w_1 and $\sim M$ a disjunction of $k - w_1 = w_2$ Q-properties.

Let j be a sentence which asserts that there are N_1 individuals in the language which have the simple property M and $N_2 = N - N_1$ individuals which do not possess it. Then the measure of the sentence j is given by the combinatorial formula:

$$m(j) = \frac{\binom{N_1 + w_1 - 1}{N_1}\binom{N_2 + w_2 - 1}{N_2}}{\binom{N + k - 1}{N}}. \tag{2.6}$$

Example 18.—To exemplify this we assume a language with $N = 3$ individuals and $p = 2$ simple attributes. Hence $k = 2^p = 4$.

We give a table of the structure descriptions, state descriptions and their measures:

Structure description	*State description*	*Q-properties*			*Measure*
		a_1	a_2	a_3	
S_1	1	1	1	1	1/20
S_2	2	1	1	2	1/60
	3	1	2	1	1/60
	4	2	1	1	1/60
S_3	5	1	1	3	1/60
	6	1	3	1	1/60
	7	3	1	1	1/60
S_4	8	1	1	4	1/60
	9	1	4	1	1/60
	10	4	1	1	1/60
S_5	11	1	2	2	1/60
	12	2	1	2	1/60
	13	2	2	1	1/60
S_6	14	1	2	3	1/120
	15	1	3	2	1/120
	16	2	1	3	1/120
	17	2	3	1	1/120
	18	3	1	2	1/120
	19	3	2	1	1/120
S_7	20	1	2	4	1/120
	21	1	4	2	1/120
	22	2	1	4	1/120
	23	2	4	1	1/120
	24	4	1	2	1/120
	25	4	2	1	1/120
S_8	26	1	3	3	1/60
	27	3	1	3	1/60
	28	3	3	1	1/60
S_9	29	1	3	4	1/120
	30	1	4	3	1/120
	31	3	1	4	1/120
	32	3	4	1	1/120
	33	4	1	3	1/120
	34	4	3	1	1/120
S_{10}	35	1	4	4	1/60
	36	4	1	4	1/60
	37	4	4	1	1/60
S_{11}	38	2	2	2	1/20
S_{12}	39	2	2	3	1/60
	40	2	3	2	1/60
	41	3	2	2	1/60
S_{13}	42	2	2	4	1/60
	43	2	4	2	1/60
	44	4	2	2	1/60

Structure description	State description	Q-properties a_1	a_2	a_3	Measure
S_{14}	45	2	3	3	1/60
	46	3	2	3	1/60
	47	3	3	2	1/60
S_{15}	48	2	3	4	1/120
	49	2	4	3	1/120
	50	3	2	4	1/120
	51	3	4	2	1/120
	52	4	2	3	1/120
	53	4	3	2	1/120
S_{16}	54	2	4	4	1/60
	55	4	2	4	1/60
	56	4	4	2	1/60
S_{17}	57	3	3	3	1/20
S_{18}	58	3	3	4	1/60
	59	3	4	3	1/60
	60	4	3	3	1/60
S_{19}	61	3	4	4	1/60
	62	4	3	4	1/60
	63	4	4	3	1/60
S_{20}	64	4	4	4	1/20

Now let the simple property M be:

$$M = P_1 \vee P_2.$$

In terms of Q-properties this can be written: $M = Q_1 \vee Q_2 \vee Q_3$ (Example 3). M is a disjunction of 3 Q-properties. The logical width of M is $w_1 = 3$. But the negation of M is: $\sim M = \sim(P_1 \vee P_2) = Q_4$. Hence, $\sim M$ is a disjunction of 1 Q-property. Its logical width is: $w_2 = 1$.

It should be noted that the sum of the logical widths of M and $\sim M$ is: $w_1 + w_2 = k$. Here $k = 4$.

Consider now the sentence j which asserts that among the 3 individuals there are 2 which have the simple property M' and one which does not possess it. Hence we have: $N_1 = 2$, $N_2 = 1$.

We seek all the state descriptions given in the table above which satisfy our sentence j. We must have all state descriptions which have two individuals with the Q-properties 1, 2 or 3 and only one individual with the Q-property 4.

By direct enumeration we see from the table that all the state descriptions corresponding to the following 6 structure descriptions fulfil this condition: S_4, S_7, S_9, S_{13}, S_{15}, S_{18}. The total measure of each structure description in the table is 1/20. Hence the measure of the sentence j is:

$$m(j) = 6/20 = 3/10.$$

Alternatively we could have used the above formula, which gives the measure of a sentence referring to simple properties. There we have:

$$m(j) = \frac{\binom{2+3-1}{2}\binom{1+1-1}{1}}{\binom{3+4-1}{3}} = \frac{\binom{4}{2}\binom{1}{1}}{\binom{6}{3}} = 3/10.$$

3. *Applications of Carnap's Theory for Simple Properties*

Carnap's theory has been applied to a number of problems of special interest to statisticians. These are the inferences drawn from a population to the sample (direct inference); from a given sample to another unknown future sample (predictive inference); and the inferences from the sample to the population (inverse inference).

(a) *Direct Inference*

Direct inference is the inference from the population to the sample. The evidence e is here that in a population consisting of n individuals there are n_1 which have the simple property M. There are $n_2 = n - n_1$ individuals which have not this property.

The hypothesis h is that in a sample consisting of s individuals there are s_1 with the simple property M; and there are $s_2 = s - s_1$ items which do not possess the property.

The results are here the same as in the established theory. The degree of confirmation c is simply a term of the well-known hypergeometric distribution:

$$(3.1) \qquad c = \frac{\binom{n_1}{s_1}\binom{n_2}{s_2}}{\binom{n}{s}}.$$

The mean of the hypergeometric distribution is:

$$(3.2) \qquad \bar{s}_1 = \frac{s\ n_1}{n}$$

and its variance:

$$(3.3) \qquad \frac{s\ n_1\ n_2\ (n - s)}{n^2(n - 1)}.$$

It is well known that for large samples the degree of confirmation or probability$_1$ c follows a normal distribution with the mean and the variance given above.*

Example 19.—Consider again the language of Example 18, with $p = 2$ and $n = 3$. Let the simple property in question now be: $M = P_1 P_2 \cdot Q_1$ (Example 3). The logical width of M is $w_1 = 1$, since it corresponds to a single Q-property. The logical width of $\sim M$ is $w_2 = 3$.

Now let the evidence be stated in a sentence that says: in a language with $n = 3$ individuals there are $n_1 = 2$ who have the property M (i.e. the Q-property one) and there are $n_2 = 1$ individuals which do not possess it (i.e. have the Q-properties 2, 3 or 4). From the table given in Example 6 we derive by direct enumeration the state descriptions which correspond to the sentence e. They are those state descriptions which have two ones among the three Q-properties: 2, 3, 4, 5, 6, 7, 8, 9, 10. Adding the individual measures of all state descriptions we derive for the measure of the evidence e: $m(e) = 9/60 = 3/20$.

The hypothesis is that among a sample of $s = 2$ individuals taken from the above population we have $s_1 = 1$ individual which has the simple property M and $s_2 = 1$ individual which has not this property.

Choosing as our sample the first two individuals a_1 and a_2 we take now as the conjunction of e and h those state descriptions in the previous table which have exactly one Q-property one in the first two columns: 3, 4, 6, 7, 9, 10.

We compute by addition the measure corresponding to the conjunction of the two sentences $e \,.\, h$: $m(e \,.\, h) = 1/10$. Hence the degree of confirmation is: $c = m(e \,.\, h)/m(e) = 2/3$.

The same result could also have been derived by the application of the classical formula given above:

$$c = \frac{\binom{2}{1}\binom{1}{1}}{\binom{3}{1}} = 2/3.$$

* M. G. Kendall: *Advanced Theory of Statistics*, vol. 1, London, 1945, pp. 126 ff.

(b) *Predictive Inference*

Predictive inference is the inference from a given to a future unknown sample. Here our evidence e is a sample of size s. There are s_1 items possessing the simple property M and $s_2 = s - s_1$ individuals without this property. The simple property M has the logical width w_1.

The hypothesis is that in a second sample of s' individuals there are s'_1 individuals with the property M and $s'_2 = s' - s'_1$ individuals without this property.

The degree of confirmation for predictive inference is now:

$$c = \frac{\binom{s_1 + s'_1 + w_1 - 1}{s'_1} \binom{s_2 + s'_2 + w_2 - 1}{s'_2}}{\binom{s + s' + k - 1}{s'}}. \tag{3.4}$$

Note that in this formula w_1 is the logical width of M. The quantity $w_2 = k - w_1$ is the logical width of $\sim M$. Hence predictive inference, in distinction to direct inference, depends upon the semantical properties of the language. They determine the logical widths of the various properties.

In the above formula, let $s_1 = s_2 = s = 0$ and $s'_1 = s' = 1$, $s'_2 = 0$. We obtain then

$$c = \frac{w_1}{k}. \tag{3.5}$$

Hence the relative logical width of the property M, i.e. w_1/k, may be considered as the *a priori probability* of this property. It should be noted that this is established on the single basis of the logical structure of the language considered, without the intervention of experience. The probability$_1$ or degree of confirmation will be modified, if $s > 0$, i.e. if we have a sample available.

We can establish the *mode* for s'_1. This is for large samples, where the Stirling formula is valid:

$$\hat{s}'_1 = \frac{s'(s_1 + w_1 - 1)}{s + k - 2}. \tag{3.6}$$

It is seen that the mode involves now the logical nature of the problem in question. If, however, the sample size s becomes large compared with k (and w_1) we have:

$$\lim_{s \to \infty} \hat{s}'_1 = \frac{s's_1}{s}. \tag{3.7}$$

We see that for sample sizes which are large enough the mode of s'_1 is independent of the nature of the language and of the logical width of the property in question.

Next we consider the *mean*. This is easily computed from the factorial moment of the distribution of s'_1. For the mean we obtain the following:

$$\bar{s}'_1 = \frac{s'(s_1 + w_1)}{s + k}. \tag{3.8}$$

It should be noted that here the mean is not identical with the mode. It is also dependent upon the logical structure of the language.

It is again interesting to note the relationship of this result to the classical theory of Bayes' theorem. The mean given above is also the mean of a binomial distribution of s' trials where the *a priori* probability of a success is:

$$\frac{s_1 + w_1}{s + k}. \tag{3.9}$$

Note that this *a priori* probability involves both the empirical experience of the first sample (s_1 and s) and the logical structure of the language in question (w_1 and k).

If, however, the sample is large, i.e. if s and s_1 are large compared with k (and w_1), we obtain:

$$\lim_{s \to \infty} \bar{s}'_1 = \frac{s' s_1}{s}. \tag{3.10}$$

For large samples the mode and the mean are identical. Both become independent of the logical structure of the language.

The variance of s'_1 can also be computed by factorial moments. It is:

$$(3.11)\qquad \frac{(s_1 + w_1)(s_2 + w_2)(s + s' + k)\,s'}{(s + k)^2 (s + k + 1)}$$

For large samples, i.e. for large s, we obtain in the limit:

$$(3.12)\qquad (s_1/s)(s_2/s)s'.$$

Hence the variance tends for large samples to be independent of the logical nature of the language in question.

It can again be shown that under certain conditions the distribution c becomes normal for large samples with the mean and variance given above.*

Example 20.—Consider the data in Example 18 and the simple property $M = Q_1 \vee Q_2 \vee Q_3$. Hence its logical width $w_1 = 3$; and the logical width of the property $\sim M$ is: $w_2 = 1$.

The evidence e says that in a sample of $s = 2$ items there is one which possesses the property M (i.e. has the Q-properties 1, 2, or 3). Hence $s_1 = 1$. There is also one item which does not have the property M (i.e. has the Q-property 4). Hence $s_2 = 1$.

In the table given in Example 18 consider the first two individuals a_1 and a_2 as the ones which form the sample. We list the state descriptions which have in the first two columns only one Q-property 4. These state descriptions have been taken from Example 18: 9, 10, 21, 23, 24, 25, 30, 32, 33, 34, 35, 36, 43, 44, 49, 51, 52, 53, 54, 55, 59, 60, 61, 62.

The measure corresponding to the sentence e is by addition: $m(e) = 3/10$.

The hypothesis h is a sentence which asserts the following: In a future sample of $s' = 1$ items, there is $s'_1 = 1$ item which has the simple property M and $s'_2 = 0$ items which have the property $\sim M$.

To find the measure of the conjunction $e\,.\,h$ we regard the individual a_3 as the representative of the new sample. Here we select from the above table the items which have 1, 2 or 3 as the Q-property of the last column.

The state descriptions which correspond to the conjunction $e\,.\,h$ are: 9, 10, 21, 23, 24, 25 30, 32, 33, 34, 43, 44, 49, 51, 52, 53, 59, 60.

The sum of the measures of the state descriptions corresponding to the conjunction $e\,.\,h$ is $m(e\,.\,h) = 1/5$. Hence the degree of confirmation of h on e is:

$$c = m(e\,.\,h)/m(e) = 2/3.$$

Using Carnap's formula given above we have also:

$$c = \frac{\binom{1+1+3-1}{1}\binom{1+0+1-1}{0}}{\binom{2+1+4-1}{1}} = \frac{\binom{4}{1}\binom{1}{0}}{\binom{6}{1}} = 2/3.$$

(*c*) *Inverse Inference*

This type of inference is probably the most important one for statistical practice. It is the inference from the sample to the whole population.

A sample of s individuals is taken from a population with n individuals. The simple property in question is M, its logical width w_1. Hence the logical width of $\sim M$ is: $w_2 = k - w_1$. The evidence says that among the s individuals in the sample there are s_1 who possess the property M and $s_2 = s - s_1$ who have not the property in question.

But the hypothesis says now that among the n individuals in the population there are n_1 who show the property M, and $n_2 = n - n_1$ who do not have the property.

Evidently this is a special case of the predictive inference treated in section b. We have only to replace s' by $n - s$, s'_1 by $n_1 - s_1$, s'_2 by $n_2 - s_2$ in the formulae given there.

* H. Jeffreys: *op. cit.*, pp. 56 ff.

The degree of confirmation for inverse inference is:

$$c = \frac{\binom{n_1 + w_1 - 1}{s_1 + w_1 - 1} \binom{n_2 + w_2 - 1}{s_2 + w_2 - 1}}{\binom{n + k - 1}{n - s}}. \tag{3.13}$$

The mode is for large samples by Stirling's approximation:

$$\hat{n}_1 = \frac{n(s_1 + w_1 - 1) + s_1(w_2 - 1) - s_2(w_1 - 1)}{s + k - 2}. \tag{3.14}$$

If n is large compared with s and both n and s large compared with k, we obtain a limiting value of the mode:

$$\lim_{n \to \infty} \hat{n}_1 = \frac{n\, s_1}{s}. \tag{3.15}$$

This is, of course, the classical result. It becomes independent of the logical structure of the language in question.

The mean of n_1 is now:

$$\bar{m}_1 = \frac{n(s_1 + w_1) + w_2 s_1 - w_1 s_2}{s + k}. \tag{3.16}$$

Interpreting this again in terms of the classical Bayes' theory we note that the mean is also approximately the mean of a series of n independent trials with *a priori* probability:

$$\frac{s_1 + w_1}{s + k} \tag{3.17}$$

if $(s_1/w_1) - (s_2/w_2)$ are negligible.

It should be noted that the *a priori* probability is the same as with predictive inference above.

In the limit, under the conditions stated above, we obtain the following value for the mean:

$$\lim_{n \to \infty} \bar{n}_1 = \frac{n\, s_1}{s}. \tag{3.18}$$

This is the classical result.

The variance of n_1 is:

$$\frac{(s_1 + w_1)(s_2 + w_2)(n + k)(n - s)}{(s + k)^2 (s + k + 1)}. \tag{3.19}$$

If both n and s tend to infinity this becomes

$$n(s_1/s)(s_2/s). \tag{3.20}$$

This is again a classical result.

For large samples the probability distribution c becomes normal with the mean and variance given above.*

Example 21.—We utilize again the data of Example 18. We consider the simple property $M = Q_1 \vee Q_2$. Hence its logical width is $w_1 = 2$, and the logical width of $\sim M$ is $w_2 = 2$.

The evidence e says now that in a sample consisting of $s = 2$ individuals a_1 and a_2 there is $s_1 = 1$ which has the simple property M' (i.e. the Q-properties 1 or 2) and $s_2 = 1$ which does not possess it.

We take from the data in Example 18 all the state descriptions where the Q-properties 1 or 2 appear once and the Q-properties 3 or 4 appear once in the first two columns: 6, 7, 9, 10, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 29, 30, 31, 33, 35, 36, 40, 41, 43, 44, 45, 46, 48, 49, 50, 52, 54, 55.

* *Ibid.*

The sum of the measures of all these state descriptions is: $m(e) = 2/5$.

The hypothesis h says that in the population of $n = 3$ items from which the sample is taken there are $n_1 = 2$ items which have the property M and $n_2 = 1$ items which do not have it.

If we consider the first two columns in the above table as our sample, the state descriptions which correspond to the conjunction $e \,.\, h$ are the following: 6, 7, 9, 10, 15, 17, 18, 19, 21, 23, 24, 25, 40, 41, 43, 44.

The measure corresponding to the conjunction $e \,.\, h$ is: $m(e \,.\, h) = 1/5$. Hence the degree of confirmation of h on e is: $c = m(e \,.\, h)/m(e) = 1/2$.

The same result follows from Carnap's formula. The degree of confirmation is:

$$c = \frac{\binom{2+2-1}{1+2-1}\binom{1+2-1}{1+2-1}}{\binom{3+4-1}{3-2}} = \frac{\binom{3}{2}\binom{2}{2}}{\binom{6}{1}} = 1/2.$$

4. *Multiple Attributes*

The theory indicated above is designed to deal with simple attributes, i.e. attributes which can only assume the two values P and $\sim P$. An example of this situation is the throwing of a coin, where the outcome is either head (say P) or tail ($\sim P$).

We need a theory of multiple attributes if we want to treat situations which involve more than one single alternative. We will consider a language irvolving p multiple attributes.

The attribute P_i ($i = 1, 2, \ldots p$) will be assumed to be able to have r_i distinct values. In the special case of simple attributes we have $r_i = 2$. We denote the j^{th} value of the i^{th} attribute by P_{ij} ($i = 1, 2, \ldots p$, $j = 1, 2, \ldots r_i$). In Carnap's terminology multiple attributes are called: Families of more than two related properties.

We denote now by $Q_{j_1 j_2 \ldots j_p}$ the Q-property corresponding to the combination of the multiple attributes $P_{1j_1} P_{2j_2} \ldots P_{pj_p}$. The total number of Q-properties is $k = r_1 r_2 \ldots r_p$.

In a language with N individuals $a_1, a_2, \ldots a_N$ and p multiple attributes $P_1, P_2 \ldots P_p$ the total number of possible state descriptions is k^N and the number of structure descriptions is:

$$m = \binom{N+k-1}{N}. \tag{4.1}$$

If we follow Carnap's example and give each structure description the same measure $1/m$ then all the formulae given in section 3 for simple attributes remain valid for multiple attributes. It should only be noted that k is now differently defined.

5. *Multiple Properties*

An obvious generalization of the theory given above consists in the introduction of multiple properties. This situation is called in Carnap's theory: A division or classification with more than two properties. Up to now we have discussed simple properties. In this case an individual in question was classified according to whether it possessed a given property (disjunction of certain Q-properties) or not.

Now we divide the total range of all Q-properties $Q_1, Q_2 \ldots Q_k$ into t mutually exclusive multiple properties $M_1, M_2 \ldots M_t$. No Q-property which appears within a given partition of the multiple property M, say in M_1, can appear in any other partition. The t mutually exclusive partitions exhaust the total of all k Q-properties. The sum of the logical width of all multiple properties $w_1 + w_2 + \ldots w_t = k$.

The formulae given by Carnap can be easily generalized for this case. It should perhaps be remarked that of course multiple attributes and multiple properties may also be combined.

(a) *Measure of a Sentence Regarding Multiple Properties*

Consider the language described above. What is the measure of the sentence which says that there are in a language of N individuals exactly N_1 with the property M_1, N_2 with the property $M_2 \ldots N_t$ with the property M_t ($N_1 + N_2 + \ldots N_t = N$)?

An easy generalization of Carnap's formula yields:

$$(5.1)\qquad \frac{\binom{N_1 + w_1 - 1}{N_1}\binom{N_2 + w_2 - 1}{N_2}\cdots\binom{N_t + w_t - 1}{N_t}}{\binom{N + k - 1}{N}}.$$

(b) *Direct Inference*

The evidence is that in population with n items the following numbers belong to the multiple properties $M_1, M_2 \ldots M_t$: $n_1, n_2 \ldots n_t$ $(n_1 + n_2 + \ldots n_t = n)$.

The hypothesis says that in a sample taken from the above population there are s items. Among those s individuals there are s_1 belonging to the property M_1, s_2 belonging to the multiple property $M_2, \ldots s_t$ to the multiple property $M_t(s_1 + s_2 + \ldots s_t = s)$. The degree of confirmation is here

$$(5.2)\qquad c = \frac{\binom{n_1}{s_1}\binom{n_2}{s_2}\cdots\binom{n_t}{s_t}}{\binom{n}{s}}.$$

(c) *Predictive Inference*

The evidence is here that in a sample of s items there are s_1 items with property M_1, s_2 items with property $M_2 \ldots s_t$ items with property M_t. $s_1 + s_2 + \ldots s_t = s$.

The hypothesis is about a new sample of size s'. There are s'_1 items with M_1, s'_2 items with $M_2 \ldots s'_t$ items with M_t, where $s'_1 + s'_2 + \ldots s'_t = s'$.

The degree of confirmation of the hypothesis on the evidence is:

$$(5.3)\qquad c = \frac{\binom{s_1 + s'_1 + w_1 - 1}{s'_1}\binom{s_2 + s'_2 + w_2 - 1}{s'_2}\cdots\binom{s_t + s'_t + w_t - 1}{s'_t}}{\binom{s + s' + k - 1}{s'}}.$$

(d) *Inverse Inference*

The evidence says here that in a sample of s items there are $s_1, s_2 \ldots s_t$ items corresponding to the multiple properties $M_1, M_2 \ldots M_t$. $s_1 + s_2 + \ldots s_t = s$.

The hypothesis asserts that the sample comes of a population with n items, n_1 of which belong to M_1, n_2 to $M_2 \ldots n_t$ to M_t. $n_1 + n_2 \ldots n_t = n$.

The degree of confirmation is here:

$$(5.4)\qquad c = \frac{\binom{n_1 + w_1 - 1}{s_1 + w_1 - 1}\binom{n_2 + w_2 - 1}{s_2 + w_2 - 1}\cdots\binom{n_t + w_t - 1}{s_t + w_t - 1}}{\binom{n + k - 1}{n - s}}.$$

6. *Applications to Statistical Prediction and Estimation*

The degree of confirmation may be used in statistical inference. The ideas described above for predictive inference may be utilized for statistical prediction. The concepts introduced for inverse inference are relevant for statistical estimation.

It is tentatively suggested that the following procedures may be useful in statistical inference:

(a) *Maximum Degree of Confirmation Value*

This is the particular value of the parameter in question which makes the degree of confirmation c a maximum. This idea is similar to R. A. Fisher's maximum likelihood.*

* M. G. Kendall: *op. cit.*, vol. 2, pp. 1 ff. H. Cramer: *op. cit.*, pp. 498 ff.

(b) *Mean Degree of Confirmation Estimate*

This is the weighted arithmetic average of the parameter in question with the degrees of confirmation as weights. It is analogous to the mean value or mathematical expectation in statistical theory.*

(c) *Median Degree of Confirmation Estimate*

This estimate is chosen in a way so that one-half of the probability$_1$ lies on each side of it. It has the desirable property that it is invariant for any single-valued transformation of the variable in question. For instance, if the square of the variable is substituted for the original variable the median remains unaffected: the median of the square of the variable is the square of the median of the original variable, etc. The median has frequently been used in non-parametric statistical inference.†

(d) *Degree of Confirmation Limits*

Interval estimation may be attempted by seeking limits of a parameter in such a fashion that the probability$_1$ (degree of confirmation) that the parameter will lie in the interval assumes a given value. This idea is related to R. A. Fisher's fiducial limits and the confidence limits of J. Neyman and E. S. Pearson.‡ Similar methods may also be used for tests of significance.

These ideas will be illustrated in the following examples:

Example 22.—We consider again a simple property M whose logical width is $w_1 = 3$. The logical width of the property $\sim M$ is $w_2 = k - w_1 = 1$. The evidence says the following: In a sample of $s = 8$ individuals there are $s_1 = 4$ individuals who have the property M. There are $s_2 = 4$ items which have the property $\sim M$.

The hypothesis concerns a second sample of $s' = 8$ individuals. It says that among these 8 individuals there are s'_1 who have the property M and s'_2 who do not possess this property. The degree of confirmation is (*predictive inference*):

$$c = \frac{\binom{6 + s'_1}{s'_1}\binom{4 + s'_2}{s'_2}}{\binom{19}{8}}; \quad s'_1 + s'_2 = 8$$

$s'_1, s'_2 = 0, 1 \ldots 8.$

The probabilities$_1$ of various values of s'_1 computed from this formula are given in the following table:

s'_1		*Probability*$_1$
0	.	0·007
1	.	0·031
2	.	0·077
3	.	0·140
4	.	0·194
5	.	0·214
6	.	0·183
7	.	0·114
8	.	0·040

The mode of this distribution corresponding to the maximum confirmation value is 5. By the approximation formula (3·7) using Stirling's approximation we have $\hat{s}'_1 = 4{\cdot}8$. The median is (by linear interpolation) 4·74.

* H. Cramer: *op. cit.*, pp. 170 ff.

† H. Scheffé: "Statistical Inference in the Non-Parametric Case," *Annals of Mathematical Statistics*, **14**, 1943, pp. 305 ff. S. S. Wilks: *Mathematical Statistics*. Princeton, 1943, pp. 200 ff. "Order Statistics," *Bulletin American Mathematical Society*, **45**, 1948, pp. 6 ff. M. G. Kendall: *Rank Correlation Methods*. London, 1948.

‡ M. G. Kendall: *op. cit.*, pp. 62 ff.

The mean of the distribution is 4·67 and the variance 2·99. We see from the table that the 95 per cent. confirmation limits are one to seven. These limits include about 95 per cent. of the probability$_1$. We can also find confirmation limits if we make the assumption of a normal distribution. If the confidence coefficient is 95 per cent. the approximate limits are: 1·28 and 8·06.

Example 23.—Consider the following situation (*inverse inference*): The evidence says that in a sample of $s = 10$ individuals there are $s_1 = 5$ items which have the simple property M with logical width $w_1 = 3$. There are also $s_2 = 5$ items which have the property $\sim M$ with logical width $w_2 = 1$.

The hypothesis concerns a population with $n = 16$ items from which the sample is taken. It says that there are n_1 items with the property M and $n_2 = n - n_1$ without the property. The degree of confirmation is:

$$c = \frac{\binom{n_1+2}{7}\binom{n_2}{5}}{\binom{19}{6}}; \quad n_1 + n_2 = 16,$$

$n_1, n_2 = 5, 6 \ldots 11$.

The various probabilities$_1$ are given in the following table:

n_1		*Probability*$_1$
5	.	0·017
6	.	0·074
7	.	0·167
8	.	0·248
9	.	0·256
10	.	0·175
11	.	0·063

The mode is here at $\hat{n}_1 = 9$. The approximation formula (3.15) gives 8.5. The median is 8.48 by linear interpolation.

The mean is 8·43 and the variance of the distribution is 1·96. The 95 per cent. confirmation limits are five to ten. The approximate 95 per cent. confirmation limits are 5·66 and 11·17, assuming a normal distribution.

7. *Theory of Testing Statistical Hypotheses*

We present in this section some of Carnap's ideas which seem to be important in connection with problems arising in the theory of testing statistical hypotheses. The hypothesis postulated is of the form: *If* an individual in a given language has a certain property, it possesses also another property.

(a) *Universal Inference*

Assume we have two simple properties M and M'. Define a new property as $M_1 = M \,.\, \sim M'$; the property M_1 has the logical width w_1. Hence M_1 corresponds to a disjunction of w_1 Q-properties.

Let there be a law l which asserts that in a language consisting of N individuals *all* individuals have the property $\sim M_1$. The law l claims that there are no individuals in the whole population who have the property M_1. In other words, every individual who has the property M has also the property M'.

The evidence e consists of a report that in a sample of s individuals there are s which have the property $\sim M_1$, i.e. there are none which contradict the law l.

The degree of confirmation (probability$_1$) of the law l on the evidence e is:

$$(7.1) \qquad c = m(e \,.\, l)/m(e) = \frac{\binom{s+k-1}{w_1}}{\binom{N+k-1}{w_1}}.$$

Example 24.—Consider the attributes defined in Example 3. Let $M = P_1$ and $M' = P_2$. Hence we have: $M_1 = M \,.\, \sim M' = P_1 \,.\, \sim P_2 = Q_2$. This corresponds to just one single Q-property, Q_2. Hence the logical width of M_1 is $w_1 = 1$.

Now consider the language described in Example 18. Here $N = 3$. Let the evidence be that in a sample of $s = 2$ individuals a_1 and a_2 there are $s = 2$ individuals which conform to the law l (all the individuals in the sample have the property $\sim M_1$). We select the state descriptions in Example 18 which have in the first two columns (a_1 and a_2) the Q-properties 1, 3 or 4.

It is easy to see that all the state descriptions in the following structure descriptions are of this form: S_1, S_3, S_4, S_8, S_9, S_{10}, S_{17}, S_{18}, S_{19}, S_{20}. In addition, the following state descriptions in other structure descriptions conform also to the conditions postulated by the evidence e: 2, 15, 18, 21, 24, 47, 51, 53, 56. The total measure of all these state descriptions is: $m(e) = 3/5$.

The law l asserts that the third individual a_3 also has the property $\sim M_1$. Hence for the conjunction $e \,.\, l$ we have to choose from the state descriptions listed above those which have either 1, 3 or 4 in the *last* column. Only the structure descriptions $S_1 \,.\,.\,.\, S_{20}$ listed above conform to this condition. Hence the measure of the conjunction of the evidence and the postulated law is: $m(e \,.\, l) = 1/2$.

From the definition of the degree of confirmation we have now:

$$c = (1/2)/(3/5) = 5/6.$$

But formula (7.1) also gives for the degree of confirmation under universal inference:

$$c = \frac{\binom{2+4-1}{1}}{\binom{3+4-1}{1}} = 5/6.$$

(b) *Modified Universal Inference*

The method of universal inference is applicable as long as we obtain in our sample, which forms the evidence e, only individuals which *conform* to the law l (i.e. which have the property $\sim M_1$). As soon as we obtain a single individual in our sample which contradicts the law l (i.e. which has the property M_1) then the degree of confirmation $c = 0$.

Carnap has modified the universal inference in the following way: Assume that we have an evidence e' which consists of a sample of s individuals. There are now in this sample s_1 individuals which *violate* the law l (i.e. which have the property M_1); and there are $s_2 = s - s_1$ individuals which conform to the law (i.e. which have the property $\sim M_1$ postulated in the law l).

The hypothesis l' is now as follows: In a language consisting of N individuals the $N - s$ individuals *not comprised in the sample* have the postulated property $\sim M_1$. All the remainder of the population is in agreement with the assumed law.

The formula for the degree of confirmation under modified universal inference is now:

$$(7.2) \qquad c = m(e' \,.\, l')/m(e') = \frac{\binom{s+k-1}{s_1+w_1}}{\binom{N+k-1}{s_1+w_1}}.$$

This formula reduces itself to formula (7.1) if $s_1 = 0$, i.e. if there are in the sample no cases in contradiction to the postulated law l.

Example 25.—Consider a situation similar to the one in the previous example 24, based on the data in Example 18.

Let e' be the evidence which says that in a sample of $s = 2$ individuals there are $s_1 = 1$ individuals which *violate* the postulated law (i.e. have the property M_1); and there are $s_2 = 1$ individuals which conform to the law (i.e. have the property $\sim M_1$ postulated by the law l in Example 24).

We have to select from the table of state descriptions in Example 18 those which have exactly one Q-property 2 and one Q-property 1, 3 or 4 in the first two columns. The following state descriptions fulfil this condition: 3, 4, 11, 12, 14, 15, 17, 19, 20, 22, 23, 25, 40, 41, 43, 44, 45, 46, 48, 49, 50, 52, 54, 55. The sum of the measures of these state descriptions is: $m(e') = 3/10$.

The modified law l' asserts that the individual a_3 which is the only one not comprised in the sample has the postulated property $\sim M_1$. Hence for the conjunction $e' \,.\, l'$ we select from the state descriptions in e' those which have 1, 3 or 4 in the last column. These are: 3, 4, 14, 15, 17, 19, 20, 22, 23, 25, 45, 46, 48, 49, 50, 52, 54, 55. The sum of the measures of these state descriptions is: $m(e' \,.\, l') = 1/5$. Hence we have for the degree of confirmation:

$$c = (1/5)/(3/10) = 2/3.$$

But formula (7.2) also yields:

$$c = \frac{\binom{2+4-1}{1+1}}{\binom{3+4-1}{1+1}} = \frac{\binom{5}{2}}{\binom{6}{2}} = 10/15 = 2/3.$$

(c) *Instance Confirmation*

Instance confirmation deals with a hypothesis not about the whole population (universal inference), or the part of the population not in the sample (modified universal inference), but concerning only one *single* individual which is not in the sample.

Let the assumed law say as before that all individuals which have a simple property M also possess the simple property M'. Define the property $M_1 = M \,.\, \sim M'$. This is the property which contradicts the assumed law. Let its logical width be w_1.

The evidence e'' consists of a sample of s individuals. There are s_1 individuals with the property M_1. These individuals violate the postulated law. The remaining $s - s_1$ individuals in the sample satisfy the postulated law. They have the property $\sim M_1$.

Let now l'' stand for the hypothesis that an individual which is not in the evidence e will have the property postulated by the law, i.e. will not have the property M_1. The instance degree of confirmation is, according to Carnap:

$$c_i = 1 - \frac{s_1 + w_1}{s + k}. \tag{7.3}$$

It is interesting to note that formula (7.2) becomes identical with (7.3) if $N = s + 1$, i.e. if the population assumed for modified universal inference consists of one more individual than the sample.

If the sample is large relatively to k (number of Q-properties in the language) formula (7.3) becomes:

$$c_i = 1 - (s_1/s). \tag{7.4}$$

The quantity s_1/s is the relative frequency in the sample of the cases which violate the assumed law.

Example 26.—We use again the attributes introduced in Example 3. Let $M = P_1 = Q_1 \vee Q_2$. Also, $M' = P_2 = Q_1 \vee Q_3$.

The property which contradicts the postulated law is: $M_1 = M.\ (\sim M') = Q_2$. There is a single Q-property corresponding to this situation. Hence the logical width of M_1 is $w_1 = 1$.

Consider now a language with $N = 4$ individuals and the 4 Q-properties Q_1, Q_2, Q_3, Q_4. Space does not permit to give a list of all the structure descriptions and state descriptions. We may, however, indicate some of the relevant information by listing some structure descriptions and a typical state description for each structure description as well as the number of state descriptions which are isomorphic to the given state description:

Structure description		*Q-properties* a_1	a_2	a_3	a_4		*Number of isomorphic state descriptions*
S_1	.	1	1	1	1	.	1
S_2	.	1	1	1	2	.	4
S_3	.	1	1	1	3	.	4
S_4	.	1	1	1	4	.	4
S_5	.	1	1	2	2	.	6
S_6	.	1	1	2	3	.	12
....							
S_{35}	.	4	4	4	4	.	1

(The reader can easily construct the missing structure descriptions.)

The evidence consists of three individuals, a_1, a_2, a_3. ($s = 3$.) Let there be among the sample of three $s_1 = 1$ individual which contradicts the law. This individual has the Q-property Q_2, which corresponds to M_1. There are also $s - s_1 = 2$ individuals which satisfy the postulated law.

The evidence e'' consists of three individuals. We select the state descriptions which are as follows: In the first three columns (a_1, a_2, a_3) we have once Q_2 and twice either Q_1, Q_3 or Q_4.

The table of structure descriptions indicated above shows that the following state descriptions correspond to the evidence: 3/4 of all the state descriptions in the following structures, S_2, S_6, S_7, S_{14}, S_{15}, S_{16}, S_{27}, S_{28}, S_{29}, S_{30}; also one-half of all the state descriptions in the following structures, S_-, S_{12}, S_{13}, S_{24}, S_{25}, S_{26}, satisfy the same condition. The sum of the measures of all the state descriptions which belong to the evidence is: $m(e'') = 3/10$.

The hypothesis l'' says the following: The individual a_4 which was not included in the sample forming the evidence e'' must conform to the postulated law; i.e. for the conjunction $e''\ l''$ we select from the state descriptions listed above for e'' those which have either 1, 3 or 4 in the last column. (Our law excludes only the Q-property Q_2.) It is easily seen that all the state descriptions listed above fulfil this condition, except those belonging to the following structure descriptions: S_5, S_{12}, S_{13}, S_{21}, S_{25}, S_{26}. The sum of the measures of all the state descriptions which correspond to the conjunction of evidence and hypothesis is: $m(e''\,.\,l'') = 3/14$.

Hence we have for instance confirmation:

$$c_i = (3/14)/(3/10) = 5/7.$$

Formula (7.3) yields also:

$$c_i = 1 - \frac{1+1}{3+4} = 5/7.$$

(d) *Qualified Instance Confirmation*

Here we consider the same law as before; if an individual which is not in the evidence has the property M it has also the property M'. For universal inference, modified universal inference and instance confirmation we have assumed that the law says categorically that the individual in question has not a certain property (which is itself a conjunction of two properties).

The evidence is here a conjunction of two components: (1) e''' defined similarly as under 7 (*c*) for instance confirmation; (2) a sentence j which says that a new individual not in the sample which forms the evidence e''' has the property M.

The hypothesis l''' asserts that this individual has *also* the property M'. According to Carnap the qualified instance degree of confirmation of the law that all M are M' is defined as the degree of confirmation of the hypothesis l''' with respect to the combined evidence $e'''\,.\,j$:

$$\text{(7.5)} \qquad c_{qi} = m(e'''\,.\,j\,.\,l''')/m(e'''\,.\,j)$$

$$= 1 - \frac{s_1 + w_1}{s_1 + w_1 + s_2 + w_2}.$$

Here w_1 is as before the logical width of the property $M_1 = M\,.\,{\sim}M'$. This is the property which contradicts the assumed law. But w_2 is the logical width of the property $M_2 = M\,.\,M'$.

This is the property postulated by the law. s_1 is the number of cases in the sample with the property M_1; and s_2 the number of individuals with the property M_2.

By comparison of (7.5) with (7.4) we see that

$$c_i > c_{qi}.$$

If the sample is large relatively to the number k (number of Q-properties in the language considered) then (7.5) becomes—

$$c_{qi} = \frac{s_2}{s_1 + s_2}.$$

This is the relative frequency of the cases which correspond to the assumed law among all cases in the sample which possess the property M.

Example 27.—Consider a problem similar to the one discussed in Example 26. The language is the same as in this example. The assumed law is also the same.

The property which contradicts the assumed law is: $M_1 = M . (\sim M') = Q_2$. There is a single Q-property corresponding to this situation. Hence the logical width of M_1 is: $w_1 = 1$.

Next we have the property which corresponds to the assumed law: $M_2 = M . M' = Q_1$. Again, there is a single Q-property which corresponds to M_2. Its logical width is: $w_2 = 1$.

It should be noted that in Example 3, $k = 4$. But evidently $w_1 + w_2 = 2$. The properties M_1 and M_2 do not exhaust all possible situations with a language with two attributes. The remaining two Q-properties are covered by: $\sim M = Q_3 \vee Q_4$.

Consider now the language in Example 26 (N 4). The evidence consists of the three individuals a_1, a_2, a_3; $s = 3$. Let there be in the sample of three $s_1 = 1$ individual which contradicts the law (i.e. has the property M_1). This individual has the Q-property Q_2 which corresponds to M_1. There is also $s_2 = 1$ individual which possesses the property postulated by the law, i.e. which has the property $M_2 = Q_1$. Finally, there is $s - s_1 \quad s_2 = 1$ individual which has the property $\sim M = Q_3 \vee Q_4$. This individual has neither the property M nor the property M_2.

The evidence e''' consists of three individuals a_1, a_2, a_3. For the evidence we select the state descriptions which are as follows: In the first three columns (a_1, a_2, a_3) we have once Q_1, once Q_2 and once either Q_3 or Q_4.

The table of structure descriptions indicates that the following state descriptions correspond to the evidence e''':—one-half of all the state descriptions in the following structures: S_6, S_7, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}.

The sentence j says that the individual a_4 which does not appear in the sample constituting e''' has the property M, i.e. possesses the Q-properties Q_1 or Q_2.

Hence for the conjunction $e''' . j$ we have to select from the state descriptions given above for e''' those which have also 1 or 2 in the *last* column. It is seen from the table that one-half of the state descriptions in the following structure descriptions correspond to the conjunction $e''' . j$: S_6, S_7, S_{12}, S_{13}. The sum of the measures of these state descriptions is: $m(e''' . j) = 6/35$.

The hypothesis l''' says now that a_4 has the property $\sim M_1$. This means the following: a_4 has the Q-properties Q_1, Q_3 or Q_4. It should be remembered that our law excludes only the Q-property Q_2.

For the conjunction $e''' . j . l'''$ we have to select from the state descriptions corresponding to $e''' . j$ given above those which have 1, 3 or 4 in the last column. We see that one-half of the state descriptions corresponding to the structure descriptions S_6 and S_7 satisfy this condition. Hence the sum of the measures of the state descriptions which correspond to the conjunction $'' . j . l'''$ is $m(e''' . j . l''') = 3/35$.

The qualified instance degree of confirmation is then:

$$c_{qi} = (3/35)/(6/35) = 1/2.$$

This is the probability$_1$ on the basis of the knowledge of the observed sample (e''') and the fact (j) that a certain individual not belonging to the sample has the property M for the hypothesis that this individual will also have the property M'.

But Carnap's formula gives also

$$c_{qi} = 1 - \frac{1+1}{1+1+1+1} = 1/2.$$

We have $w_1 = 1$ as in Example 26. The logical width of $M_2 = M \,.\, M' = Q_1$ is also: $w_2 = 1$.

It should be noted that a similar evidence was used in Example 27 as in Example 26; but in Example 27 the evidence contains in addition to e''' the statement j. It appears that the qualified instance degree of confirmation (1/2) is smaller than the instance degree of confirmation (5/7).

8. *Testing Statistical Hypotheses.*

Carnap's methods described in Section 7 may also be used for testing statistical hypotheses. In testing statistical hypotheses we select (arbitrarily) a certain level of probability$_1$ p' (say $p' = 0{\cdot}95 = 95$ per cent.). This corresponds to the level of significance in statistics. We will reject all hypotheses whose degree of confirmation is less than p' and not reject those whose degree of confirmation is larger than p'.

The application of Carnap's theory to the testing of statistical hypotheses will be illustrated in the following examples:

(a) *Universal Inference*

Example 28.—Consider again two simple attributes, P_1 and P_2 described in Example 3. We use now the interpretation given there and denote by P_1 nickel head, by $\sim P_1$ nickel tail, by P_2 dime head and by $\sim P_2$ dime tail.

The "law" in question asserts that in a population of 10 throws the property $P_1 \,.\, \sim P_2$ cannot occur. This is to say, it is claimed that in a throw of a nickel and a dime it is impossible to obtain simultaneously: nickel head and dime tail.

This "law" may be considered a particular form of a gambling system. It is, perhaps, worth while noting that propositions not very different from this have been proposed by statisticians and seriously discussed in the literature.*

The evidence consists of a sample of 5 throws with the two coins. All 5 throws confirm the "law," i.e. we have not once obtained simultaneously nickel head, dime tail.

According to Carnap's formula (7.1) the degree of confirmation of our law under universal inference is:

$$c = \frac{\binom{5+4-1}{1}}{\binom{10+4-1}{1}} = 8/13 = 0{\cdot}615.$$

Since this is less than $0{\cdot}95$, we see no reason why the hypothesis that the "law" holds should not be rejected.

(b) *Modified Universal Inference*

Example 29.—We consider again a situation as described in Example 28. We note that in universal inference the degree of confirmation would become zero if in our sample there was a single instance contradicting the "law" of Example 28. Hence we must use modified universal inference if there are in the evidence cases which contradict the "law."

Let us now assume that in a sample of 5 there is one case which contradicts the "law" and there are 4 cases which conform to it. This is to say: among the five throws with the two coins we have once obtained: nickel head, dime tail.

* K. Marbe: *Grundfragen der angewandten Wahrscheinlichkeitsrechnung*. Munich, 1934. *Das Ausgleichungsprinzip in der Statistik*. Munich, 1938. L. von Bortkiewicz: *Die Iterationen*. Berlin, 1917. L. V. Furlan: *Das Harmoniegesetz der Statistik*. Basel, 1946.

The hypothesis is that in the remaining five cases not included in the sample the "law" holds. The degree of confirmation is now, according to (7.2) in this case of modified universal inference:

$$c = \frac{\binom{5+4-1}{1+1}}{\binom{10+4-1}{1+1}} = \frac{\binom{8}{2}}{\binom{13}{2}} = 0{\cdot}359.$$

Since this probability$_1$ is less than 0·95 we see again no reason not to reject the hypothesis that the "law" holds in the remainder of the population of 10.

(c) *Instance Confirmation*

Example 30.—Using again the attributes of Example 3 we have now a situation which is as follows:

Assume the evidence consists again of 5 throws. Of these we have one which violates the "law" of Example 28 (i.e. shows nickel head, dime tail). There are four cases which do not contradict the "law."

The instance degree of confirmation concerns the probability$_1$ for the hypothesis, that a new throw not among the five constituting the sample will conform with the "law." This is to say, this new throw cannot give: nickel head, dime tail.

Carnap's formula (7.3) gives here the following value for the instance degree of confirmation:

$$c_i = 1 - \frac{1+1}{5+4} = 7/9 = 0{\cdot}778.$$

This is again less than 0·95. Hence there is no reason not to reject the hypothesis formulated above.

(d) *Qualified Instance Degree of Confirmation*

Example 31.—We assume here a situation like that in Example 30. The hypothesis concerns a throw with two coins which is not included in the five throws which form the observed sample. However, it is already known that *this new throw shows nickel head.* The hypothesis says that it also has dime head according to the "law."

Let the evidence consist of 5 throws. One result violates the "law" (i.e. shows nickel head, dime tail). Three cases have nickel head, dime head, i.e. possess the exact property postulated by the "law." The last case has nickel tails. Hence we have $s_1 = 1$, $s_2 = 3$.

The formula (7.4) given by Carnap yields the qualified instance degree of confirmation as follows:

$$c_{qi} = 1 - \frac{1+1}{1+1+3+1} = 2/3 = 0{\cdot}667.$$

This is again less than 0·95. Hence there is no reason not to reject the hypothesis formulated above.

9. *Choice between Statistical Hypotheses*

In this section we will treat the application of Carnap's methods to the choice between hypotheses or theories. We will choose the hypothesis with the highest degree of confirmation. This can again be done by using universal inference, modified universal inference, instance confirmation or qualified instance confirmation.

These ideas will be illustrated in the following example:

Example 32.—We consider again a language like the one described in Example 3 above. We assume two exclusive "laws" or hypotheses:

l_1: If P_1, then P_2; if nickel head, then dime head.
l_2: If P_1, then not P_2; if nickel head then dime tail.

The two "laws" l_1 and l_2 are evidently incompatible.

The evidence obtained in five throws with the two coins is presented in the following table:

Q_1 nickel head, dime head	1 time.
Q_2 nickel head, dime tail	0 times.
Q_3 nickel tail, dime head	1 time.
Q_4 nickel tail, dime tail	3 times.

(a) *Universal Inference*

In order to treat the problem of the choice between the two hypotheses by universal inference, we list below for each "law" the simple property M_1; this is for each hypothesis the property which is *excluded* by the "law" in question:

"Law"	*Excluded property*
l_1 .	Q_2
l_2 .	Q_1

Using Carnap's formula for universal inference we remember that $w_1 = 1$, since M_1 for each "law" corresponds to exactly one Q-property. Let the population consist of ten items. We have for the various "laws" the following degrees of confirmation:

$$c_1 = \frac{\binom{5+4-1}{1}}{\binom{10+4-1}{1}} = 8/13 = 0 \cdot 617.$$

$$c_2 = 0.$$

The last result follows immediately from the consideration that the property excluded by the "law" l_2 has actually occurred. Hence, in the circumstances, we would from the point of view of universal inference prefer the hypothesis l_1 to the other. It is the only one which has a positive degree of confirmation.

(b) *Modified Universal Inference*

The conclusion just stated was derived from the principle of universal inference, which postulates the property in question for the whole population, and hence permits no exceptions in the sample. We want here to use modified universal inference, which is less stringent. We list below for each "law" the number of cases in which it has been violated in the sample of five throws just given:

"Law"	s_1 *number of violations*
l_1 .	0
l_2 .	1

Let the population again consist of 10 individuals. We have then for the degrees of confirmation of the "laws" or hypotheses:

$$c_1 = 8/13 = 0 \cdot 617$$

$$c_2 = \frac{\binom{8}{1+1}}{\binom{13}{1+1}} = 28/78 = 0 \cdot 364.$$

Under modified universal inference the hypothesis postulates only that with the remaining individuals in the population not included in the sample the "law" shall not be violated. On this basis we would also prefer the "law" l_1 to the other, according to the evidence given. It has again the highest probability$_1$.

(c) *Instance Confirmation*

Instance confirmation concerns a prediction for one of the items in the population which is not in the sample. The probability$_1$ (degree of confirmation) of the various "laws" in these circumstances is:

$$c_1 = 1 - \frac{1}{5 + 4} = 8/9 = 0{\cdot}889$$

$$c_2 = 1 - \frac{1 + 1}{5 + 4} = 7/9 = 0{\cdot}778.$$

It follows that on the basis of the idea of instance confirmation we would again, given the evidence, prefer the "law" l_1 to l_2.

(d) *Qualified Instance Confirmation*

With qualified instance confirmation we make a prediction about a member of the population which is not in the sample. But here we assume that the individual in question is already known to possess one definite property.

We list below for the "laws" the number of cases in which the postulated property has actually occurred in the sample:

"Law"	*Number of confirmations* s_2
l_1	1
l_2	0

We compute for the various hypotheses the probability$_1$ (degree of confirmation) on the basis of the idea of qualified instance confirmation:

$$c_1 = 1 - \frac{1}{1 + 1 + 1} = 2/3 = 0{\cdot}667$$

$$c_2 = 1 - \frac{1 + 1}{1 + 1 + 1} = 1/3 = 0{\cdot}333.$$

On the basis of the evidence, we would again prefer the first "law" to the second, if we use qualified instance confirmation. It has the higher probability$_1$.

10. *Limitations of Carnap's Theory*

The theory constructed by Carnap has been sketched and illustrated. It has also been applied to various problems arising in statistical inference. From the practical point of view of the statistician the most important limitation is this: it cannot be applied to situations which involve continuous variables. Hence a problem like the celebrated Fisher-Behrens problem is as yet out of its reach. There is some hope that the theory can eventually be extended to questions of this type.

But I believe that the illustrations given above indicate the great power of the theory and the possible usefulness for statisticians in the limited field for which it is already applicable. It has been possible to construct a "pure" theory of prediction, estimation, testing hypotheses and choice between statistical hypotheses which is independent of pragmatic considerations. Such considerations may, however, be of importance in statistical practice. They can probably be dealt with by methods similar to the theories developed in mathematical economics when dealing with non-static problems.*

* G. Tintner: "The theory of choice under subjective risk and uncertainty," *Econometrica*, vol. 9, 1941, pp. 208 ff. "The pure theory of production under technological risk and uncertainty," *ibid.*, pp. 305 ff. "A contribution to the nonstatic theory of production." Lange *et al.*, ed. *Studies in Mathematical Economics and Econometrics*, Chicago, 1942, pp. 92 ff. "A contribution to the nonstatic theory of choice," *Quarterly Journal of Economics*, vol. 56, 1942, pp. 274 ff. "The theory of production under nonstatic conditions," *Journal of Political Economy*, vol. 50, 1942, pp. 645 ff. L. Hurwics: "Theory of the firm and investment," *Econometrica*, vol. 14, 1946, pp. 153 ff.

Discussion on Professor Tintner's Paper

Professor G. A. Barnard: It is a great pleasure to move a vote of thanks to Professor Tintner. I agree with his opinions on the pragmatic theories of statistical inference that have been put forward recently. One had come to think that scientific people believed in the existence of the objective world and that the purpose of scientific theory was to reflect the objective properties of the objective world, and that the status or otherwise of a scientific theory did not and should not depend on the particular purpose which the individual holding the theory might have in mind. One had thought that there was such a thing as objective scientific truth which did not depend on the particular aims, feelings, or other intentions of the observer. Pragmatism appears to conflict with this idea and should, accordingly, have no place in a sound scientific theory.

The other point of major agreement with Professor Tintner concerns his list of the current theories of inference and his insistence that not all of these can be right. Another variation of the pragmatic belief in discussing inferences is the view that it does not really matter which theory you accept—that theories are tools and different people can use different tools and get the same sort of answer with them. That attitude contains an element of truth, namely, that no theory put forward in a finite space or time can hope to cover all the situations with which a statistician may be confronted in the course of his practical work, so that if a theory claims that it is one method of analysing all statistical problems, that theory would be wrong. But it remains true that if we have a given concrete problem there will be only one correct method of dealing with it. But that will not necessarily be the method for dealing with another problem.

Now I come to the points of disagreement, not so much with Professor Tintner as with Professor Carnap, whose views he is explaining. The Rev. Thomas Bayes considered that he had shown how to solve a problem providing the prior probabilities could be evaluated. He had one view and Professor Carnap has another, and there is very little to be said about Professor Carnap's views which cannot also be said of Mr. Bayes's. The crux of the theory is in the allocation of equal measures to the different structure descriptions. Why should one do this? As far as I can see one can get a clue to the answer by considering that the allocation of prior probabilities should be invariant under permutations of the basic "properties," $P_1, P_2, \ldots, P_p$. This means, roughly, that we are not giving any preference to one of the properties over another. I am not sure whether this makes the choice of the prior probabilities unique, but it certainly limits the choice.

It also brings out the arbitrariness of the choice, because it is far from obvious that the basic properties should be treated on an equal footing. Quite apart from pragmatic considerations, which admittedly should be excluded, one can imagine grounds on which distinctions between the different basic properties could be justified. For example, some of the properties might be regarded as more stable than others. The property of the head of a coin of being shiny is much less stable than the property of its being a head. Such a difference in stability might be used to justify a treatment in which these properties were not put on an equal footing. If we admitted this, Carnap's theory would be inapplicable.

None the less, the theory does show how one can lay down in a systematic and non-contradictory way the prior probabilities. It amounts to saying that once the set of properties which an individual can have or not have is restricted to a finite number, the totality of possible hypotheses is fixed, and is finite in number. Within this totality one can find a basic set of mutually exclusive hypotheses, such that any possible hypothesis is formed by disjunction from members of the basic set, and the problem of the prior probabilities is solved once the probabilities have been allocated to the hypotheses of the basic set. But the difficulty remains that the choice of probabilities for the hypotheses of the basic set is still arbitrary.

One final point. As Professor Tintner pointed out, throughout the paper the notion of Probability$_1$ has been used, and the notion of Probability$_2$ has not been used at all. It seems to me that this implies that this theory, while it may be regarded as a theory of induction, cannot be regarded as a theory of *statistical* inference. If when we repeat an experiment we always get the same result, we have a problem of induction, but no statistical problem. Statistical problems arise because sometimes, when we repeat an experiment, we do not get the same result. The essence of a theory of statistical inference should lie in the way in which it relates Probability$_1$, the degree of confirmation, with Probability$_2$, the frequency. It is valuable to have these two concepts distinguished. The distinction is not altogether new, but it is useful to emphasize it. Having made it, however, it is necessary to relate the two concepts in some way in order to get a genuine theory of statistical inference.

We must all thank Professor Tintner for his very clear exposition of Professor Carnap's theory and, if I may say so, a further cause for congratulation is that this exposition is to some extent an exposure.

Mr. F. J. ANSCOMBE (in seconding the vote of thanks): Professor Tintner has rendered us a valuable service in giving this clear and easily intelligible account of Professor Carnap's theory of inductive logic. One has heard a good deal of this theory, but Carnap's own papers are couched in logical phraseology which I for one have found unfamiliar and difficult, and I was pleased and relieved when Professor Tintner first showed me his paper and the whole system was clearly revealed. Through his direct contact with Carnap, Professor Tintner has been able to give us an exposition which we can accept as authoritative.

The theory itself I do not like. To begin with, it sets out to be a "pure theory of induction, independent of pragmatic considerations." I cannot see any reason to look for such a theory. We only accept a formal system of deductive logic if it appears to generalize arguments that we agree are "sound" or "reasonable." No internal consistency will identify a branch of mathematics as logic. Sooner or later we must answer the question: does this formal system represent what we believe to be valid forms of argument? The situation is the same with induction, except that now there is very much less general agreement as to what constitute sound arguments. But still, of any proposed formal system of inductive logic we must ask whether it leads, in particular cases, to arguments that we can agree to be good, if not necessarily the best.

It seems to me that in scientific work the problem never arises of basing an inference strictly on a certain well-defined body of data and on nothing else. In particular, remarks by Professor Pearson, and discussion with Dr. Daniels, have convinced me of the importance of risk considerations even in "pure" scientific research. If a researcher draws a wrong inference from his experiments he may waste much time before he finds out his mistake. He will therefore have some preference for inferences that can quickly be either confirmed or disproved.

Granted, however, that we try to find an ideal non-pragmatic theory of inference, is the one that Professor Tintner gives us acceptable? Despite its different formulation, it seems to work out very like Professor Jeffreys's probability theory when the prior probabilities are chosen to denote ignorance. If Carnap's theory is developed to deal with continuous variables, I think this similarity will become even clearer. But whereas the arbitrariness of Jeffreys's prior probabilities is open and obvious, in Carnap's theory the arbitrariness is hidden in the structure of the language. But it is there just the same. When Professor Tintner talks about the "language," he means, not the language of the science to which the observations are relevant, but a particular subset of sentences in that language which has to be carefully chosen and for the choosing of which we are given absolutely no guidance. If we wish to look at a certain set of observations from more than one point of view, we have to compose a special language for each question we wish to ask.

Suppose, for example, that we have some bags of grain that may be infested by the larvae of an insect pest, the number of larvae per bag being possibly anything from 0 to, say, 10,000. We select three bags at random and determine the number of larvae in them, and find the answer 0 twice and 2 once. But if we are interested in whether the fourth bag is infested, we may ask a question in a language which has two sentences concerning any bag, "it is infested" and "it is not infested." We can then work out the degree of confirmation that the fourth bag contains no larvae. If, on the other hand, we are simply interested in estimating the number of larvae in the fourth bag we presumably use a language which has 10,001 sentences concerning any bag, namely, "it contains n larvae" ($n = 0, 1, 2, \ldots$ 10,000). We shall now get an entirely different degree of confirmation that the fourth bag has no larvae. We shall get a different answer again if we are interested in whether the infestation is light, say under 20 larvae, or heavy, with 20 larvae or more. If we use Professor Jeffreys's methods, there is an analogous shuffling of the prior probabilities, but the conventional character of the procedure is much more obvious.

To sum up, it seems to me that the intention of this theory is mistaken, and that, moreover, it has succeeded in removing certain difficulties from the domain of inference only by inserting them at full strength into the domain of language construction. I do not, therefore, expect that the theory will lead to the solution of any of the philosophical problems with which the statistician is faced. But the theory is still young, and its approach is new. It is, however, interesting that such an approach should be tried, and I am sure that everyone will wish all possible success to those who are labouring to develop it.

The vote of thanks was put to the meeting and carried unanimously.

Professor BARTLETT added his appreciation of this paper. One or two of the points he wished to make had to some extent been covered by previous speakers. With regard to Professor Tintner's question whether only one theory could be "right," he himself did not think that one theory need necessarily be right in contrast with others. That was, of course, one of the great difficulties.

Coming to the question of probability$_1$ *versus* probability$_2$, Carnap had stressed this distinction between probability$_2$ or frequency, and probability$_1$ or degree of belief or confirmation. As

Professor Barnard had noted, this was not new; Carnap himself had referred to the late F. P. Ramsey's ideas, which stimulated his (the present speaker's) own distinction, when in 1933 he suggested that one should use capital P for the first kind and small p for the second.*

The attraction of Carnap's probability$_1$ was, as Carnap had pointed out, that it was perhaps rather unjustifiable to think of it as a degree of belief as, for example, it had been thought of by Jeffreys. Carnap's own phrase, "degree of confirmation," tended to recognize that. This was to some extent attractive, but care must be taken not to confuse logical clarity and simplicity with the complete solution of the induction problem.

The third point in its favour was connected with the fact of considering individuals as members of groups quite on a par with each other, and not in themselves. That to his mind was in one sense the very essence of the statistical method, but he was not sure how far this was relevant to the question of induction with which they were concerned here.

He now came to the difficulties of the theory, which were very familiar. As other speakers had pointed out, this theory did not really (apart from its greater logical clarity) differ much from other theories—for example, that of Jeffreys. First, the measure which one assigned was, of course, arbitrary. The second point, as mentioned by the last speaker, was associated with the difficulty of logical width. The arbitrariness of the measure could here be compared with the non-invariance of prior probabilities with change of scale. If he just had "not the ace of spades," he would have, if he understood it rightly, a logical width of 1; if he had "not the ace and not spades," the logical width would be 3; and if he thought of all the other cards in the pack, the logical width jumped up to 51. Alternatively, if this example seemed artificial, he would instance Mendel's genetical experiments in which one had short plants and tall plants; then the logical width which one would assign would not have any particular relation to the 3 to 1 genetically important ratio. It could be argued that the logical width was related to the *logical* structure, and that it was rational to have a logical width which changed, but it was going to cause a considerable amount of trouble.

The last point was that Carnap worked all the time with probability$_1$; he never got to probability$_2$, namely, the frequency, which was of interest to statisticians, and this was really unfortunate because, supposing his inference was considered in relation to the sample, all he considered was the number of individuals in that sample. He never asked "Is this sample a random sample?" whereas that was a vital question to the statistician. In connection with probability$_2$ it would be the crucial point.

He might recall the "confidence statement" type of inference which the statistician sometimes preferred to use, because in this simple case of ordinary attributes, the confidence statement became a very simple kind of inference. As he had pointed out in the discussion in the Research Section recently on Professor Barnard's paper, such inferences did not avoid induction, but in this particular type of problem it became a very minimum of induction that one used, namely, that the sample was in fact a random one from the finite population. If that assumption were made it was possible to make a confidence statement about a further sample or about the total population.

In Example 23 Professor Tintner had noted that logical width 1 should be logical width 3, as he had noticed himself; but in either case if the 95 per cent. zone were considered for the value of n_1 (for simplicity he had avoided any ambiguity due to considering both tails of the distribution) one merely excluded the first value given, $n_1 = 5$. If one worked out the probability$_2$ zone on a confidence statement basis—and he recalled that he considered this particular application of confidence intervals in a paper† in the *Supplement* to the *Journal* in 1937—one would find that 95 per cent. of the probability$_2$ zone also excluded $n_1 = 5$ (or less than 5).

He was aware that he was not using quite a fair example, and that in general the confidence statement zone would, especially for small samples, be apparently less efficient than that by Carnap's method, but with the gain that it was independent of any arbitrariness in measure. It would not be affected by changing the logical width from one value to another. It was expressed in terms of probability$_2$, the associated 5 per cent. representing an upper bound of the direct probability$_2$ of misstatement.

Mr. W. Perks said that as an unrepentant supporter of inverse probability he would remind the Fellows of Professor Jeffreys's statement that the effect of a normal change in the prior probability distribution was equivalent to the effect of only one observation. He would support the primary idea of Professor Carnap's approach, namely, his desire to produce a "pure" theory, but, as previous speakers had said, the actual method seemed to be largely a restatement of the Bayes-Laplace theory in terms of modern logic. It was therefore subject to a number of objections.

* *Proc. Roy. Soc.*, A. 141, p. 518.
† *J. R. Statist. Soc.* (*Suppl.*) 4, p. 131.

In particular, it produced results similar to those produced by a uniform prior probability distribution, and did not take account of the invariant indifference rules developed by Professor Jeffreys in the second edition of his book, and by the speaker in a recent paper in the *Journal of the Institute of Actuaries.*

A further objection lay in the treatment of multiple alternatives, Q-properties, and the inability to deal with continuous variables. The Bayes-Laplace rule for multiple alternatives gave inconsistent results, as was pointed out in his own paper. He proposed later to suggest a modification of Carnap's method which seemed to meet all the objections.

Professor Tintner had resurrected a mistake which had dogged the footsteps of many writers on inverse probability. This appeared on p. 259, where a reference was made to the principle of indifference in what the author called the classical theory. Suppose they had four urns with three balls in each; in the first urn there was no white ball; in the second, one; in the third, two, and in the fourth three. The structure descriptions were then as stated in the paper for Example 1. It was true that a ball chosen at random had a probability of one-half of being white, but the classical theory did not suggest the use of the binomial theorem to obtain the probability of the structure descriptions of three balls because the conditions required the three balls to be from the same urn. The figures given by the author of $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$ and $\frac{1}{8}$ depended on the independence assumption of the binomial which did not apply. Therefore, on p. 259, line 13, it would seem more appropriate to substitute the word "abuse" for "use."

He then returned to the question of multiple alternatives, and took as an illustration Example 2, where there were four alternatives. If these were divided into two pairs the logical widths as defined on p. 261 were $2 + 2 = 4$. If, however, they started with each of the pairs as simple alternatives, which seemed equally justifiable, there were only 2 Q-properties and the logical widths would be $1 + 1 = 2$. The two approaches gave different results and previous speakers had referred to the point, but he had repeated it because in his own paper he gave a rule which overcame these difficulties. Suppose they had a die with six faces; they usually thought of these faces being numbered 1 to 6, but three of them might be coloured red and three white. If they now imagined a die having two million sides, half coloured red and half coloured white, it would require an enormous sample before a degree of confirmation differing from one-half would be obtained. If one thought of the million red sides and the million white as two simple alternatives one did get reasonable results. He would suggest that that difficulty could be overcome by defining the logical width, not as the *number* of Q-properties corresponding to the property M in the disjunction treatment on p. 261, but more appropriately as the *proportion* of Q-properties in the disjunction. Formula 2.6, which, as had already been pointed out, in principle merely defined the prior probability distribution, would be modified so that W_1 and W_2 would become complementary fractions and k would be unity. The formula would then be interpreted in terms of gamma functions and the inconsistencies he had mentioned would disappear. The results shown on p. 265 would then become more reasonable. They would seem to be consistent with his own rule for multiple alternatives, and in the limit when N tends to infinity the invariant binomial rule resulted.

With this modification the method did give very interesting new results for a finite universe with the higher values for the prior probabilities at the extremes of the distribution which seemed to be required. Perhaps even more important than this was the fact that the use of proportions for the width showed the way in which continuous variables could be brought within the system. In his own paper he did not deal with the general question of invariant rules for two or more parameters at a time. For one parameter his rule, in terms of standard error of the corresponding sufficient statistic, was, as Professor Jeffreys had acknowledged, equivalent to Jeffreys's rule. A similar standardizing rule in terms of the volume of the Ellipsoid of Concentration of the distribution applicable to the set of corresponding sufficient statistics, as defined in Professor Cramer's book, would give a basis for two or more parameters at a time equivalent to Jeffreys's general rule, and would elucidate some of the remaining problems in this theory.

It seemed to him clear that a pure theory of probability which dealt satisfactorily with direct probability, including biased coins and human choice in calling to a tossed coin and also with statistical inference, must contain at least one undefined idea and one arbitrary rule. He recognized the need for special treatment in both these respects to deal with the problem of infinity. He preferred the undefined idea of "equally likely" combined with an appropriate prior probability rule.

It had been pointed out that the two corresponding ideas of Carnap's theory were given on p. 258, which, in contradistinction to inverse probability, had also to state a rule corresponding to Bayes's theorem. In the frequency theory, the limit of relative frequency involved the idea of a random experiment which could be analysed into "equally likely" plus "independence," and therefore seemed to be unnecessarily complex. The Neyman-Pearson confidence theory involved

a "rule of conduct" on the assumption that there was no prior knowledge. If confidence coefficients were used in the sense of posterior probabilities it seemed that this implied the invariant prior probability rule. This was indicated in the discussion on the speaker's paper, but something similar seemed to be involved in a recent paper by Professor Gini of Italy.

Mr. STONE said that there were two closely related points he would like to make in connection with Professor Tintner's most interesting paper. First he would like to point out that the assignment of prior probabilities on the basis that each structure was equally likely, as given in Example 10, was identical with D'Alembert's solution in his celebrated article "Croix ou Pile" in the *Encyclopédié ou Dictionnaire Raisonné* (1754). The question proposed there was to find the chance of throwing a head in the course of two throws with a coin. Letting H stand for a head and T for a tail, D'Alembert detected three cases which he considered to be equally likely, namely

H (game terminates)
TH
TT

whence the required (prior) probability was 2/3 as against the ordinarily accepted value of 3/4. Similarly with three throws we should have according to D'Alembert four cases, namely

H (game terminates)
TH (game terminates)
TTH
TTT

whence the required (prior) probability was 3/4 as against the ordinarily accepted 7/8. It was only necessary to note that in this case D'Alembert was equating the prior probabilities of getting 0, 1, 2,... tails to see that the cases distinguished by him were nothing other than Carnap's structures.

D'Alembert was severely taken to task for expressing these views. For example, in his *History of the Theory of Probability* (1865) Todhunter wrote (p. 258) : "This great mathematician is known in the history of the Theory of Probability for his opposition to opinions generally received; his high reputation in science, philosophy and literature have secured an amount of attention for his paradoxes and errors which they would not have gained if they had proceeded from a less distinguished writer."

He did not know enough about D'Alembert to say how far these strictures were justified in his case, but he submitted that as a general statement and outside a particular context they were not justified at all so far as the present example was concerned. Indeed, from some points of view the D'Alembert-Carnap solution of the above problem was more interesting than the classical one, because it did not involve smuggling in so many assumptions about the actual world.

His second point was that the classical solution assumed that we knew enough about coins, which no doubt we did, to justify the assignment of prior probabilities on a specific assumption about the bias in any coin we should be likely to encounter. Thus, if p were the prior probability of throwing a head in a single throw, the probability of obtaining a head in n throws or the probability of not obtaining a tail at each throw was

$$1 - (1 - p)^n$$

With $p = \frac{1}{2}$, i.e. on the usual assumption that the possibility of bias was unimportant to the point of non-existence, and $n = 2$, this yielded the classical 3/4. If, on the other hand, we took the standpoint that we postulated bias but did not know what bias to assume, then it would be reasonable to suppose that p might take any value between 0 and 1 with equal probability. The solution to our problem would then be, as Professor Tintner had suggested in his discussion of Example 10,

$$\int_{p=0}^{1} [1 - (1 - p)^n]\, dp.$$

With $n = 2$ this yielded

$$\int_{p=0}^{1} (2p - p^2)\, dp = [p^2 - \tfrac{1}{3}p^3]_0^1 = 2/3$$

while with $n = 3$ it yielded 3/4 and so on.

On the assumption that we had no empirical knowledge, in this case that in practice coins are more or less unbiased, which was the effective justification for concentrating the whole weight of the distribution of p at the value $p = \frac{1}{2}$, it seemed to him that the D'Alembert-Carnap solution was the more appropriate one. Objection might of course be raised to the particular method of expressing our lack of knowledge of the bias we expected to encounter, but in all ordinary

cases some method must be found of doing this. The exceptional cases were two in number. In the first place we might intend to undertake such an immense enquiry, and in general this would not be practically possible, that we should be led empirically to the correct assessment of chances independently of our expectations at the outset. In the second place we might already have so much empirical knowledge that we had good empirical grounds for particular expectations. In any actual case we had to try to express the state of our expectations, but if we had no reason to expect lack of bias to the virtually complete exclusion of other possibilities we had no right to make the simple assumption that $p = \frac{1}{2}$.

Dr. J. O. IRWIN (the following contribution was received after the meeting): Most of the more philosophical and logical criticisms which I intended to make have been made by other speakers, but I would like to add a few words about the particular "model" or "set up" adopted. This is a set of $k = 2p$ Q-properties to which N individuals are to be allocated. In the terminology of that excellent old book, Whitworth's *Choice and Chance*, which managed to cover most of the ground that matters in the application of probability to attributes without ever being highbrow, this is the problem of dividing N different "objects" into k different "parcels." This can be done in k^N ways, which is the total number of state descriptions, if some of the Q-properties are permitted to be absent. If exactly r of them are to be present this can be done in $\Delta^r (0^N)$ ways, and this is the number of state descriptions involving a particular set of r of the Q properties. These r can be selected in $\binom{k}{r}$ ways so that we have the identity

$$k^N = \sum_{r=1}^{k} \binom{k}{r} \Delta^r (O^N) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which is in fact a particular case of Newton's forward difference formula and so serves to verify the reasoning.

Now consider that the N individuals are indifferent or indistinguishable, the number of ways of allocating them to r different "parcels" (with no blanks) is $\binom{N-1}{r-1}$, and this is the number of structure descriptions involving r particular Q properties.

The number of structure descriptions involving r Q-properties is therefore $\binom{k}{r} \binom{N-1}{r-1}$ and the grand total

$$\sum_{r=1}^{k} \binom{k}{r}\binom{N-1}{r-1} = \binom{k+N-1}{N} = T \text{ say} \quad . \quad . \quad . \quad . \quad (2)$$

which provides another verification.

Any particular one of these structure descriptions can be represented by the partition

$$N = (p_1^{\pi_1} p_2^{\pi_2} \ldots\ldots p_\lambda^{\pi_\lambda}) \text{ with } \left.\begin{array}{l} \Sigma p\pi = N \\ \Sigma\pi = r \end{array}\right\} \quad . \quad . \quad . \quad (3)$$

provided we specify which of the r parts is to be allocated to each quality, and the number of state descriptions in this particular structure description is easily seen to be

$$\frac{N!}{(p_1!)^{\pi_1} (p_2!)^{\pi_2} \ldots\ldots (p_\lambda)!^{\pi_\lambda}} = U (p_1^{\pi_1} p_2^{\pi_2} \ldots\ldots p_\lambda^{\pi_\lambda}) \text{ say.}$$

The number of state descriptions corresponding to this particular partition is therefore

$$\frac{r!}{\pi_1! \, \pi_2! \ldots\ldots \pi_\lambda!} \cdot \frac{N!}{(p_1!)^{\pi_1} (p_2!)^{\pi_2} \ldots\ldots (p_\lambda!)^{\pi_\lambda}} = V(p_1^{\pi_1} p_2^{\pi_2} \ldots\ldots p_\lambda^{\pi_\lambda})$$

and therefore $\Sigma \, V(p_1^{\pi_1} p_2^{\pi_2} \ldots\ldots p_\lambda^{\pi_\lambda}) = \Delta^r (O^N)$,

the summation being over all partitions of N into r parts, a known identity which provides another verification.

The measure of any state description included in a structure description of type (3) is therefore $1/TU$ which provides a general formula.

Professor TINTNER said he would reply to most of the points raised in the discussion in writing, but would deal now with one point made by Professor Bartlett—the relation between probability,

and probability$_2$. The various writers he had quoted had indicated a number of relationships between these two concepts. Carnap had stressed the differences rather than the relationships because it was his aim to bring out the contrast between these concepts. It seemed to him that in a great many of the discussions the confusion did not arise so much because people held different views about probability, but because they did not make a clear distinction between the two concepts. Probability$_1$, he thought, could be applied also to statements on probability$_2$.

Another relationship which could be established between the two ideas and was evident from the formulæ given in the paper was the following: Any system of inductive inference must have this property, that if more and more information was obtained, the probability$_1$ must tend towards the relative frequency (probability$_2$). This was certainly true for Carnap's concept of degree of confirmation. But the particular way of assigning fundamental measures was of importance in small samples where there was little information, and here he believed that Carnap's theory as presented to-day could make a contribution.

Professor Tintner subsequently wrote as follows:

I am very grateful for the contributions made by those who participated in the discussion. It is indeed gratifying to have the ancestry of the ideas presented traced back to such a distinguished lineage as d'Alembert and Ramsey. The analogies with Jeffreys's theory are obvious, and the reference to Whitworth's combinatorial calculus will be most helpful. Carnap's forthcoming book will deal with the historical background of his ideas.

The particular way in which Carnap has assigned the *same* measure to each structure can only be justified by the simplicity of the procedure.* He also demonstrates that this method is superior to the assignment of the same measure to each state description (Wittgenstein). If the second method is adopted the probability$_1$ is independent of all previous experience. This is evidently not a desirable property for the degree of confirmation.

Another justification can be given for an analysis of predictive inference (see above, section 3b).† Carnap's rule, which corresponds to Laplace's rule of succession, is: $c = (s_1 + w_1)/(s + k)$. This is the probability$_1$ that an item which is not in the sample will have the property M. It can be shown that $w_1/k \leq c \leq s_1/s$ or $s_1/s \leq c \leq w_1/k$. Hence the probability$_1$ for predictive inference lies always between the relative logical width and the relative frequency of cases which have the property M in the sample. It tends for large samples towards the second quantity. Hence the particular way of assigning the fundamental measures is irrelevant in large samples.

Invariance properties will doubtless be of great importance for the theory when it becomes applicable to continuous variables. Discussion of this question had better be postponed until such a theory has been constructed.

Certain requirements have been formulated by Carnap‡ regarding the language underlying his system of inductive inference. The atomic sentences should be logically independent. The individuals should be different and entirely separate. The logical predicates (attributes) should be logically independent of each other. The qualities and relations designated by the primitive predicates should be simple, i.e. it must not be possible to analyse them into simpler components. (There is still a certain amount of difficulty connected with this concept.) The set of primitive predicates must be complete.

In computing the degree of confirmation the *total evidence* available should be taken into account. The only evidence which can be neglected is irrelevant evidence, i.e. evidence which does not change the degree of confirmation. Carnap also sketches the possibility of taking order (e.g. temporal relations) into account. This amplification of the theory may make it possible to deal with random temporal sequences.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

John Ernest Adamson.	Kenneth George Odell.
Leslie Thomas Andrew.	John Gold Ratcliffe.
Cecil Gilbert Burgess.	Joseph Arthur Saltmarsh.
George Cork.	Samuel Jeyasingam Somasundaram.
William Fletcher.	Charles Arthur Stebbing.
Sir Geoffrey Heyworth.	Anthony Charles Stonell.
Harold Lampkin.	George Alfred Tunnell.
William Joseph Massey.	Frank Leslie Wheeler.

* R. Carnap, "On inductive logic," *op. cit.*, pp. 80 ff.
† *Loc. cit.*, pp. 86 ff.
‡ R. Carnap. "On the application of inductive logic," *op. cit.*, pp. 135 ff.

Some Uses of Statistics in the Building Industry: An Investigation into the Erection Times of Nine Types of Non-Traditional House

(Made by the Statistical Section of the Chief Scientific Adviser's Division of the Ministry of Works)*

Read before the Royal Statistical Society, April 27th, 1949,

by Dr. J. Bronowski

the President, Dr. David Heron, in the Chair

1. *Introduction*

This paper presents the statistical results of a part of an investigation† made by the Chief Scientific Adviser's Division of the Ministry of Works mainly during the years 1946 and 1947. The object of the investigation was to discover whether economies could be made in man-power and scarce materials by using new methods of house construction, and if so, to estimate the trend in cost. The statistical techniques developed and used in the analysis presented here were applied only to the labour requirements of the non-traditional houses.

The results presented are interim results, and are based on an analysis which in some respects is simplified and tentative, applied to data which are neither as complete nor as systematic as might have been wished. Nevertheless, it was thought worth while to attempt this analysis for two reasons, one practical and one theoretical. On the practical side, it was believed that the data collected by the Division already sufficed to show conclusively that certain of the non-traditional methods of construction offered a very substantial saving in man-hours over traditional methods. On the theoretical side, it was believed that a practical analysis now would serve to develop the methods required for future analyses, and single out necessary improvements in technique and in design.

The basis of our findings, quantitative and qualitative, was the analysis of man-hours on ten sites employing nine different methods of non-traditional construction. A basis of comparison was provided by a parallel analysis on four sites employing traditional methods of construction. Attention was naturally concentrated on those operations in which non-traditional methods differ fundamentally from traditional methods: construction of the external walls, the party wall and the internal partitions, which in the traditional house are built of brick and in the non-traditional house usually of larger and partly pre-fabricated units. These three categories, which we shall bracket together as "Walls," absorb about one-quarter to one-third of the man-hours used on the site. They are, in effect, the only categories to which the present experiment in non-traditional methods was applied.

Since these results are confined to man-hours spent on the site, they necessarily give only an incomplete picture. They require to be augmented on the one hand by costs, and on the other hand by similar observations at the works, the components factory and the brick yard. Nevertheless, the present results make a necessary contribution to our knowledge of the economics of non-traditional house building.

* The senior officers of the Statistical Section of the Chief Scientific Adviser's Division, during the time that this work was carried out, were Messrs. J. Bronowski, N. H. Carrier, J. R. Illingworth, W. J. Reiners, S. Rosenbaum and W. Rudoe, all of whom contributed to it at one time or another. The present account of the work has been written by J. Bronowski, J. R. Illingworth and W. J. Reiners, and they alone are responsible for the presentation, both of methods and of results, which follows.

† The whole experiment has been described in National Building Studies, Special Report No. 4, "New Methods of House Construction," H.M.S.O., 1*s*.

2. *Description of the Experiment**

Groups of houses, generally more than 50 in number, were built in semi-detached pairs by new methods of construction on sites situated in various parts of the country. These Development Groups were under constant observation by specially trained staff who made the necessary recordings. Parallel observations were made on sites where traditional brick houses were being built. It was expected, therefore, that a fair comparison might be made between new and traditional methods, since both would have experienced similar difficulties in labour and material supplies, which were characteristic of the period.

Since the main purpose of the experiment was comparison between the various methods of construction, it was apparent that reliable data could not be obtained from the usual methods of costing current in the building industry. The sites under observation employed widely differing types of construction, and it was therefore necessary to use a method of observation which would apply to all sites and forms of construction, and which would provide data suitable for analysis in accordance with the aims of the research. It was further essential that the data obtained should be sufficiently detailed to allow direct comparison not only between types of house but between individual components of the houses.

It was realized from the outset that trends of improvement would be met in both site organization and acquired skills; and that in the essence of the problem these would have to be estimated and allowed for. In order that mean times estimated should allow for this factor and also that variability might be studied, it was essential to allocate labour expenditure on each operation to each house individually.

The method of observation developed is new to the building industry, and it had, therefore, to be adjusted to meet novel situations as they arose on the different sites. It is based upon the preparation of a Schedule of Operations. This shows the house divided into major groups of operations, each of which is subdivided into a number of constructional operations. If it is necessary for any purpose, an operation may be divided still further if each part can be separated and observed. For the present analysis the groups of operations have been consolidated again into nine main categories of house components. (See Table I.)

The preparation of the Schedule of Operations for each site was carried out in collaboration with the respective sponsors and builders, after examination of their plans and specifications. While the number and description of operations in any group may vary from site to site or scheme to scheme, the corresponding groups are directly comparable. There are a few cases where this is not immediately possible; e.g. plastering to internal, external and party walls must be recorded as one operation and later apportioned on a *pro rata* basis. And again, labour expenditure on certain operations cannot be directly allocated to individual pairs of houses. Such general operations, e.g. site clearance, are included in the later groups of the Schedule, and their man-hour totals finally allocated on a *pro rata* basis to all houses on the site.

A team of observers was stationed on each site. Their primary duty was to record, at frequent intervals, man-hours and machine-hours spent on each operation on each block or pair of houses. Times of general operations were recorded separately and, as already stated, finally allocated to individual blocks. Experience showed that recording at 15-minute intervals was possible when the observer had not more than 20 men to watch. Towards the end of the site development, when the majority of the operations under observation were internal, it became difficult to maintain a regular observation of each operative at 15-minute intervals.

There was much unproductive time on the sites, as a result of bad weather, the experimental nature of the types of house, shortages of materials, use of trainees, and the need to repeat unsatisfactory work. Recording of non-productive time is more complete where observation is continuous; consequently, that recorded is mainly due to bad weather and the repetition of bad work, where the amounts are considerable. Every effort was made, however, to separate out the productive time, since any comparison between types of house must obviously be based on times from which have been eliminated the effects of factors not inherent in the schemes.

* Before giving a description of the experiment, it should be explained that the Statistical Section had no opportunity to participate in its initial design or control. In fact, the experiment was in an advanced stage of development before the statistical group of workers was formed. This explains why the statistical treatment was subject to some of the handicaps which will be met later.

TABLE I

Definition of Categories

Main Category	*Category*	*Definition*
Sub-structure	Foundations	..
	Ground Floor	..
Superstructure	Walls:	
	External Walls	All walls left ready for decoration. All external walls from damp proof course to eaves level, including door frames, and windows ready for glazing.
	Internal Partitions	All internal walls, including door frames.
	Party Wall	..
	First Floor	Includes ceiling.
	Roof	All work above eaves level, including stacks, etc. Includes ceiling. (Each category in super-structure includes scaffolding.)
Finishings	Joinery	Includes staircase.
	Plumbing	..
	Electrical	..
	Heating	Includes Ministry of Works unit.
	Gas	..
	Glazing	..
	Decorating	..
Ancillary Work	Drainage and Main Services	Drainage; water, gas and electrical mains.
	Site Works	Site clearance, gardens. Roads, paths and walls.
	Welfare	..
	Site Office Staff	Timekeeper, clerks, watchmen.
	Sundries	Plant maintenance. Erection of offices and stores.

Unloading, handling and preparatory work done on site have been allocated to the category concerned. Supervision (foreman, walking gangers and storemen) has been allocated on a *pro rata* basis to all categories.

A daily sheet giving all the productive times and non-productive times for each trade for each block operation, together with times for mechanical plant, was sent to the group analysing the data. Before leaving the site, this was balanced against the contractor's records, to ensure that the total man-hours agreed with his pay-sheet figures.

3. *Treatment of the Data*

The method of summarizing the data was evolved during the experiment, and has been dictated to some extent by frequent calls for interim information. At the beginning of the experiment the man-hour records were entered by hand on cards, but the bulk of the recording later was done by accounting machines. For each site a set of cards was kept covering each operation on each pair or block of houses on the site. Each card showed the site, block number, operation

number, and description. Each entry showed the date, site record sheet number, trade classification, and the hours expended under productive time and the specific sub-groups of non-productive time. A cumulative total was kept of productive and non-productive times to date irrespective of trade. Periodically the cards were summarized and balanced with the total recorded hours. Man-hour totals for completed operations only were used in this analysis.

This method of recording and summarizing the data was slow, and required a considerable clerical staff. The obvious advantage of recording the data on punched cards and making the subsequent summaries on sorting machines was recognized, and data from the last five of the sites were dealt with in this way. Any future experiments of this nature will be planned with pre-coding for punched card analysis as one of the fundamental requisites.

The data required some preliminary processing before analysis. First, it was necessary to confine the analysis to a manageable number of categories, and for this purpose the whole house was divided into two Sub-structure categories (Foundations and Ground Floor), five Super-structure categories (External Walls, Internal Partitions, Party Wall, First Floor and Roof), and Finishings and Ancillaries. It was therefore necessary to consolidate the detailed observations on individual processes into these nine categories appropriately. The scheme on which this was done, with the advice of those technically expert in building practice, is given in Table I. It is to be noted that while ancillaries contribute labour cost to each house, they have not been used in the comparative analysis, since they are unrelated to the purpose of the experiment.

Second, in order to make the work manageable, it was necessary to reduce the number of minor variants within each scheme or method of construction on a site. Technical advice was sought to establish for each scheme one standard house—so far as possible that type of house on which, it was believed, the particular scheme was coming progressively to concentrate. The technical experts also estimated the difference in man-hours to be expected where a variant feature or operation in a house differed from that in the standard house on that site. We had hoped, where necessary, to apply these corrections immediately to the man-hour figures as observed, in order to convert them into parallel figures for standard houses erected in the same order and under similar conditions. In practice, however, there were a number of design or method differences where the experts were in doubt as to the correction which should be applied; and these uncertainties lent force to the general objection to accepting such crude corrections, which is made in Section 6 below. Generally, therefore, it was also necessary to obtain a statistical estimate of the differences from the site observations. The simplest statistical technique was used for the purpose, based on the familiar method of maximum likelihood: and no differences were considered unless they exceeded 10 man-hours.

4. *Outline of the Statistical Method*

It is the aim of our analysis to predict the erection times in the future of each type of house tested in the experiment. For this purpose we have from each site recordings of the productive times, under each of the nine category headings, for the erection of the houses. We can use the data from a particular site to calculate average category and total erection times of the houses on this site, together with the variability of these times. It is therefore possible to construct confidence intervals which forecast the category and total erection times of the houses when built on the same site and under identical conditions. This process constitutes the first stage of the statistical analysis, and will be described in more detail under the heading, "The Primary Analysis."

It will be noticed that the predictions of the primary analysis are limited in their application, not merely each to its own non-traditional scheme, but each to its own scheme when built on a site identical with that on which it was observed. Or to put the matter somewhat differently: the figures derived in the primary analysis for any non-traditional scheme are functions of two consolidated variables, the scheme and the site. There is nothing in the primary analysis to distinguish the influence of one from the other.

It is the first problem of any further analysis to separate, in the results of the primary analysis, the influence of the scheme proper from the influence of the particular site conditions under which it was tested. The problem is somewhat simplified if the schemes do not respond differentially to the varying conditions on the different sites. We have technical advice which confirms

that this is so. It is then sufficient to compare estimates for the erection of the non-traditional schemes on a standard site, say one corresponding in conditions to the average of those met in the experiment. Under this assumption the comparison will hold on all sites, and can be considered as measuring the relative performance of the schemes in the future. The problem is to obtain a numerical measure of the effect of the differences from site to site, either systematic (giving a measure of the efficiency of the site), or random; and then to construct new confidence intervals which include an allowance for site-to-site variation, and possibly an adjustment for systematic site-to-site differences in efficiency.

In a statistically planned experiment, the estimation of the site effect would be a major objective of the design, which ideally would replicate the same scheme on different sites. In the present analysis we do possess, potentially, a certain amount of such replication. In the first place we have four similar traditional schemes on different sites, and two Type 3 schemes on different sites. Secondly, we have information that the non-traditional methods have been applied to the Wall categories only, and that the non-Wall categories are, in the main, physically alike. We have then data from which a measure of site-to-site differences may be constructed, provided that they can be shown to be sufficiently homogeneous among themselves. Accordingly, we begin by using the duplication of the four traditional and two Type 3 sites to prove that the Wall categories within the same scheme and the non-Wall categories form a homogeneous race with respect to inter-site variation. This then becomes the basis for an Analysis of Variance, in which we compare the contribution to the total variability made by the non-Wall categories, which represents inter-site variation, with the contribution made by the Walls, which represents inter-site plus inter-scheme variation.

This analysis proves to correspond to the physical facts in a striking manner. It shows that, on different sites, the Wall categories in the non-traditional schemes do not belong to a homogeneous race, while the non-Wall categories do. Indeed, if the categories are not consolidated into Wall and non-Wall, but are analysed separately, there are indications that the Wall categories, and they alone, have the suspected heterogeneity of race.

The data from the non-Wall categories are now available for the detailed study of inter-site variation, in which we attempt at first to evaluate a figure of merit for each site representing a measure of efficiency of all work on the site. The analysis shows, however, that the variations from category to category are of a purely random nature, with no systematic tendency over any one site. The evidence, therefore, is that the sites are a random sample from a homogeneous (though internally highly variable) race. To predict performances on an average site, therefore, the confidence intervals found in the primary analysis must be enlarged by an allowance for random site-to-site variation.

There is one more technical remark which deserves a place in this general description. We have said that the Analysis of Variance which tests for these heterogeneities must assume the variability of the same scheme on different sites to be roughly the same for Wall and non-Wall categories. And we have said that this assumption can be tested and confirmed over the more restricted data given by the duplicated parts of the experiment. There is an independent test furnished indirectly by another statistical approach. This is to rank the figures for the Wall and non-Wall categories on the different sites, and to derive a statistic from ranking only. This method, which is effectively free of all assumptions, shows the Wall categories to be related to the scheme and site together with high significance, and the remaining categories to be unrelated. It thus confirms the findings of the Analysis of Variance, and, indirectly, its assumptions.

5. *The Primary Analysis*

In the primary analysis each site is treated as an entity of its own and no reference is made to conditions on any other site. For each site separately we require to estimate the expected times for the nine categories when constructed on a site and under conditions similar in all respects to those of the experiment.

The data for this analysis have been described above, and consist of the productive time on each block for each category. Associated with each of these figures we have the date of starting and of finishing the work, so that it is possible to isolate a trend with time in the man-hour figures for each category. A common feature of this trend is a "prototype" effect, an exceptionally high

TABLE II.—*Actual Average and Improved Average (These Results are only Applicable*

	Poured Concrete		*Pre-cast Concrete*			
	Type 1	*Type* 2	*Type* 3 (*Site* a)	*Type* 3 (*Site* b)	*Type* 4	*Type* 5
Foundations						
Actual	200±10	144± 4	178± 9	335± 10	No data	372± 38
Improved	186±11	135± 5	178± 9	335± 10	,,	372± 38
Ground Floor						
Actual	55± 5	39± 3	88± 7	22± 2	,,	44± 5
Improved	55± 5	31± 4	88± 7	22± 2	,,	44± 5
Substructure Total						
Actual	255±10	183± 5	266±12	357± 10	,,	416± 41
Improved	241±10	166± 5	266±12	357± 10	,,	416± 41
External Walls						
Actual	787±28	417±13	259± 8	283± 10		591± 16
Improved	725±37	377±15	244±12	283± 10		554± 20
Internal Partitions						
Actual	226± 9	133± 5	191± 6	223± 19	546±13	214± 10
Improved	221± 9	116± 6	184± 9	214± 19	530± 19	190± 12
Party Wall						
Actual	145± 5	63± 2	55± 2	79± 3		113± 9
Improved	136± 7	57± 2	54± 2	76± 4		98± 11
First Floor						
Actual	86± 7	95± 2	95± 3	115± 8	84± 3	184± 12
Improved	77±10	91± 3	93± 4	111± 8	84± 3	138± 14
Roof						
Actual	336±12	209± 6	193± 7	213± 7	167± 7	345± 18
Improved	311±12	197± 6	192± 9	205± 7	165± 9	252± 16
Superstructure Total						
Actual	1,580±42	917±14	793±19	913± 42	797±14	1,447± 44
Improved	1,470±52	838±16	767±23	889± 57	779±18	1,232± 49
Finishings						
Actual	654±19	777±20	640±29	905± 49	No data	891± 28
Improved	646±19	739±20	619±32	905± 49	,,	789± 29
Ancillaries						
Actual	657	401	620	620	,,	829
Improved	657	401	620	620	,,	829
TOTAL HOUSE						
Actual	3,146±77	2,278±31	2,319±66	2,795±102	,,	3,583± 83
Improved	3,014±93	2,144±34	2,272±79	2,771±102	,,	3,266±103

Notes:

1. The "Actual" times are the means of all blocks observed including prototypes and abnormalities.
2. The "Improved" times refer to the mean time excluding prototypes, abnormalities and "intermediate" blocks, i.e. blocks constructed while improvement was taking place.

figure for the first and perhaps second block constructed, followed by a steady decrease for the succeeding blocks until about half the houses are completed. That is, for a time there is a progressive improvement of work on the site. The exact nature of this trend is not relevant to the comparison between schemes, as its form is dictated by the quality of the labour and site organization and not by a particular system of construction. It is necessary to study the trend in some detail in the primary analysis, however, in order to obtain an efficient estimate of the variability of the man-hour figures.

The "actual average" productive times have been listed in Table II. Each is calculated as the mean time of all the houses on one site, including both prototypes and abnormalities. Thus, our statement that the actual average productive time for a complete house of Type 9 is, in round

Productive Times per House (in man-hours).
for each Scheme on its Specific Site)

Pre-cast Concrete			Timber Framed	Traditional Brick			
Type 6	*Type 7*	*Type 8*	*Type 9*	*Site B*	*Site A*	*Site D*	*Site C*
237± 8 190± 9	101± 4 101± 4	No data ,,	133± 8 133± 8	233±18 233±18	214±11 195±11	155±12 142±12	155± 7 147± 7
43± 2 42± 2	23± 2 22± 2	,, ,,	22± 2 22± 2	42± 4 42± 4	16± 1 10± 1	70± 9 70± 9	56± 2 55± 2
280± 9 232±10	124± 4 123± 4	,, ,,	155± 8 155± 8	275±25 275±25	230±10 205±10	225±10 212±11	211± 8 202± 9
771±18 723±18	344± 9 301±12						
224±10 189±14	101± 3 95± 4	589±23 576±33	566±24 522±30	1,108±49 1,108±49	707±16 669±16	800±26 771±26	935±14 912±15
81± 5 81± 5	72± 2 50± 2						
143± 7 129± 7	102± 3 97± 2	57± 2 58± 3	56± 3 56± 3	89± 5 89± 5	30± 2 30± 2	91± 4 88± 5	87± 4 86± 4
204± 6 197± 6	163± 6 154± 8	189± 5 193± 8	101± 6 96± 6	283±11 283±11	166± 5 150± 7	259±13 245±14	295±10 291±10
,423±27 ,319±38	782±13 697±15	835±26 827±34	723±27 674±38	1,480±57 1,480±57	903±14 849±14	1,150±28 1,104±28	1,317±15 1,289±16
,307±26 ,240±29	993±18 892±22	No data ,,	685±28 685±28	823±26 823±26	346± 8 324±11	690±43 690±43	531±13 506±22
569 569	520 520	,, ,,	405 405	658 658	313 313	977 977	814 814
,579±40 ,360±51	2,419±41 2,232±43	,, ,,	1,968±38 1,919±47	3,236±70 3,236±70	1,792±26 1,691±27	3,042±72 2,983±72	2,873±36 2,811±40

3. 90 per cent. confidence intervals are defined by the figures after the ± sign.
4. The Ancillary times were observed for the site as a whole and apportioned equally to all blocks on the site. No confidence limits can therefore be given for them as their accuracy is not known.

figures, 1,950 ± 50 man-hours, means that we are able to assert, with 90 per cent. confidence, that if Type 9 house is built on other sites under conditions similar in all respects to those which obtained on this site, the average over all these sites of the productive time, per complete house, will lie between 1,900 and 2,000 man-hours (in round figures).

Productive time is a better basis for comparison between schemes than total time, but it is not the best comparison at our disposal. The existence of the trend with time described above makes it possible to calculate an improved figure for the man-hours in any category, in which the disproportionate times expended on this category on the first houses of the site have been discounted. These "improved averages" represent the figure at which a category settles down on a site of 50 houses when improvement tails off. They are not affected in magnitude by the duration of the

improvement, which is largely governed by the organization of the site. They form the best basis for comparisons which imply prediction on future sites, and are therefore listed separately in Table II.

6. *The Data for the Secondary Analysis*

We may briefly characterize the data as a rectangular array in which the columns (say) represent the fourteen sites, and the rows represent the various building categories, such as Foundations, First Floor, Roof, etc. A typical entry (i, j) is the improved productive time observed for category i on site j. These times refer to the standard house which has been selected for each site as described above.

We recall that the specifications of the standard houses differ from site to site in two ways. First, there are the fundamental differences between the non-traditional schemes which appear on the ten sites, e.g. in one scheme the walls are concrete poured *in situ*, and in another are of pre-cast concrete posts and panels. It is the effects of this type of difference in construction with which the experiment is concerned. The non-traditional methods were in general confined to the three Wall categories—External Walls, Internal Partitions and Party Wall. The houses of Type 5 and Type 9 were exceptional in that the design of both the First Floor and Roof were non-traditional, and were related to the method of construction of the Walls.

Second, there are a number of variations in design of a minor nature, which are not connected with the non-traditional system of construction. An example of this type of difference is the use of one coat of plaster in the houses on some sites and two coats in those on other sites. These latter variants in design serve only to confuse the discrimination between the non-traditional schemes. It was therefore our intention to reduce such variants to a common standard, applicable to all sites for each category. We had experimented in the primary analysis with the use of corrections proposed by those expert in building technique, to adjust for variants within each site in otherwise similar designs; and it was tempting to follow the same procedure here. But to attempt quantitative corrections based on such advice supposes us to possess in advance the very estimates of difference which the analysis seeks, and does so with no estimates of the errors which they carry. In failing to isolate differences which exist, the statistical analysis of course introduces errors; but the theory of statistics is substantially the study of such errors, and a proper analysis covers them in the confidence limits which it obtains. Adjustments whose margins of error are unknown, far from improving our predictions, strip them of any measure of confidence and in effect deprive them of value.

It was, however, possible in certain cases to make corrections based entirely on observations, the variability of which was known. Where there was no basis for such a correction, the technical hypotheses were submitted to statistical test and accepted only when supported by the data. This treatment was also applied to test the effect of abnormal conditions which occurred on certain sites. An example of conditions described as abnormal is given by the Foundations category of Type 5 houses, when the laying of foundations was impeded by the remnants of war-destroyed houses on the same site.

7. *The Test of Ranking Concordance*

We have stated in the preceding paragraphs that the differences between the standard houses on the different sites were confined to the Wall categories, and that the remaining categories were in the main physically alike. This was the advice of the technical experts, and it was our first objective to seek some confirmation for this from the data. This was obtained initially by means of a test of ranking concordance, which is free from any assumption concerning the frequency distributions of the man-hour figures. For this test the sites were "ranked" within each category, that is, were arranged in order of ascending man-hours. If there is no relation between the times for any set of categories and the sites on which these times have been observed, the rankings of the sites in the various categories will show no resemblance; and the sums of the ranks of each site in the different categories will tend to be equal, apart from the fluctuations of random sampling. If a relation between the category times and the sites does, however, exist,

this will be revealed by certain sites having, say, a low ranking in all the categories considered, so that the rank totals will show a significant discrepancy from equality as we go from one site to another. The resemblance of ranking can be defined by a statistic W, known as the Coefficient of Concordance,* and tested for significance by Fisher's Z test. This test was applied separately to both the Wall categories (External Walls, Internal Partitions and Party Wall) and the non-Wall categories (Foundations, Ground Floor, First Floor, Roof and Finishings). The value of W ($W = 0{\cdot}797$) obtained for the Wall categories indicated a significant resemblance between the rankings at the 1 per cent. level, whereas for the non-Wall categories the value of W ($W = 0{\cdot}213$) showed no departure from a random distribution. It appeared immediately, therefore, that the man-hours expended in the Wall categories were not independent of the combination site-plus-scheme, whereas the times for the non-Wall categories were quite unrelated to site-plus-scheme. This important result indicated (although it could not prove) the following two conclusions:

(i) there were significant differences between the schemes only in respect of the Wall categories;

(ii) the variations in the non-Wall categories could be taken as a measure of inter-site variation alone.

Both these logical deductions served as a most useful pointer to the lay-out of the quantitative analysis by which they were finally established.

8. *The Logarithmic Transformation*

The ranking tests of the previous Section pointed immediately to the use of the technique of Analysis of Variance, to compare the inter-site variances of the Wall and the non-Wall categories as a means of obtaining numerical demonstration of results (i) and (ii) above. It was clear, however, both from previous work and from the crude calculations, that categories with a large man-hour figure tended in any case to have a large variance, whatever other factors were operating; so that no direct comparison of the variances of categories having different means could lead to results of any value. In addition, the following known characteristics of man-hour data seemed to indicate that an assumption of normality was false:

(i) Standard deviations frequently occurred which were from 30 per cent. to 70 per cent. of the mean. A normal distribution fitted to such data would give a high probability for obtaining negative man-hours.

(ii) Distributions of man-hour data were skew, figures larger than the mean being noted more frequently than figures smaller than the mean.

(iii) Although variances fluctuated widely, the coefficients of variation tended to fall within narrow limits.

These properties all seemed to indicate that the logarithms of the man-hour data might be distributed normally, rather than the data themselves. This was, in fact, taken to be the case without an exhaustive examination of the original and transformed data, for which the time was not available. The result has, however, been confirmed since. It would have been preferable, of course, to have taken logarithms of the basic data in the primary analysis, but unfortunately the logarithmic analysis was not introduced until it was too late to undertake the extensive re-computations which would have been required.

The stability of the variance of the transformed data in the "traditional" categories is illustrated in Table III. The Ground Floor category has been omitted from this table, and also from the following analysis, as being too small in magnitude to give any reliable measure of the inter-site variation. The homogeneity of this set of variances has been tested by the L test of Neyman and Pearson. The value obtained, $L = 0{\cdot}855$, does not indicate a significant departure from homogeneity.

* Kendall, M. G., *The Advanced Theory of Statistics*, **1**, 411; **2**, 215.

TABLE III.—*Comparative Sets of Variances*

Source of Variation within the Categories	*Estimates of Variance*	*Degrees of Freedom*
Foundations	0·007,227	8
First Floor	0·011,104	10
Roof	0·008,065	10
Finishings	0·002,528	9
Between Type 3 Walls	0·002,828	1
,, Traditional Walls	0·003,426	3

A further merit of the logarithmic transformation is that it makes additive those effects which were multiplicative in the untransformed data. For example, one would expect good or bad site organization to reduce or increase the man-hours required for a given category by a certain percentage, rather than by a certain amount. Thus site and scheme or category effects could now be regarded as additive.

These two factors—additiveness of the class effects, and stability of the variance—satisfied the necessary conditions for the study of the transformed data by the methods of the Analysis of Variance.

9. *The Statistical Technique*

We have stated that the purpose of the logarithmic transformation was to permit the treatment of the data by the method of the Analysis of Variance. This technique, however, can be applied in its simple form only when the data are balanced or orthogonal, a condition that is certainly not satisfied here, owing to the presence of missing values and of alternative means in the same category. The presence of missing values would not alone have prevented the use of the standard methods of the Analysis of Variance, since an approximate treatment is available which is sufficiently accurate if the proportion of missing values is not excessive. The further complication of alternative category means, however, corresponding to different methods or designs, cannot be treated accurately within the framework of a simple analysis of variability into row and column components; and such analyses were only used as an approximation in preliminary and exploratory investigations.

The nature of the statistical hypotheses which were being tested was fortunately such that even with the unbalanced data available, it was possible to derive exact tests by identifying the data with a normal regression system—a technique that includes the standard Analysis of Variance as a special case.

The most general hypothesis has the form

$$x_{ij} = N\{\mu_{ik} + \lambda_j;\ \sigma^2\} \qquad (1)$$

where x_{ij} is the observation in the cell (i, j); μ_{ik}, λ_j and σ^2 are constants, σ^2 having a unique value for the system. The λ_j represent the site effect, independent of the category means, and the μ_{ik} (having a number of different values for each i) correspond to the category means for the different methods or designs, independently of the site effect. The nature of this lay-out may best be appreciated by reference to Table IV. This table shows the expected values of the observations x_{ij} in each cell, blanks in the array indicating missing values (including some values rejected because work was sub-standard).

Comparing the expression (1) with the standard form of a normal linear regression

$$x_{ij} = N\{\Sigma a_q z_q;\ \sigma^2\} \qquad (2)$$

it is clear the (1) and (2) may be identified; in fact, the a_q represent the values μ_{ik}, λ_j while the z_q take the values 1 or 0. The array of values x_{ij} has thus been identified with a sample from a normal linear regression system.

For data of this kind the estimates of the parameter values together with the appropriate tests of significance, based on the principle of maximum likelihood, can be expressed in the form

TABLE IV

Site or Type	Category: Foundations	First floor	Roof	Finishings	Walls
1	$\mu_1 + \lambda_1$	$\mu_2 + \lambda_1$	$\mu_3 + \lambda_1$	$\mu_4 + \lambda_1$	$\mu_{01} + \lambda_1$
2	$\mu'_1 + \lambda_2$	$\mu_2 + \lambda_2$	$\mu_3 + \lambda_2$	$\mu_4 + \lambda_2$	$\mu_{02} + \lambda_2$
3*a*	$\mu_1 + \lambda_3$	$\mu_2 + \lambda_3$	$\mu'_3 + \lambda_3$	$\mu_4 + \lambda_3$	$\mu_{03} + \lambda_3$
3*b*	..	$\mu_2 + \lambda_4$	$\mu'_3 + \lambda_4$	$\mu_4 + \lambda_4$	$\mu_{03} + \lambda_4$
4	..	$\mu_2 + \lambda_5$	$\mu_3 + \lambda_5$	..	$\mu_{04} + \lambda_5$
5	..	$\mu'_2 + \lambda_6$	$\mu''_3 + \lambda_6$	$\mu_4 + \lambda_6$	$\mu_{05} + \lambda_6$
6	$\mu_1 + \lambda_7$	$\mu_2 + \lambda_7$	$\mu_3 + \lambda_7$	..	$\mu_{06} + \lambda_7$
7	..	$\mu_2 + \lambda_8$	$\mu_3 + \lambda_8$	$\mu_4 + \lambda_8$	$\mu_{07} + \lambda_8$
8	..	$\mu_2 + \lambda_9$	$\mu_3 + \lambda_9$	..	$\mu_{08} + \lambda_9$
9	$\mu_1 + \lambda_{10}$	$\mu''_2 + \lambda_{10}$	$\mu'''_3 + \lambda_{10}$	$\mu_4 + \lambda_{10}$	$\mu_{09} + \lambda_{10}$
B	$\mu'_1 + \lambda_{11}$	$\mu_2 + \lambda_{11}$	$\mu_3 + \lambda_{11}$	$\mu_4 + \lambda_{11}$	$\mu_0 + \lambda_{11}$
A	$\mu'_1 + \lambda_{12}$	..	..	..	$\mu_0 + \lambda_{12}$
D	$\mu_1 + \lambda_{13}$	$\mu_2 + \lambda_{13}$	$\mu_3 + \lambda_{13}$	$\mu'_4 + \lambda_{13}$	$\mu_0 + \lambda_{13}$
C	$\mu_1 + \lambda_{14}$	$\mu_2 + \lambda_{14}$	$\mu_3 + \lambda_{14}$	$\mu'_4 + \lambda_{14}$	$\mu_0 + \lambda_{14}$

of determinants. It is simpler in practice, however, not to use such formulae but to evaluate the quantities required directly from the likelihood equations, basing ourselves on the general theory only for the proof that the sums of squares arising in the test ratios are distributed as χ^2, and that therefore their ratio is distributed as Snedecor's F.

Tests of Significance

As an example of how the significance tests are formulated, consider the tests for the systematic differences between the sites, given by the λ_j.

Following the method of Wilks* we may define the permissible subspace Ω of the parameter hyperspace by

$$-\infty < \mu_{ik} < +\infty \qquad \text{all } i, k.$$

$$-\infty < \lambda_j < +\infty \qquad \text{all } j.$$

$$0 < \sigma^2 < +\infty$$

together with the condition

$$\sum_j \lambda_j = 0$$

which stipulates that the level of the category means is that appropriate to the "average" site of the experiment.

The subspace ω of Ω is defined by

$$\lambda_j = \lambda \qquad \text{for all } j.$$

Now define

$$S^2 = \sum_{ij} (x_{ij} - \mu_{ik} - \lambda_j)^2$$

and denote by S_Ω^2 and S_ω^2 the minimum values of S^2 for variations of the parameters in Ω and ω

* S. S. Wilks (1944), *Mathematical Statistics*, Chapters VIII and IX. Princeton University Press.

respectively. If there are N values of x_{ij}, p_i alternative methods or designs in the i^{th} category, and q sites, the required test of the hypothesis $\lambda_j = \lambda$ reduces to a test of the Snedecor F ratio defined by

$$F = \frac{S_\omega^2 - S_\Omega^2}{q-1} \cdot \frac{N - \Sigma p_i - q + 1}{S_\Omega^2} \cdot$$

The tests for the differences between the alternative category means follow the same method.

10. *The Secondary Analysis*

(*a*) We can now perform the quantitative analogue of the ranking test to confirm the technical advice, that the differences between the schemes were confined in the main to the Wall categories. We may further extend the test to confirm the validity of the description of the abnormal conditions and differences in design existing on certain sites. Indeed, it was found preferable to examine this latter information first, with the aim of establishing from the outset the data to be used in the subsequent analysis.

These preliminary tests are listed in Table V below. The table shows the components of variance contributed by the differences in conditions, methods and design, compared with the residual variance, calculated from the deviations of the x_{ij} from their category means in the non-Wall categories. The data on conditions and methods were proved to be of value even by this crude analysis, and were incorporated into the subsequent analysis, while those on the design did not seem to be relevant. Nevertheless, it was thought that the differences in design might prove to be of value at a later stage of the analysis, when the estimate of the residual variance had been refined by the removal of any possible systematic site effect.

TABLE V.—*Test of Data on Conditions, Methods and Designs*

Source	*Sum of Squares*	*D.F.*	*Estimate of Variance*	*F.*	*Significance*
Abnormalities in conditions	0·556,969	7	0·079,567	10·63	0·1%
Non-traditional methods	0·207,736	4	0·051,934	6·94	0·1%
Differences in Design					
Foundations	0·000,244	1	0·000,244	<1	Insignificant
Roof	0·000,162	1	0·000,162	<1	,,
Finishings	0·002,260	1	0·002,260	<1	,,
Residual	0·285,518	38	0·007,514	..	..
Pooled Residual	0·288,184	41	0·007,027	..	..

(*b*) It is an essential requirement of the analysis that there be obtained some measure of the effect of the differences in conditions from site to site on the times observed for the non-traditional schemes. We have anticipated that such a measure might be provided by the replications of the same scheme which occur in the experiment in the houses of traditional brick design built on four sites, in the Type 3 houses on two sites, and in the non-Wall categories of the non-traditional schemes. It would appear possible to construct a measure of the site effect, either systematic or random, from the variations in the non-Wall categories on the fourteen sites, which might then be applied to the non-traditional schemes. Before proceeding with this analysis, however, we must be satisfied that both Wall and non-Wall categories form a homogeneous race with respect to inter-site variation; and further, that both the traditional and non-traditional sites, in the non-Wall categories, form such a homogeneous race. That these two conditions are satisfied may be seen from Table VI below, which lists the inter-site variances separately for each of the categories, further subdivided into traditional and non-traditional sites. For the purpose of this and the following table alone, the two Type 3 sites have been included with the four traditional

brick sites under the heading of "Traditional sites," as representing sites on which a true replication of scheme exists for the whole house.

TABLE VI.—*Division of Variation between Categories and between Traditional and Non-Traditional Sites*

Category	*Traditional Sites*		*Non-Traditional Sites*	
	Estimate of Variance	*D.F.*	*Estimate of Variance*	*D.F.*
Foundations	0·008,015	4	0·007,229	3
First Floor	0·008,533	4	0·015,376	5
Roof	0·002,166	4	0·011,749	5
Finishings	0·002,570	4	0·001,656	4
External Walls	0·002,074	1	0·025,256	4
Internal Partitions	0·002,152	1	0·025,004	4
Party Walls	0·011,011	1	0·031,012	4
All Walls	0·003,276	4	0·020,083	7

The homogeneity of the variances of the Wall and non-Wall categories on "traditional" sites and of the variances of both "traditional" and non-traditional sites for non-Wall categories may perhaps be more clearly seen in the summary of the results of Table VI given in Table VII.

TABLE VII.—*Summary of Results of Table VI*

Source	*Sum of Squares*	*D.F.*	*Estimate of Variance*
Wall Categories			
Non-Traditional Sites			
External Walls	0·101,024	4	0·025,256
Internal Partitions	0·100,016	4	0·025,004
Party Walls	0·124,048	4	0·031,012
All Walls	0·140,581	7	0·020,013
Traditional Sites			
All Walls	0·013,105	4	0·003,276
Non-Wall Categories			
Non-Traditional Sites	0·163,934	17	0·009,643
Traditional Sites	0·090,990	16	0·005,689

It can be seen immediately in these tables that the greatest variation occurs in the Wall categories of the non-traditional sites. These categories in fact make up the non-traditional experiment, since any isolated exceptions were removed in the analysis recorded at Table V. We may therefore regard the Wall categories on the non-traditional sites as representing inter-site and inter-scheme variations together; while all the non-Wall categories, together with the Wall categories on the traditional sites, are available for the study of inter-site variation alone.

(*c*) Having isolated the appropriate data, we made an exhaustive study of the inter-site variation with the object of identifying a systematic between-site effect. The different causes which give rise to the variations between sites are not yet well understood, but it may be confidently assumed that the most important factors are productivity of labour and efficiency of site organization. An attempt was made, therefore, to incorporate into the analysis independent measurements of these two factors, such as Time and Motion study records of effort rates, which, it was thought, might give additional measures of the systematic differences between the sites. This investigation did not give any positive result.

The more direct approach to the problem was to seek, from the data itself, systematic differences between the sites, common to all the categories. The nature of the statistical hypothesis together with the derivation of the tests of significance have been discussed for this case in Section 9 on "The Statistical Technique." The results of the test, however, which are given in Table VIII, together with a re-examination of the differences in design, show the systematic effect to be not significant.

TABLE VIII.—*Test of Systematic Inter-site Variation*

Source	*Sum of Squares*	*D.F.*	*Estimate of Variance*	*F.*	*Significance*
Differences in design	0·003,696	3	0·001,232	<1	Insignificant
Systematic inter-site effect	0·130,351	13	0·010,039	1·63	,,
Residual	0·154,137	25	0·006,165	..	..

The interpretation to be given to this result is that the site effect operates differentially from category to category; in other words, there is an interaction between the category and the site effect. It is therefore not valid to apply to the non-traditional scheme on a given site a correction for the site effect calculated from the non-Wall categories. This is not to say that we cannot obtain an estimate of the between-site variation. If we regard the effect of the site on each of the non-traditional schemes as randomly selected from an infinite population of such effects, then we may use our between-site variance from the non-Wall categories as an estimate of the variance of this population. In fact the best estimate is given by combining the separate components of the variance in Table VIII. We may now test for the existence of the differences between the schemes by comparing the variance of non-traditional Walls, representing site-plus-scheme variation, against the pooled inter-site variance from Table VIII. This test, given in Table IX below, demonstrates the reality of the scheme effect in the (non-traditional) Wall categories.

TABLE IX.—*Test of Non-Traditional Schemes*

Source	*Sum of Squares*	*D.F.*	*Estimate of Variance*	*F.*	*Significance*
Between Non-Traditional Schemes	0·158,450	8	0·019,806	2·818	Sig. at 2·5%
Residual	0·288,184	41	0·007,029	..	..

We may now proceed to examine individual differences between the schemes by Student's "t" test, using the residual variance above as our estimate of variance. These tests have been illustrated in Fig. 1, it being understood that there is a significant difference between the schemes where the intervals do not overlap. The method employed here will perhaps be of interest. The intervals are calculated as $\bar{x} \pm t\sigma/\sqrt{2}$, where $\bar{x}$ is the average time for the scheme and σ is the common estimate of variance. Where two intervals do not overlap we have

$$\bar{x}_1 - t\sigma/\sqrt{2} > \bar{x}_2 + t\sigma/\sqrt{2}$$

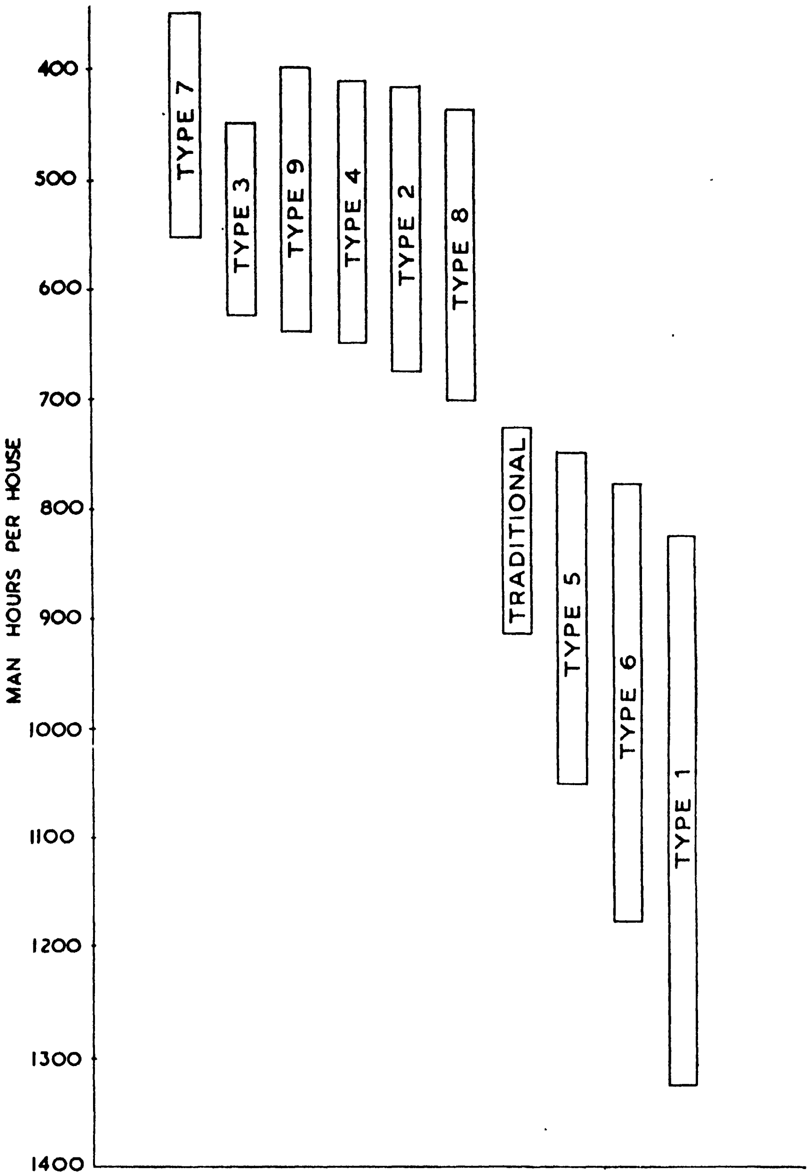
$$\text{or} \quad \{\bar{x}_1 - \bar{x}_2\}/\sqrt{2}\,\sigma > t$$

which has the form of Student's "t" test.

Where the variance of the values compared differed, as in the case of the traditional house and Type 3, based on observations from four and two sites respectively, the appropriate allowance has been made in calculating the intervals. The comparison between Type 3 and traditional houses, however, cannot be shown accurately on the same diagram with the other comparisons, but in this case the result of the test is not affected by the slight inaccuracy introduced.

It will be recalled that at Table IV, in certain isolated cases, non-traditional methods were identified in the First Floor and Roof categories. A final comparison of the schemes, therefore, requires that these categories should be taken into account.

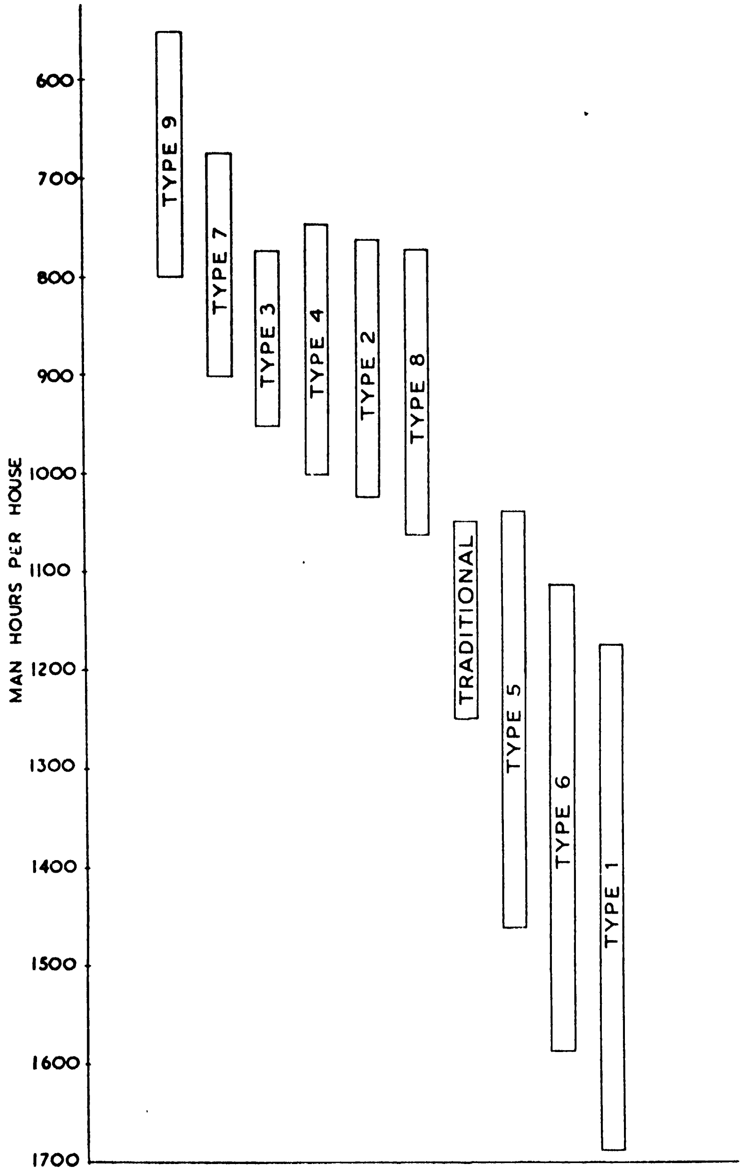
FIG. 1.—*Productive Times for Walls*



NOTE.—Where two ranges do not overlap, it can be stated (with 90% confidence) that the difference in man hours between the two corresponding schemes is significant. Where two ranges do not overlap, this cannot be stated.

Accordingly, a further analysis was performed to evaluate Superstructure times (corresponding to Walls + First Floor + Roof), using common figures for the First Floor and Roof where the traditional scheme was employed. The comparisons between the Superstructure times of the different schemes are illustrated in Fig. 2.

FIG. 2.—*Productive Times for Superstructure*



NOTE. Where two ranges do not overlap, it can be stated (with 90% confidence) that the difference in man hours between the two corresponding schemes is significant. Where two ranges do not overlap, this cannot be stated.

To complete the analysis, confidence intervals were estimated for the erection of each component of a house and for the complete house on the standard "average" site. These figures are given in Table X.

TABLE X

Scheme	*Foundations*	*Ground Floor*	*Walls*	*First Floor*	*Roof*	*Superstructure*	*Finishings*	*House Totals (ex. Ancillary)*
Type 9			520±170	55±20	95±30	670±175		1,660±195
,, 7			445±145			780±150		1,770±170
,, 3			520±120			855±125		1,845±150
,, 4			530±175	100±10	235±25	865±175		1,855±195
,, 2	160±20	30±5	550±180			885±180	800±80	1,875±200
,, 8			575±190			910±195		1,900±210
Trad.			815±135			1,150±140		2,140±160
Type 5			840±280	140±45	250±85	1,230±295		2,220±305
,, 6			995±325	100±10	235±25	1,330±325		2,320±335
,, 1			1,080±355			1,415±355		2,405±365

Conclusions

This paper has tried to go beyond previous work in the building industry, in that it has devoted a major effort of analysis to obtaining comparisons from which the effect of local site conditions has been removed. It was not to be expected that this effort would immediately meet with complete success. On the one hand, the confidence limits which could be obtained from an unplanned experiment were bound to be coarse. On the other hand, we have seen that it has not been possible to analyse the differences between sites to form a systematic pattern.

In spite of these limitations, our results are valuable practically as well as theoretically. They do give a valid appreciation, and the first such appreciation, of the man-hours expended in various schemes of non-traditional construction, with a comparison against traditional methods. In doing so, they record and use explicit confidence limits, and these as given in Table X include allowances both for the random fluctuations of experimental data and observation and for normal site-to-site variation.

We can state, with 90 per cent. confidence, that five of the non-traditional schemes (Types 2, 3, 4, 7 and 9) will employ fewer man-hours for the erection of the superstructure than current traditional construction. The remaining four schemes (Types 1, 5, 6 and 8) cannot with confidence be separated from traditional construction.

Naturally, these results are applicable directly only under the conditions of the years 1946 and 1947 during which our observations were made, and make no allowance for trends which might appear in the future. For example, if it is thought that the conditions of the two years observed were abnormally difficult, then the statements of future performance must be scaled down accordingly; but they must be scaled down uniformly, and the comparisons between schemes which we have obtained will continue to hold.

But the major interest of this work lies, not in the first place in the practical results obtained, but in the development and application of the statistical method in the field of building economics, and in the contribution of a scientific measure of reliability to building estimates and comparisons.

In future research we shall obviously have a fully planned experiment in which the same scheme will be duplicated on some different sites and some different schemes erected on the same site. The analysis will then be at once simpler and more precise. In the present analysis there was markedly higher precision of results in the Type 3 and Traditional schemes. Efforts will also have to be made to minimize minor variations in design within a scheme, since these render the analysis highly complicated and lessen its clarity. Where alternative variants are genuinely to be tested, they will have to be apportioned to the houses on a site in a manner designed to facilitate the most powerful statistical decision.

The building cost estimator and contractor will also realize from a study of this work the danger of comparing different methods of construction on different sites as though they existed in isolation and were independent of site effects. It is in such examination and clarification of the basis of conventional estimating that the analysis throws into bold relief the value of imaginative statistical thought within industry.

DISCUSSION ON DR. BRONOWSKI'S PAPER

DR. M. G. KENDALL: It is a great personal pleasure as well as an official one to propose this vote of thanks to Dr. Bronowski. With one exception I think we have not met for twenty years. The last occasion when I remember meeting him was when we sat looking at each other for hours over a chess-board and fought each other to a bitter draw. After that he went to Spain and I to Whitehall, but I am glad to say that we have survived the experience. I was particularly glad to see him again in such good form to-night.

We are glad to have this paper and appreciate the interesting way in which it was read and summarized. I know nothing about building, and my comments fall into two groups: one is a request for additional information, and the other a comment on the statistical technique of analysis.

One of the points which struck me on reading through the paper was that certain information could have been included with advantage. For example, in the big table on page 292 the averages are given for various houses, but not the actual number of houses on which those averages are based. Similarly, although at later stages in the analysis the variances are given, the actual means in a number of instances are not. Here again we have the situation in which certain things are said to be significant or not significant, but we are not given the actual mean figures to which they relate.

Dr. Bronowski in his verbal comments cleared up one of my difficulties, that is to say, the swamping effect of the inter-site variation. At the bottom row of Table II, for example, one thing which stands out is that the average time for the improved total house on Site A—the last column but three—1,691 hours, is, in fact, lower than any other of the times taken on the non-traditional types. That in itself is some indication of the enormous influence of the inter-site variation, and I wonder whether that figure has any particular explanation. As I understand it, Dr. Bronowski says an analysis of the individual items shows that the variations are scattered at random, so that this is in fact a member of a homogeneous group, but I think perhaps that the figures on that particular site might be looked at again. All the individual items cannot be scattered at random; otherwise the total time would not be so different from the total of the others.

Personally I am convinced by what Dr. Bronowski says about the analysis of the normal categories, but there are two passages in the paper which arrested my attention on the question of interaction between site and type. On page 290 he says, "The problem is somewhat simplified if the schemes do not respond differentially to the varying conditions on the differing sites. We have technical advice which confirms that this is so." That means that there is no interaction between scheme and type. On the other hand, on page 300 he says, "The interpretation to be given to this result is that the site effect operates differentially from category to category; in other words there is an interaction between the category and the site effect." I am a little puzzled as to whether these two statements are consistent. I should have thought that if there was an interaction in the non-wall categories there must have been an interaction for the whole house; but that is a point which, no doubt, Dr. Bronowski can clear up for us.

There are two other points of technique on which I would like information. The first to know exactly how the learning curve was eliminated from the data. Cases in which there is an analysis of variance of material which is moving through time are sometimes rather complicated. Was it, for example, removed by inspection, or was it removed by isolating a time polynomial in the variance? When such factors are isolated there is danger of producing interaction effects and the point is therefore of some importance. The other point is perhaps asking a little too much: in material of this kind, as Dr. Bronowski has said very truly, particularly when the statistician is called in when the investigation is nearly over, it is extremely difficult to do a complete analysis of variance, particularly for more than a two-way classification, because of non-orthogonality of the data; but since interactions are very important, in this case I wonder whether his team has given any attention to giving a three-way analysis by site, by type, and by category.

That is all I have to raise on the technical aspects, and as to the significance of the results I cannot express an opinion, but I would like to conclude with one general comment, namely, that I very much welcome the reading of this type of paper to the Society and the introduction of these techniques into practical business and industrial affairs. Most of us who have to deal with this kind of thing know perfectly well that there is a very strong feeling among practical men that the statistician cannot offer them more than they know already, and that if they succeed in eliciting new facts they are probably wrong anyway. From what Dr. Bronowski has told us I think that a convincing case has been made out for the application of these techniques irrespective of the act[illegible]lts.

Mr. [illegible]ER: I am glad to join with Mr. Kendall in congratulating the authors and their c[illegible]mpleting so successfully such a laborious and important task and describing

it in such a succinct form. The authors have pointed out that the final analysis would have been simple had it been possible to design the experiment in a more orthodox manner, but the time so saved would, I imagine, have been trivial, in comparison with that necessarily involved in collecting and reducing the basic data. As the Stationery Office publication (*Special Report No.* 4) mentioned by the authors points out, time-recording is a job that calls for tact as well as perseverance if it is to result in reliable data. Although it is perhaps not relevant, I might mention that during the war parties were sent from Command Headquarters to various Air Force stations to record how the personnel there spent their time, and one station adjutant retaliated by appointing two of his men to record how the time-recorders spent *their* time.

I remember Dr. Irwin telling me, some years ago now, that he felt that the difficulty with mathematical statistics was not in understanding the theory, but in knowing which bit of it to apply to any particular set of figures. Dr. Bronowski and his colleagues seem to me to have solved their problem by a judicious process of questioning the technical experts, and then doubting their answers, until they arrived at a mathematical model, set out in Table 4, that would provide the basis of a standard least-squares process. I rather doubt whether the authors were justified in retaining for as long as they apparently did their hope of applying the particularly simple technique of the analysis of variance—I think that that hope went overboard directly it became necessary to adopt the rather awkward design that was used. It may be worth while to remark, however, that if we do want to regard data modelled on Table 4 as susceptible of a variance analysis, the presence of different category means and the absence of certain of the readings are difficulties that are logically equivalent. There are two unknown constants μ_1 and μ_1' in the "Foundations" column, for example, and we could replace this column by two, one containing data only for the μ_1 sites, the other only for the μ_1' sites, and thus throw the problem back to the familiar "missing readings" one.

I was puzzled by the number of missing readings indicated in Table 4 (there are 11 of them, for 7 of which figures are given in Table 2), and wish for two reasons that the authors could explain in greater detail why these 7 readings were omitted from the final analysis. My first reason is that the data omitted must have taken many man-months of observers' time to record, and it is a pity to see so much work discarded. My second reason is, that these 7 readings are usually either the largest or the smallest in their category, and might therefore be expected to influence fairly heavily the test of the hypothesis of equal logarithmic variances, on which the final analysis rests.

I have much pleasure in seconding the vote of thanks to Dr. Bronowski and his colleagues.

Dr. F. M. LEA said that Dr. Bronowski's paper could not fail to be of interest to all concerned with the economics of building. In the building industry it was difficult to get true comparisons of the relative man-hours or costs of similar operations on different sites or of dissimilar methods on the same site. The existence of these difficulties was illustrated by the examination Dr. Bronowski had made of data from an experiment which in the statistical sense was not fully controlled. But full statistical control of experiments involving considerable numbers of operatives working on different sites was not easy to achieve. Dr. Bronowski was to be congratulated on the attempt he had made, even though, as he himself said, the confidence limits which resulted were coarser than he would wish.

Dr. Bronowski was probably least satisfied with the estimate of the variation between sites which arose from differences in physical conditions and of labour productivity. This was a difficult variant to eliminate, and it could be of considerable magnitude. The estimate of time completely lost by bad weather could be assessed, but the slow-up caused by inclement weather which did not actually stop building operations was less susceptible to measurement. Again, much time was lost by a succession of minor rests by the craftsmen, some legitimate, and some not. To what extent such losses could be recorded in the work Dr. Bronowski had described was not clear.

Evidence of the considerable sum-total of minor rests was obtained in some work carried out at the Building Research Station, in which a detailed time study was made of the building of the shells of 20 houses. There had been an observer to each bricklayer—which would be quite impracticable on the scale of building involved in Dr. Bronowski's experiment—but it had enabled a complete time analysis down to each minute to be made, and correspondingly close statistical estimates to be formed. That work was published last year under the title "Work Studies in Blocklaying."

He might also give one example of how large the variation in individual operations might be between different sites. The Building Research Station had been interested in the use of concrete piles for house foundations. This involved boring holes of say 10–14 inches in diameter in the

ground to a depth of 8 or 10 ft. and filling them up with concrete. At one site two men took half an hour to bore a hole to a depth of 8 ft., at another two men took three-quarters of an hour to bore to a depth of 3 ft. 6 in. The allocation of that great difference between physical conditions, i.e. the nature of the ground, and labour productivity, illustrated the problem that always arose in studies of building operations.

The comparison of different systems of construction that Dr. Bronowski had undertaken involved just this sort of difficulty, and in a sense it was disappointing that subjective estimates of effort rating had proved unhelpful. But from the Building Research Station's experience in its own studies on effort rating in building, that did not surprise him.

He thanked Dr. Bronowski for his paper, which should prove of much help in planning future experiments designed to elucidate economic problems in building.

Mr. R. Fitzmaurice, after paying a tribute to Dr. Bronowski for the paper, said he had taken part in this work, and although not a statistician was entirely converted to the use of the statistical method. Dr. Lea had indicated some of the difficulties which arose in handling data obtained from a building site. One difficulty in applying scientific analysis to the work on building sites derived from the fact that the normal kind of recording was extremely variable, from no accurate records at all, through extremely crude and unreliable records to full and reliable records; but he had to admit that in this old and traditional industry the majority of records erred on the meagre side.

Another unfortunate factor was that the conventional work of assembling building records was a very complex and highly traditional matter which had grown up literally through the centuries. The whole basis of the judgment of the correct remuneration of the builder for his work was done in a form which from the point of view of analysis of this sort was one most calculated to conceal the real truth. The figures as they usually appeared in a bill of quantities, the document normally used for the settlement of accounts of a building site, were assembled by trades, not by operations, and everybody dealing with this work was normally accustomed to recording his figures in such a form that they could be brought back to this very old established and somewhat out-of-date method of recording. The result was that even if builders wished to help it was difficult, because all the time they were trying to get back to their old way of recording. It was a major difficulty, and the major reason for the incompleteness of the normal records.

Since this was a first venture he thought it not unlikely that certain errors might have slipped in because the practical people on the sites had had to assemble their records in a form which was new to them in order to get out an analysis at all.

He must confess that he enjoyed a thrill when Dr. Bronowski brought the completed analysis. He had from the practical point of view formed quite definite conclusions about a number of sites. At some of the sites it was extremely difficult to arrive at anything like a conclusion, but at others one could make estimates from a practical point of view, and when he exchanged notes with Dr. Bronowski it was very gratifying to find a large degree of accord in the results. Considering the difficulty of the whole matter they felt that a good job had been done.

Since this work was completed, the builders who took part had continued to build houses as a normal matter of bread and butter for a very large number of local authorities in many parts of the country. Five of the builders were known to be people who kept careful records of costs, and he was glad to say that they had taken the trouble since this work was completed to write to the team to say that the results that they had been achieving in the field since the work was done by the Ministry had in fact accorded very well with the results of the analysis. That again was a justification for this type of statistical analysis and the statistical treatment of results in an industry which was highly traditional. He had been convinced that an investigation of this sort should not be attempted in future without preparing in advance a very careful statistical plan.

He was very glad to have the opportunity of testifying to the Ministry's appreciation of this particular piece of work. It was hoped it would initiate the application of statistical method in what, judged by modern ideas, had been a rather backward industry in some ways.

Professor R. G. D. Allen also thanked Dr. Bronowski for the kind of paper which he would like to see more often in the *Journal*. He would like to ask for more information. The first point was one which happened to interest him about the fit of the log normal distribution which Dr. Bronowski mentioned in Section 8. Dr. Bronowski said that he had tested this fit since the paper was written and that it was fully established. He would like to have a little more data on that, particularly in view of all that had been said for, and particularly against, the log normal distribution. He was anxious to find examples of a good fit of this particular distribution.

He was perfectly aware in reading the paper that Dr. Bronowski had not been able to influence the design of the particular enquiry. He was given the job of isolating the scheme variation and therefore had to eliminate the site variation in the data, and it seemed to the speaker that in the end

the site variation was as important to analyse as the scheme variation. He would like some more indication of the factors involved in the site variation, which must include topographical factors as well as labour productivity and efficiency of organization. There was also the effect of bonus and other incentive schemes, which might perhaps be included under the factor of labour productivity. Paragraph (c) of Section 10 said that the different causes which gave rise to variations between sites were not yet well understood, and that the particular investigation had no positive result. It seemed important to work out the effect of the various factors, and perhaps Dr. Bronowski could give a little guidance out of his experience as to the factors to look for and their relative importance in this field.

Mr. L. T. WILKINS said that whilst one might be able to observe microbes without causing them to change their behaviour, it was generally impossible to place an observer in the field of human relations without changing the behaviour patterns it was desired to measure. Opinions, attitudes and behaviour of man could not be measured as they really were if the subjects were aware that observation or inquiry was taking place.

In the field of attitude measurement, with which he was associated, it was known that the opinions expressed by samples of the population could only be extended to suggest (within various error limits) what the residual population would also have said had they been similarly observed or interviewed.

He felt that there was a wealth of material in official and unofficial files which might be used in a manner similar to that demonstrated by the speaker. More use could be made of data already available in Local Authorities' records. Although these were not usually designed for purposes of statistical analysis, practical statistical methods might be devised to force these data to yield valuable information.

Dr. BRONOWSKI replied in writing as follows: My colleagues and I, who worked together in this analysis, greatly appreciate the kind things which have been said about it in the discussion. We are particularly grateful for the way in which several speakers have stressed the point that more practical papers of this kind should be published and discussed. It is because we ourselves felt just this, that practical investigations of this kind, with their special problems and interests, too often remain forgotten in the files of firms and Ministries, that we undertook the rather formidable task of presenting a mass of practical material on a single industry to statisticians in general.

It was inevitable that, however great the detail we could give, some factual information of interest would remain neglected. Several speakers have drawn attention to information which they would have liked to have, and I can only apologize for the omissions. It would be difficult now to remodel the paper to include it all, and, indeed, there are some pieces of information which could not now be extracted from the original data without great labour. We shall therefore treat these suggestions for more information rather as advice on what to include in the future, which we shall take to heart when we have similar material to present on another occasion.

There is, however, one thing which we neglected to show in the body of the paper, about which I am very contrite. We do not seem to have stated clearly that, although we listed the findings on all sites and categories in Table II, not all the work for which man-hours are given there reached the uniform standard laid down by the building experts. On several sites, some categories were sub-standard or otherwise not comparable, and no correction could be made to the man-hours so as to make them comparable with work of the uniform standard. We therefore had to reject the man-hours for these categories from the analysis, and this is why there are gaps in Table IV at places where there are no gaps in Table II. In particular, I regret to say that the bulk of the work on site A was sub-standard and most of the man-hours from this site had to be rejected. We did, in fact, consider rejecting site A entirely; but since we were so short of replications, we finally accepted on all sites all work which came up to standard, and rejected only what could not pass.

Dr. Kendall has drawn attention to a minor confusion of language in the paper round the use of the word "interaction": a word which I have never cared for, and which I am sorry that we used at all. When we say, towards the end of Section 9, not very happily, that there is an interaction between the category and the site effect, our meaning is that any possible site effect shows no consistency in the way it varies from category to category. Now, a non-traditional scheme so far as we are concerned in this experiment is an alternative method of construction within one category—the walls. So when we say in Section 4 that the schemes do not respond differentially to the varying conditions on different sites, this is in harmony with the other statement. It says

that there is no evidence that the site effect on any one site in the wall category would have been different if a different method of construction of the walls had been employed.

Dr. Kendall's suggestion that we might consider not a two-way but a three-way analysis is a natural one, which we ourselves entertained. Alas, in the absence of replications of the non-traditional schemes on different sites we had to abandon the notion. Indeed, our analysis is really an attempt to get round just this difficulty, and to improvise usable replications where none were provided which would allow a full analysis. A last point of Dr. Kendall's: how was the learning curve fitted to the data? It was fitted by inspection, after we had done a good deal of work on it on earlier data, which we have described in another publication. Dr. Kendall is right in warning against spurious effects which such a curve may introduce. But on the scale of our analysis, which covered improvements in well over a hundred different operations, there is little else that we could do, and we are satisfied that no sizeable bias or error was introduced.

I am very grateful to Dr. Lea for the examples which he quotes which show some of the difficulties we had to get round. In part, of course, these difficulties are more serious when observations are few and intended to be exceptionally accurate, as in Time and Motion study and effort rating. But certainly we did miss time lost in short rests, since our recordings were only at 15-minute intervals. However, the advantage of methods like the analysis of variance is that such errors fall into the residual variance instead of remaining to bias the comparisons. In swelling the residual variance, they do, of course, increase the chance that significant differences may be missed; and, naturally, they make the confidence intervals coarser.

The point made by Mr. Wilkins that workers under observation do not work normally, is also more appropriate, I think, to Time and Motion study than observations of the kind we describe. On most of our sites, observation was continuous for a period of 1–2 years. I think everyone on the site was used to it by then.

Finally, we are grateful for Mr. Wilkins's reference to material in the records of Local Authorities, and to the stress which Dr. Lea and Professor Allen have laid on the importance of site factors, including productivity. In our view these form an important subject of study, but they should be studied by the methods not of experiment but of field survey. The method of the controlled experiment is appropriate in such cases as that which we have been discussing where new methods of building are to be investigated—though we could have wished for more control and less lighthearted experiment. The experiment has thrown up the importance of site factors, and we share the feeling of several speakers that we ought to go ahead and look into this. We are, in fact, in process of doing so by examining man-hours and costs on a stratified sample of building sites all over the country on which traditional houses are being built. We hope that this sample survey, which in some ways is a complement to the present experiment, will produce results of equal value which we may have the privilege of discussing with the Society on a future occasion.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

Albert Baines.
Margaret Anne Clapp.
Edwin John Davis.
Eileen Drover.
Charles Balfour Clephan Hunter.
Doris Hyman.
Eric Gordon Janes.
Peter William Meredith John.
Anna Elizabeth Parkinson.
James Charles Lacy Rowe.
Leonard Percy Woolger.

Corporate Representatives

Leonard Frank Cheyney, representing the Institute of Municipal Treasurers and Accountants (Inc.).

John Charles Richard Clapham, representing the British Boot, Shoe and Allied Trades Research Association.

A Note on Carnap's Theory of Probability

By S. F. James
(Department of Applied Economics, Cambridge)

Connection with the Classical Theory

It has been pointed out by Professor Tintner, and emphasized as of possible importance by Mr. Stone, that in the special case of a finite language system with one simple attribute, the classical theory may under certain assumptions yield prior probabilities equal to Carnap's measure for a structure description.*

Briefly, let the attribute be P and its negation $Q = \sim P$, occurring with probabilities p and $q = 1 - p$ respectively. Then the classical formula for x appearances of P and $y = n - x$ appearances of Q out of n individuals is :

$$\binom{n}{x} p^x q^{n-x}.$$

Now we suppose the ratio of p to q to be completely unknown, and so will assume all values of p between 0 and 1 (and hence of all values of q in the same range) to be equally likely, following Bayes' procedure. With this assumption our formula for the probability becomes:

$$\int_0^1 \binom{n}{x} p^x(1-p)^{n-x}dp$$

or $\dfrac{1}{n+1}$. This is the same as Carnap's measure for the corresponding structure description, which may be denoted by P^xQ^{n-x} as a convenient shorthand.

Case of a 3-Ply Attribute

We next wish to consider whether a similar approach to the above will lead to the same value as Carnap's measure when the attribute is not simple. We begin with a 3-ply attribute, i.e. one with alternatives P, Q and R, with which we will associate the (unknown) probabilities p, q and r, where $p + q + r = 1$.

If we use the classical formula, the probability of x appearances of P, y of Q and z of R out of n $(= x + y + z)$ individuals, is:

$$\frac{n!}{x!\,y!\,z!} p^x q^y r^z, = F(n : x, y, z) \text{ say.}$$

If nothing is known about the ratios $p : q : r$, then following the method of our previous example we wish to give equal weights to all positive values of p, q and r satisfying $p + q + r = 1$. Also the sum of these weights must be 1. Hence we require $\dfrac{A(n)}{A(o)}$, where $A(n)$ is the integral of $F(n: x, y, z)$ over the region : $p + q + r = 1$; $p, q, r \geqslant 0$. To find $A(n)$ we make the transformation:

$$\begin{aligned} p + q + r &= u \\ q + r &= uv \\ r &= uvw \end{aligned} \qquad \text{so that} \quad \begin{aligned} p &= u(1 - v) \\ q &= uv(1 - w) \\ r &= uvw \end{aligned}$$

and also:

$$\frac{\partial(p, q, r)}{\partial(u, v, w)} = u^2 v.$$

The new integrand is:

$$\frac{n!}{x!\,y!\,z!} u^{x+y+z+2} v^{y+z+1} (1 - v)^x w^z (1 - w)^y$$

and the region of integration $o \leqslant v, w \leqslant 1, u = 1$.

* See Professor Tintner's paper, Part 2, Section f, p. 258, and Mr. Stone's comments, p. 284.

$$\therefore A(n) = \frac{n!}{x!\,y!\,z!}\int_0^1 v^{y+z+1}(1-v)^x dv \int_0^1 w^z(1-w)^y dw$$

$$= \frac{n!}{x!\,y!\,z!}\cdot\frac{\Gamma(y+z+2)\,\Gamma(x+1)}{\Gamma(x+y+z+3)}\cdot\frac{\Gamma(z+1)\,\Gamma(y+1)}{\Gamma(y+z+2)}$$

$$= \frac{n!}{(n+2)!}$$

Hence $\dfrac{A(n)}{A(o)} = \dfrac{1}{\binom{n+2}{n}}$.

Carnap's measure is $\frac{1}{m}$, $m = \binom{n+k-1}{n}$, where k = no. of alternatives of attribute = 3.

Hence again this measure is equal to the prior probability obtained by integration.

General Formula for one k-Ply Attribute

The proof of the general formula for the probability obtained by this integration method, with one attribute, follows by a generalization of the above transformation, which is due to Dirichlet. (See e.g. Goursat, *Cours d'analyse*, vol. i, ch. viii, par. 150.) Thus, if the alternatives of the attribute are $P_1, P_2 \ldots P_k$ with probabilities $p_1, p_2 \ldots p_k$, the probability of obtaining $P_1^{x_1} P_2^{x_2} \ldots P_k^{x_k}$ with $n = \sum_{i=1}^{k} x_i$ individuals is:

$$n!\prod_{s=1}^{k}\frac{p_s^{x_s}}{x_s!}$$

Our value for the prior probability is again $\frac{A(n)}{A(o)}$ where $A(n)$ is the integral of this over the region

$$\sum_{s=1}^{k} p_s = 1\ ;\ p_s \geqslant 0, \text{ all } s.$$

The transformation which we apply is—

$$\sum_{i=s}^{k} p_i = \prod_{i=1}^{s} u_i,$$

$s = 1, 2 \ldots k$, so that—

$$p_s = (1-u_{s+1})\prod_{i=1}^{s} u_i,$$

$s = 1, 2 \ldots k-1$,

$$p_k = \prod_{i=1}^{k} u_i.$$

Then

$$\frac{\partial(p_1 \ldots p_k)}{\partial(u_1 \ldots u_k)} = \prod_{s=1}^{k-1} u_s^{k-s}$$

and the new region of integration is: $u_1 = 1$; $o \leqslant u_s \leqslant 1$, $s = 2 \ldots k$.
Hence,

$$\frac{1}{n!}A(n)\prod_{s=1}^{k} x_s! = \prod_{s=2}^{k}\int_0^1 u_s^{x_s+\ldots+x_k+k-s}(1-u_s)^{x_{s-1}} du_s$$

$$= \prod_{s=2}^{k}\frac{\Gamma(x_s+\ldots+x_k+k-s+1)\,\Gamma(x_{s-1}+1)}{\Gamma(x_{s-1}+\ldots+x_k+k-s+2)}$$

$$= \frac{\Gamma(x_k+1)}{\Gamma(x_1+\ldots+x_k+k)} \prod_{s=1}^{k-1} (x_s+1).$$

Therefore since

$$\sum_{s=1}^{k} x_s = n,$$

$$A(n) = \frac{n\,!}{(n+k-1)\,!}$$

and the prior probability

$$= \frac{A(n)}{A(o)}$$

$$= \frac{n\,!\,(k-1)\,!}{(n+k-1)\,!}$$

$$= \frac{1}{\binom{n+k-1}{n}}$$

Thus in the case of one multiple attribute, with k alternatives, we always agree with Carnap's measure for the corresponding structure description.

Two Simple Attributes

We will revise our notation slightly to deal with the case of two simple attributes, which we will suppose to be completely uncorrelated, i.e. the occurrence or non-occurrence of one attribute in no way affects that of the other. We will denote the first attribute by P_1 and $Q_1 = \sim P_1$, the second by P_2 and $Q_2 = \sim P_2$, with probabilities p_1 and $q_1 = 1 - p_1$, p_2 and $q_2 = 1 - p_2$ respectively. With our foregoing notation, Carnap's Q-properties can then be denoted by A's (to avoid confusion of notation) as follows:

$$A_1 = P_1.P_2$$
$$A_2 = P_1.Q_2$$
$$A_3 = Q_1.P_2$$
$$A_4 = Q_1.Q_2$$

If we proceed by our former method, the probability of A appearing, with one individual only, will be:

$$\int_0^1 dp_1.\int_0^1 dp_2\,.\,p_1p_2 = \frac{1}{4}.$$

This is exactly analogous to our approach with only one attribute; we are giving equal weights to all possible values of p_1 and to all possible values of p_2, only since the occurrences of the two events are independent, the limits of the range of integration of p_2 are independent of p_1 and *vice versa*.

The same result is obtained for A_2, A_3 and A_4, and is equivalent to the Carnap measure.

However, it can speedily be seen that this is not always so. The cause of the discrepancies which arise is best illustrated by considering the case of two simple attributes with two individuals. Here Carnap distinguishes the following structure descriptions (no account being taken of order):

$$\begin{array}{cccc} A_1A_1 & A_1A_2 & A_1A_3 & A_1A_4 \\ & A_2A_2 & A_2A_3 & A_2A_4 \\ & & A_3A_3 & A_3A_4 \\ & & & A_4A_4. \end{array}$$

To each of these he gives an equal measure of $\frac{1}{10}$. Our method, on the other hand, gives us a probability for A_1A_1 of:

$$\int_0^1 dp_1\int_0^1 dp_2\,.\,(p_1p_2)(p_1p_2), = \frac{1}{9}.$$

The same result is obtained for the structure descriptions A_2A_2, A_3A_3, A_4A_4, A_1A_2, A_1A_3, A_2A_4, A_3A_4, while for A_1A_4 and A_2A_3 we obtain $\frac{1}{18}$. This would appear to indicate that in some way the structure descriptions A_1A_4 and A_2A_3 together form a more fundamental description, and that it is this description which should be counted equivalent to A_1A_1, etc.

What we are in fact doing is to expand the expression $(p_1 + q_1)^2 (p_2 + q_2)^2$ and to integrate each term separately. The expansion gives—

$$
\begin{array}{lll}
p_1^2 p_2^2 & \text{corresponding to} & A_1A_1 \\
+ p_1^2 q_2^2 & \quad ,, & A_2A_2 \\
+ q_1^2 p_2^2 & \quad ,, & A_3A_3 \\
+ q_1^2 q_2^2 & \quad ,, & A_4A_4 \\
 & & \\
+ 2p_1^2 p_2 q_2 & \text{corresponding to} & A_1A_2 \text{ (2 permutations).} \\
+ 2p_1 q_1 p_2^2 & \quad ,, & A_1A_3 \quad ,, \\
+ 2p_1 q_1 q_2^2 & \quad ,, & A_2A_4 \quad ,, \\
+ 2q_1^2 p_2 q_2 & \quad ,, & A_3A_4 \quad ,, \\
+ 4p_1 p_2 q_1 q_2 & \quad ,, & A_1A_4 \quad ,, \\
 & & \text{and } A_2A_3 \quad ,,
\end{array}
$$

Each of the terms on the left, when integrated over the complete ranges of p_1 and p_2, gives $\frac{1}{9}$, with total 1 as required.

In fact A_1A_4 corresponds to $(P_1P_2)(Q_1Q_2)$, which from the algebraic point of view is equivalent to $(P_1Q_2)(P_2Q_1)$ or A_2A_3. We are in fact led to the conclusion that it is possible to have what we will call homomorphic structure descriptions, when these correspond to the same term in the generating function, in this example $(p_1 + q_1)^2(p_2 + q_2)^2$.

Essentiality of Independence

It must be emphasized here that for this treatment to be possible the two attributes must be entirely independent. (Otherwise it is not possible to obtain the probabilities of the structure descriptions by simple multiplication.) This fact resolves the apparent anomaly of associating the structures A_1A_4 and A_2A_3.

For instance, consider the following example (suggested by Professor Tintner). Suppose the individuals to be balls, P_1 and $\sim P_1$ to mean " white" and "black" respectively, and P_2 and $\sim P_2$ to mean "big" and "small." Then A_1A_4 corresponds to the case of obtaining one big white ball and one small black ball with two selected balls, while A_2A_3 corresponds to the case of obtaining one small white ball and one big black ball, and at first it is difficult to see how these two cases can be associated. But once it is remembered that the attributes are independent, it can be seen that together these two structures correspond to the wider case where with two selected balls we obtain one white and one black ball, one big and one small ball, without stipulating the order in which one attribute is to occur with respect to the other. None of the remaining eight structure descriptions can be associated with each other in the same way, i.e. these two are the only "homomorphic" ones in this example.

To touch briefly on the case of two dependent attributes, it is probably best to reduce this to the case of one quadruple attribute; e.g. in our example, to consider the four possibilities "big white," "small white," "big black" and "small black" as the four possible alternatives of one attribute, A_1, A_2, A_3 and A_4. In this case A_1A_4 and A_2A_3 cannot be associated. Also if we give probabilities a_1, a_2, a_3 and a_4 to the four alternatives, we will, of course, find that integration over the region $0 \leqslant a_i \leqslant 1, i = 1, 2, 3$ or 4; $a_1 + a_2 + a_3 + a_4 = 1$ gives a probability of $\frac{1}{10}$ to each of the ten possible structure descriptions, agreeing with Carnap's measure.

Partition Descriptions

In the more general case of n individuals with two simple attributes, our generating function will be $(p_1 + q_1)^n(p_2 + q_2)^n$. This when expanded will give λ terms, of the form

$$\binom{n}{x_1}\binom{n}{x_2} p_1^{x_1} q_1^{n-x_1} p_2^{x_2} q_2^{n-x_2},$$

where λ is hence seen to be the number of partitions of the bipartite number $(n; n)$ into two ordered parts, zero being always a permissible part. Then each term will correspond to what we will call a "partition description" of the form $P_1^{x_1} Q_1^{n-x_1} P_2^{x_2} Q_2^{n-x_2}$; to each partition description could be allotted a measure $\frac{1}{\lambda}$, and $\frac{1}{\lambda}$ equals—

$$\int_0^1 dp_1 \int_0^1 dp_2 \, . \binom{n}{x_1}\binom{n}{x_2} p_1^{x_1} (1 - p_1)^{n-x_1} p_2^{x_2} (1 - p_2)^{n-x_2},$$

which is the probability of the partition description occurring if all values of p_1 and of p_2 between 0 and 1 are considered equally likely. With this definition, structure descriptions are said to be homomorphic if they belong to the same partition description.

If a partition description contains M state descriptions [$M = \binom{n}{x_1}\binom{n}{x_2}$ in this case] and a structure description belonging to it contains m state descriptions, then such a structure description can be allotted a measure $\frac{m}{M} \cdot \frac{1}{\lambda}$, which will equal its probability obtained by direct integration, viz.:

$$\int_0^1 dp_1 \int_0^1 dp_2 \, . \, mp_1^{x_1} (1 - p_1)^{n-x_1} p_2^{x_2} (1 - p_2)^{n-x_2}.$$

For our present example

$$\lambda = \binom{n + k - 1}{n}^2$$

where k is the number of alternatives of the 2 attributes $= 2$,

$$= (n + 1)^2;$$

[(for the number of partitions of $(n; n)$ into k parts $=$ (number of partitions of n into k parts)^2, and the number of partitions of n into k ordered parts, zero being a permissible part, is $\binom{n + k - 1}{n}$)].

Example

(2 simple attributes and 3 individuals.) With our preceding notation for Q-properties, Carnap would distinguish the following structure descriptions:

$A_1A_1A_1$	$A_2A_2A_1$	$A_3A_3A_1$	$A_4A_4A_1$
$A_1A_1A_2$	$A_2A_2A_2$	$A_3A_3A_2$	$A_4A_4A_2$
$A_1A_1A_3$	$A_2A_2A_3$	$A_3A_3A_3$	$A_4A_4A_3$
$A_1A_1A_4$	$A_2A_2A_4$	$A_3A_3A_4$	$A_4A_4A_4$
$A_1A_2A_3$	$A_1A_2A_4$	$A_1A_3A_4$	$A_2A_3A_4$,

i.e. $\binom{3 + 2^2 - 1}{3} = 20$ in all, so to each of these he would give a measure of $\frac{1}{20}$. Now we have found $A_1A_4 \equiv A_2A_3$ (where here we use $\equiv$ to denote "is homomorphic to"). Hence we should expect to find

$$A_1A_1A_4 \equiv A_1A_2A_3$$
$$A_2A_2A_3 \equiv A_1A_2A_4$$
$$A_3A_3A_2 \equiv A_1A_3A_4$$
$$A_4A_4A_1 \equiv A_2A_3A_4$$

to give 16 partition descriptions. In fact on expanding $(p_1 + q_1)^3 (p_2 + q_2)^3$, we get:

$$\begin{aligned} & p_1^3p_2^3 + p_1^3q_2^3 + q_1^3p_2^3 + q_1^3q_2^3 \\ & + 3p_1^3p_2^2q_2 + 3p_1^2q_1p_2^3 + 3p_1^3p_2q_2^2 \\ & + 3p_1^2q_1q_2^3 + 3p_1q_1^2p_2^3 + 3q_1^3p_2^2q_2 \\ & + 3p_1q_1^2q_2^3 + 3q_1^3p_2q_2^2 \\ & + 9p_1^2q_1p_2^2q_2 + 9p_1^2q_1p_2q_2^2 \\ & + 9p_1p_2^2q_1^2q_2 + 9p_1q_1^2p_2q_2^2, \end{aligned}$$

with 16 terms as expected. (The coefficient 9 in the last four terms is due to the 3 possible permutations of the structure $A_1A_1A_4$, say, and the six of $A_1A_2A_3$ for the first, and similarly for the others.)

These 16 terms correspond to the following 16 partitions into 2 parts of the bipartite number (3; 3):

(3, 0 ; 3, 0)	(2, 1 ; 3, 0)
(3, 0 ; 0, 3)	(1, 2 ; 3, 0)
(0, 3 ; 3, 0)	(2, 1 ; 0, 3)
(0, 3 ; 0, 3)	(1, 2 ; 0, 3)
(3, 0 ; 2, 1)	(2, 1 ; 2, 1)
(3, 0 ; 1, 2)	(2, 1 ; 1, 2)
(0, 3 ; 2, 1)	(1, 2 ; 2, 1)
(0, 3 ; 1, 2)	(1, 2 ; 1, 2).

Our formula was $\binom{n + k - 1}{n}^2$, $n = 3$, $k = 2$, which is, of course, 16, i.e. the integration:

$$\int_0^1 dp_1 \int_0^1 dp_2 \binom{3}{3 \quad x_1} p_1^{x_1} q_1^{3-x_1} \binom{3}{3-x_2} p_2^{x_2} q_2^{3-x_2}$$

gives $\frac{1}{16}$ for all permissible values of x_1 and x_2.

Formula for the General Case

First we will consider the case of r *k-ply* attributes, with n individuals. Then the number of partition descriptions is the number of partitions of the r-partite number $(n; n; \ldots n)$ into k parts, zero being a permissible part,

$$\begin{aligned} & = (\text{no. of partitions of } n \text{ into } k \text{ parts})^r \\ & = \binom{n + k - 1}{n}^r = \lambda, \text{ say.} \end{aligned}$$

Our method of allowing equal weights to all the possible combinations of probabilities of the various alternatives of each attribute gives for each partition description a probability of $\dfrac{1}{\binom{n + k - 1}{n}^r}$, which is equal to our measure of $\frac{1}{\lambda}$.

[$\dfrac{1}{\binom{n + k - 1}{n}}$ is the result for one attribute, and since the attributes are supposed independent, the result is obtained from the product of r independent integrations, each of which has this value.]

It follows in a similar manner for the completely general case of r_i k_i-ply attributes, $i = 1, 2, \ldots s$, with n individuals that the number of partition descriptions is:

$$\lambda = \prod_{i=1}^{s} \binom{n + k_i - 1}{n}^{r_i}$$

giving a measure of $\frac{1}{\lambda}$ for each, where $\frac{1}{\lambda}$ is also the probability obtained by the integration method.

Carnap's corresponding result for structure descriptions is $m = \binom{n + K - 1}{n}$, where $K = \prod_{i=1}^{s} k_i^{r_i}$. The two formulae are the same if $s = 1$, $r_1 = 1$, or if $n = 1$.

Conclusion

It would appear from the above that it would be possible to build up an alternative theory of probability, in a manner similar to Carnap's, only using partition descriptions. This would at the same time have a link with the classical theory of probability, since the measure allotted to each partition description will always be the same as the probability obtained by our method of integration from that given by the classical theory.

The development of such a theory may prove intractable; the treatment would in any case have to be far more rigorous than was permissible within the space of this note, where it has in many cases only been possible to sketch the outlines of the proofs of the general formulae. The main purpose here, however, has merely been to indicate the possibility of such an alternative theory.

Industrial Classification—National and International

By R. E. Beales

Two documents of considerable interest to those concerned with statistics classified by industry were issued during 1948. The first of these was the "Standard Industrial Classification"* which is to be used in future for United Kingdom official statistics; the second was the United Nations "International Standard Industrial Classification of all Economic Activities,"† which replaces the classification recommended in the Report, "Statistics of the Gainfully Occupied Population," issued by the League of Nations.

2. The purpose of this paper is to give an account of some of the problems which were met in preparing these two classifications, of the solutions which were adopted, and, in the case of the United Kingdom classification, a brief note of the changes introduced.

I. The Standard Industrial Classification for Use in the United Kingdom

3. The new Standard Industrial Classification is the first of its kind to be brought into use for official statistics in this country. Before the war each major Department producing statistics classified by industry had been largely free to develop its own classification, and although interdepartmental consultations did take place, it was often not possible to relate statistics published by different Departments with any certainty that the definitions and classifications were comparable.

4. There were three main classifications which it was desirable to bring into line—(*a*) the classification used by the Registrars-General in the Census of Population, (*b*) the classification used by the Ministry of Labour for statistics of employment and unemployment, earnings, hours of work, etc., and (*c*) the classification used by the Board of Trade for the Census of Production. Of these, the first two, which covered the whole field of economic activity (apart from the exclusion of private domestic service from Ministry of Labour figures), were closely, but not completely, comparable, while the classification used in the Census of Production, which had a more limited scope, differed considerably from both the others. Many of the differences were unfortunately concealed by a similarity in nomenclature, and were only brought to light by detailed study.

5. The first task in drawing up a standard classification was to obtain agreement on the general principles governing the classification. The main points to be considered were (*a*) the classification unit; (*b*) the criteria for deciding the classification of mixed businesses; (*c*) the treatment of establishments owned or operated by the Central or Local Government, or nationalized; and (*d*) the optimum number of headings to be used in the classification.

6. There was little difficulty in deciding that the unit for classification should normally be the "establishment," that is, a self-contained unit such as a farm, mine, factory, shop, or office in respect of which it would be possible to obtain all the information required at a Census of Production or Distribution. In most cases such a unit could be readily identified merely by a name and address. In some cases, however, a single address, particularly that of a factory, covers a number of different activities. Where these activities are sufficiently departmentalized for separate records to be kept for each section for labour, production, materials used, fuel and power, etc., it is obviously desirable that in a Census of Production separate returns should be obtained and each department classified under its correct industry heading. While departmental returns could be obtained from employers, it would be more difficult to classify in this way statistics based on information obtained direct from employees at a Census of Population or an exchange of insurance books. However, it was decided that the attempt should be made in those cases for which departmental returns are made in the Census of Production.

* Published by H.M. Stationery Office, February 1948.

† Supplement No. 5B to the Official Records of the Economic and Social Council (Seventh Session). Published by the United Nations and obtainable from H.M. Stationery Office.

7. Where an establishment was engaged in more than one type of activity and could not be split into departments it was agreed that the whole establishment should be classified according to its major product (or group of products allocated to the same heading). The "net output" or "value added" would be the best criterion for deciding which is the major product, but since net output is normally available only for the establishment as a whole it is necessary to use gross output instead. Similarly for distributive or service establishments the criterion will usually be the gross receipts. There are certain exceptions to these rules. Establishments which combine production with retail sale (either of their own products or of purchased goods also) or which combine wholesale and retail trade will, as far as possible, be classified according to the numbers employed in the two sides of the business rather than on financial data, which in such cases might be misleading.

8. It was recognized from the outset that a common classification would not, by itself, be sufficient to ensure comparable statistics. Differences might still arise because of different views on the correct classification of particular establishments engaged in more than one activity. The ideal solution would be a complete register or directory showing the name, address, and agreed classification of every establishment in the country. But the work involved in compiling such a register and keeping it up to date would be so great that a complete register is hardly a practicable proposition. The principle of consultation between the Government Departments concerned with the classification of "difficult" establishments has, however, been accepted and is in operation, and the records kept by local offices of the Ministry of Labour and by the Census of Production Office will between them form a substitute for the complete register which will be adequate for most purposes.

9. The three existing classifications differed considerably in their treatment of establishments owned or operated by central or local governments and by other public bodies. In general, in the Population Census and in Ministry of Labour statistics all manufacturing establishments were classified together regardless of ownership (in the Census of Population subdivisions were shown for Government, Local Government, Railways, etc.), but this principle did not extend to the service industries and professions. In the Census of Production the main classification was confined to private industry, the productive activities of Government Departments, Local Authorities and public utilities (including railways) being relegated to a separate volume. With the great increase of Government activity in industry, both direct and by nationalization, it was decided that the only logical course was to classify solely by the nature of the industry, regardless of ownership or control, and to carry this principle right through the classification.

10. In deciding on the number of separate headings in the classification several points had to be borne in mind. The classification must be sufficiently detailed to meet all the reasonable requirements of statisticians, economists, research students and business men. At the same time it must not be in such great detail that the figures produced would be misleading, or would take so long to prepare that they would be valueless as a guide for official policy. The danger of attempting too fine a subdivision of industry is not always appreciated by those interested in a particular narrow field. In industries such as engineering, for example, if there are too many subdivisions an establishment producing a variety of machines may have to be classified according to a major product which represents less than half its total output. The result will be the creation of so-called "industries" which only account for a small proportion of the total output of their "principal products" and at the same time have an unduly large output of products classified elsewhere. Statistics on such a basis may be dangerously misleading.

11. The existing Ministry of Labour classification had approximately 100 headings. The Census of Population had over 400, and the Census of Production, in its narrower field, had also far more than the Ministry of Labour. But whereas the Ministry of Labour had to use the classification for monthly statistics in which speed of tabulation was of prime importance, the Censuses could, and did, pursue a more leisurely course. In the new classification it was agreed that a Minimum List of headings should be drawn up, to which Departments could work. Further subdivisions of these headings could be made if desirable, and in order to maintain comparability at this level also, it was decided to publish recommendations for subdivisions, and to use these, as far as possible, in the Censuses of Population, Production and Distribution. The Ministry of Labour agreed to operate a Minimum List considerably longer than its own previous classification, and the number of headings eventually decided upon was 163, which were grouped

into 24 Orders.* The recommended subdivisions brought the total number of industries separately identified in the classification up to 380. These numbers were not, of course, decided upon in advance, but were those that emerged from the detailed discussions, taking into account such factors as homogeneity, size and economic or strategic importance.

12. It was not difficult to obtain agreement on these general points since, for the most part, they constituted a continuation, or a logical extension, of the principles on which the Departments were already working. Greater difficulty was anticipated in drawing up the precise definitions of the various industries or groups of industries which were to be separately identified. It was in these definitions that the main differences between the existing classifications were to be found. Any change would involve additional work for the Department concerned, and, possibly, a break in the continuity of its figures, and in these circumstances there would be a natural tendency for each Department to defend its own basis. Fortunately the value of a common classification was so well appreciated that there was a minimum of friction. It was accepted at the outset that comparability with earlier figures should be regarded as having only subsidiary importance, and while all the differences in classification were thoroughly discussed, concessions were readily made in order to secure agreement. The principal changes made are set out in the succeeding paragraphs.

13. In Order I—Agriculture, Forestry and Fishing—a considerable improvement was effected by splitting off forestry from the old Ministry of Labour heading, and by transferring jobbing and private gardeners to the private domestic service category. The figures for agriculture under the new classification will thus be very closely comparable with those of the Ministry of Agriculture, the only substantial difference being the exclusion by the latter of workers on holdings of one acre or less.

14. Order II—Mining and Quarrying—was very little changed. The only difficulty arose in the case of opencast coal production. The logical course was to treat this as a subdivision of coalmining, but the work is performed by civil engineering contractors, and no satisfactory method could be found by which this section of their work could be separately identified for the coding of insurance cards or Census of Population schedules. Opencast coal production was therefore, reluctantly, left in civil engineering.

15. In Order III—Treatment of Non-Metalliferous Mining Products—the main alterations were in the Ministry of Labour heading "Cement manufacture, limekilns and whiting works," which was restricted to cement manufacture only, and in the Census of Production Glass Trade, which was split into two Minimum List headings. The growing importance of cast stone and cast concrete products was recognized by the creation of a separate subdivision of the residual heading.

16. The classification adopted for the chemical and allied trades (Order IV) involved considerable modification of all the existing classifications. The Ministry of Labour heading for chemicals was split into two, and that for oils, greases, etc., into three, while the Census of Production trades and subdivisions were regrouped. The manufacture of pigments was transferred from the Paint and Varnish heading to the main chemical group, a Minimum List Heading was set up for mineral oil refining (mainly for purposes of international comparability), and a new subdivision was established for "synthetic resins and plastics materials."

17. Metal Manufacture (Order V) presented one very serious problem—how to deal with the "vertically integrated" establishment, particularly in the sheet and tinplate industries. While it was known that the vertically integrated establishments could provide information about the various sections of their works for the purpose of the Census of Production or "L" returns, there seemed to be no way by which corresponding figures could be obtained for the individual employees at a Population Census, or in the exchange of insurance cards. The classification was therefore drawn up on the assumption that the whole of the integrated works would have to be classified according to the final product. Later, however, the employers agreed to allocate each of their employees to his or her appropriate industry for insurance purposes, and to inform employees immediately before the next Population Census which industry they should give in their returns. With this co-operation it became possible to split the integrated works, placing the workers in preliminary processes, up to the bar or slab stage, in "Steel manufacture" and the remainder in "Sheets and tinplate." Changes in the heading "Iron Foundries" affected all the previous

* The numbers of the Minimum List headings actually start at 1 and end with 299, but within each Order some numbers have been left unused, so that if necessary additional headings can be raised at a later date without disturbing all the subsequent numbers.

classifications. The new heading included engineer's ironfounding (other than foundries attached to engineering establishments) which the Ministry of Labour had previously classified with engineering and excluded steel foundries. The latter change had the advantage of bringing together in the same heading the production of "direct" castings at steel works and the production at steel foundries.

18. The main problem in Order VI was to break down the large and unwieldy groups "General engineering" and "Electric cables, apparatus, etc.," in the Ministry of Labour classification into a number of smaller and more useful headings without introducing too much overlapping between headings. The new headings eventually adopted had, in most cases, been shown as subdivisions of the Mechanical Engineering and Electrical Engineering Trades in the Census of Production, but considerable research into the 1935 Census results was necessary to establish that these headings were, in fact, reasonably homogeneous. Three important changes from previous classifications were made. Engineers' small tools were grouped with machine tools, instead of with hand tools and cutlery. Agricultural tractors were transferred from this Order and classified with motor vehicles. Gramophones (both electrical and mechanical) were classified with wireless apparatus. All these changes were made to correspond with the actual present-day practice in the industries concerned.

19. The great expansion in the aircraft industry during recent years made the main change in Order VII (Vehicles)—the splitting of the Ministry of Labour heading for motor vehicles, cycles and aircraft—almost automatic. Two difficult problems arose in this connection, however, and it may well prove necessary to introduce further modifications when some experience has been gained in using the new classification. The first of these difficulties arose from the fact that many of the parts and accessories for motor vehicles are very similar to those used in aircraft, and may be manufactured by the same firms. It was not, therefore, considered advisable to set up headings "Motor vehicles and parts" and "Aircraft and parts," since all the parts and accessories which were common to both would tend to be classified with the motor industry as the principal user. The solution adopted was to set up a separate heading, "Manufacture of parts and accessories for motor vehicles and aircraft." The Census of Production returns from establishments classified under this heading will, however, be carefully studied, and if and when it is possible to divide them into two sections, for motor vehicles and aircraft respectively, without too much overlapping, this will be done. The second problem was the classification of "motor garages." This term may cover any or all of a number of activities which could be classified under at least four different headings of the classification, namely, (1) repair of motor vehicles; (2) distribution (of vehicles, spares, petrol, etc.); (3) passenger transport (motor coaches or private-hire cars); and (4) storage accommodation. It would obviously be extremely difficult in a Census of Production or Distribution to obtain from each of the numerous small units in this trade a return which would make possible its correct allocation to one of these four headings, and to classify correctly on the scanty information available in a Census of Population schedule would be impossible. The solution adopted was to include in a single heading all the repair garages, service stations, filling stations, etc., regardless of the relative proportions of repair work and petrol and other sales. The exclusions were the sales depots for new vehicles, the passenger transport establishments and the "lock-up" garages, all of which could be identified fairly readily.

20. Orders VIII and IX presented little difficulty, the main changes being the subdivision of the Ministry of Labour headings for metal goods and for watches, clocks and jewellery. A number of new subdivisions were established.

21. All the existing classifications for the textile industries (Order X) were fairly detailed, so that no great expansion of Minimum List Headings was required. The only additions to the existing Ministry of Labour list were those resulting from the split of "Textile industries not separately specified" into the three headings "Narrow fabrics," "Made-up textiles" and "Other textile industries." The first of these new headings included the Census of Production trade "Elastic webbing" as well as smallwares previously classified by material, while the second included the Census trade "Canvas goods and sack." The carpet industry, which already had a separate heading in the Ministry of Labour classification, but not in the Census of Production, was made a Minimum List Heading. The line of demarcation between cotton spinning and weaving caused some difficulty. Some of the processes preparatory to weaving (winding, beaming, etc.) are sometimes performed by the spinners, sometimes by the weavers, and sometimes by establishments

specializing in this work. This last group had been classified in the spinning trade in the Census of Production and Census of Population, but in the weaving trade by the Ministry of Labour. The argument advanced in favour of the former method of treatment was that the finished product was still in the form of yarn, rather than cloth. The counter argument was that, logically, the preparatory processes for weaving should be classified in the weaving industry, just as the preparatory processes for spinning (opening, picking, carding, etc.) were grouped with spinning. Eventually, after a study of such figures as were available to show the relative amounts of this work done at establishments in the spinning and weaving sections of the industry, it was agreed to follow the census practice. There were a number of other minor amendments to existing practice, the chief of which affected the Ministry of Labour, and were the transfer of cotton waste cleaning, pulling, sorting, etc., and of the flock and rag trade from cotton and wool respectively to "Other textile industries," and the transfer of soft furnishings (curtains, blinds, quilts, etc.) from "Other textiles" to "Furniture and upholstery."

22. In Order XI the only significant changes, both of which brought the Ministry of Labour classification into line with that of the Census of Production, were the creation of a separate heading for "Fur," and the transfer of establishments making leather leggings, etc., from "Leather goods" to "Boots and shoes."

23. In Order XII, on the other hand, the new classification followed fairly closely that of the Ministry of Labour and most of the changes affected the Census of Production. Separate headings were set up for tailoring and dressmaking, and the distinction between wholesale, wholseale bespoke and retail bespoke tailoring made in the Census of Production was not embodied in the standard classification. The Ministry of Labour classification was affected by the transfer of millinery from "Dressmaking" to "Hats, caps and millinery" and by the separation of boot making and repairing. A problem which had to receive attention was the treatment of establishments specialising in repairing, turning and altering clothing, which have greatly increased in number mainly as the result of clothes rationing. In general, repair establishments had been classified with the manufacture of the corresponding goods, or as separate industries in the same Order. In this case, however, since a great deal of clothing repair is also undertaken by dry-cleaning establishments, it was decided to classify the specialist clothing repairers under the same heading as the dry cleaners.

24. The food, drink and tobacco industries (Order XIII) did not present any very difficult problems. Some of the existing Ministry of Labour headings were subdivided, but without any appreciable change in their content, and most of the Census of Production trades were also retained. The Census "Preserved Foods" trade was divided, part of it forming the new heading "Preserving of fruit and vegetables," part going with slaughtering and bacon-curing to make up "Meat and meat products," and the rest going into the residual heading. The manufacture of margarine was removed from the "Milk products" heading, but ice-cream was included there instead of with bread and cakes. In view of the difficulty in distinguishing between edible and laundry starch both were included in this Order.

25. The classification adopted for Order XIV (Manufactures of Wood and Cork) departed in several respects from each of the previous classifications. In the Ministry of Labour classification the main changes were the creation of separate headings for furniture and shop-fitting, the inclusion in "furniture" of soft furnishings and rustic and basket furniture, and the grouping of boxes, baskets and cooperage products under the single heading "Wooden containers and baskets." The Census of Production classification was affected mainly by the division of "manufactures of timber" between the new headings "Timber" and "Miscellaneous wood and cork manufactures," the transfer to this Order of "Shop and office fitting" (previously classified under Building) and the transfer of Basket furniture.

26. The changes made in Order XV (Paper and printing) mainly affected the Ministry of Labour but, apart from the creation of two additional headings, were not of any great significance.

27. In Order XVI (Other manufacturing industries) the most interesting changes were the setting-up of a heading for "Production and printing of cinematograph films," and the recognition of the growing importance of the plastics industry by a subdivision for "Plastics moulding, manipulating, etc." In the older classifications only film developing, printing, etc., had been included in the manufacturing section, the film studios being grouped with theatres and cinemas.

28. The Building and contracting industry (Order XVII) gave rise to a great deal of discussion,

first on the activities to be included, and second, on the most suitable headings and subdivisions. So much constructional and repair work is done outside the building industry proper, by the direct labour employed by local authorities, Government Departments, public utilities and private firms, that it would have been useful to obtain some measure of its extent in terms of the labour employed. Consideration was therefore given to the possibility of identifying building and civil engineering establishments or departments in other industries. Except for some local authorities and Government Departments, however, the attempt proved impracticable and had to be abandoned. Even in the case of local authorities it is not always easy to distinguish between the men employed solely on building and civil engineering work of the type that might also be undertaken by contractors, and those employed on minor road repairs, street cleaning and similar duties which can be regarded as a normal function of the local authority. On the question of the individual headings and subdivisions for this Order, the possibility of making some split between the main contractors and the various types of specialist sub-contractors was considered. Here again, however, it was decided that while the information necessary for such a subdivision might be available on detailed returns obtained from the employers, it would present insuperable difficulties at a Population Census or exchange of insurance cards. The final result left the existing Ministry of Labour classification virtually unchanged, except for the exclusion of stone and slate-cutting and monumental masonry.

29. The decision to set up separate headings for gas, electricity, and water, which had previously been combined in the Ministry of Labour classification, was reached without difficulty. The only problems concerned those establishments which were not producing for sale to the general public, and the gas and electricity "showrooms." The electric power stations operated by railway and tramway undertakings were left in this Order rather than in transport, but were given a separate subdivision. Power stations operated primarily for the use of factories, mines, etc., were regarded as being merely alternatives to the ordinary boiler house and were therefore left with the parent establishment, even when they disposed of surplus current to neighbouring houses. Consideration was given to classifying gas and electricity showrooms in retail distribution, but in view of their close connection with the payment of accounts it was decided to leave them in this Order.

30. The remaining Orders of the classification covered activities which were, for the most part, outside the scope of the Census of Production. Decisions could no longer be reached on the factual basis provided by the Census of Production returns, but some assistance was available from the preliminary discussions with traders about the Census of Distribution as well as from Departments.

31. The new headings for transport follow very closely those previously used by the Ministry of Labour, apart from the subdivision of the old residual heading to show separately air transport, postal, telegraph and wireless communication, and storage. The big change was the inclusion of the bulk of the Post Office under communications instead of under Government Service.

32. The distributive trades provided a very complicated problem. The classification adopted was based largely on the preliminary work for the Census of Distribution, and may well require modification later when the results of the first Census are available. The principle was to take out first the dealers in materials in which wholesale and retail trade are frequently combined in one establishment, and then the dealers in industrial materials and machinery, and finally to divide the remainder into four main categories of food and non-food goods, wholesale and retail. A difficulty arose over the correct classification under this system of the numerous retailers who combine the sale of confectionery, tobacco and newspapers, and in the end it proved necessary to set up an additional minimum list heading for the retailing of these commodities, whether singly or in combination. Each of the seven main headings was in turn broken down into a number of subdivisions by main commodity or group of commodities. In establishing these headings and subdivisions it was necessary to take a number of more or less arbitrary decisions, since so many dealers straddle the boundaries, not merely between the commodity groups but between food and non-food goods or between wholesale and retail. In this Order, therefore, more than in other Orders, the figures for individual headings and subdivisions will need to be treated with reserve, but they should provide at least a broad picture of the very complex structure of the distributive trades.

33. The old Ministry of Labour heading of "Commerce, banking, insurance and finance"

formed the basis of the new Order XXI, but was considerably modified. A number of minor activities—mainly "agencies" of various types—were transferred elsewhere, and the Post Office Savings Department was added to the subdivision for banking. The object was to confine the Order more strictly to financial activities.

34. National and Local Government Service, which make up Order XXII, were very much reduced in scope, compared with the previous classifications, as a result of the decision to classify under their appropriate headings elsewhere Government activities in the fields of communications, education, medicine, etc., as well as in manufacturing industry. It should be noted, however, that the central departments administering these services, such as the Ministries of Health and Education (but not the Post Office), were left under the main Government heading. Nor was any attempt made to transfer educational establishments or hospitals operated by the Service Departments for serving members of the Armed Forces. This was because so many of the staff of such establishments, apart from those with professional qualifications, would be indistinguishable from other members of the Forces.

35. Order XXIII was made up of the old Ministry of Labour heading for Professional Services, together with the corresponding Governmental activities and a number of business agencies previously classified with commerce, banking, insurance and finance. The only exclusion of importance was that of the various charitable and welfare services, which were made a subdivision of "Other services" in Order XXIV.

36. The Ministry of Labour heading for "Entertainment and sport," which covered every form of entertainment from promenade concerts to pin-table saloons and everybody connected with entertainment or sport from the West End theatrical star to the bookmaker's runner, had been subject to adverse comment both in Parliament and in the Press because of this diversity. In the new Order XXIV provision was made for two Minimum List Headings and a number of subdivisions to cover these multifarious activities. In particular, the current interest in the amount of national resources devoted to gambling was recognized by the establishment of separate subdivisions for horse racing, dog racing and betting. Other innovations in this Order were the establishment of Minimum List Headings for hairdressing and manicure and for resident and non-resident private domestic service. The last two had not previously been covered by the Ministry of Labour except for a few outdoor workers, since such employment was not insurable before July 1948, although they had, of course, been provided for in the Census of Population Classification. The distinction between resident and non-resident service was introduced to throw further light on housing requirements and social habits.

II. THE INTERNATIONAL CLASSIFICATION

37. The first industrial classification prepared for international use was the work of the Committee of Statistical Experts of the League of Nations, and was published in 1938 in the pamphlet, "Statistics of the Gainfully-occupied Population." This classification had been prepared after consultation with all the member States of the League and with non-members. A number of countries had agreed to reclassify their figures as nearly as possible in the international form at their next Censuses, but before much practical experience could be gained in the use of the new classification war brought census taking to an abrupt end in the greater part of the world. Even before the war ended, however, a number of countries again began to think of censuses, not only of population, but of industries, distribution, agriculture, mining. Classifications had to be drawn up or revised and, while the League's publication was not overlooked, it was not considered suitable in all respects for post-war use by most of the countries concerned.

38. One of the post-war projects was the 1950 "Census of the Americas," which quickly developed into a formal suggestion by the United Nations that as many countries as possible should take censuses in or about 1950. In January 1947, the United Nations Statistical Commission held its first session, recorded its opinion that "the question of securing, so far as possible, comparability in the statistics of different countries with regard to what is generally called 'industrial classification'—classification of all branches of economic activity—should receive urgent consideration," and set up a Committee on Industrial Classification. At its second session, in August 1947, the Statistical Commission approved the first draft of a new classification prepared by the Committee. This draft was circulated, with a request for comments and suggestions, to

all member nations of the United Nations, to the Specialized Agencies and to some non-member nations.

39. A group of five consultants with experience in this field was attached to the United Nations Secretariat to consider the replies and comments, a revised draft was prepared for consideration by the Committee in April 1948, and was then submitted to and approved by the Statistical Commission a few days later. The final stage was approval by the Economic and Social Council, which was given in August 1948, after which the "International Standard Industrial Classification of All Economic Activities" was formally brought to the notice of all nations by the Secretary General.

40. The problems which arose in the preparation of this international classification were rather different from those encountered in the United Kingdom. In the latter case the problem was to reconcile three alternative methods of classifying the same collection of individuals or establishments; in the former it was necessary to devise a common framework into which could be fitted national classifications designed to suit very different industrial structures.

41. The first difficulty which arose—and one of the most intractable—was unexpected. It concerned the unit for classification. The League's pamphlet included a lengthy discussion of the relative advantages and disadvantages of classifying on the basis of the "enterprise," the "establishment" or the "operational unit," and arrived at a very definite decision that the most suitable unit was the establishment, which was defined in a very similar way to that adopted in the United Kingdom. In circulating the first draft of the new classification the Statistical Commission endorsed the League's view without making any comment on it. The replies received, however, revealed that national practices were far from uniform in this respect, and that some countries even used a fourth concept which had not been discussed at all by the League. This was the "technical unit," and involved the splitting of establishments making more than one product or group of allied products. The treatment of "common service" workers by the countries using this concept was not uniform, as in some cases they were all classified to the principal activity, and in others were allocated more or less arbitrarily in the same proportions as the operatives. After considerable discussion, however, the "establishment" was confirmed as the recommended unit for classification purposes.

42. Another unexpected difficulty was the title of the classification. Several countries had apparently understood "industrial" in the narrow sense of manufacturing industries only, and had difficulties in reconciling this with the inclusion of service industries. To meet this point it was eventually decided to adopt the rather cumbersome title "International Standard Industrial Classification of all Economic Activities."

43. As was to be anticipated, the great bulk of the comments in the replies from national governments and specialized agencies concerned individual headings of the proposed classification. Small countries with some special local industry urged that it should be given much more prominence, in keeping with its importance in their own economy. Conversely, they failed to see why an activity which was almost non-existent in their country should have been accorded the dignity of a "Group" or even a "Major Group." All such suggestions were considered on their merits, and in a few cases where they were supported by several respondents additional headings were raised.

44. The agricultural industry gave rise to a number of suggestions. Breakdowns by type of main product (dairy, cereal, meat, etc.) were proposed, as well as an analysis by the size of holding. But conditions in this important industry vary so much from one part of the world to another that it was found impossible to devise any split that would have been generally practicable.

45. Textiles were also difficult. While some countries wished to distinguish the different fibres (cotton, wool, rayon, etc.), others advocated a classification by process (*e.g.* spinning, weaving) and no suitable compromise could be found. All that could be done was to segregate knitting mills and rope and cordage factories, both of which seemed to be separately identified by most countries.

46. For chemicals, however, the single heading in the original draft was divided into three on the strength of numerous requests. No less numerous were the suggestions for subdividing the metal and engineering industries. Here, unfortunately, there was again no common basis to work on, and only a very limited number of groups proved to be practicable. Much the same occurred in the wholesale and retail trade: some countries worked only on type of

commodity sold, others segregated wholesale and retail trade, others adopted various combinations of these two ideas, and it proved impossible to suggest any breakdown by commodity which would be acceptable to more than a few countries.

47. One suggestion which was included in the comments from a number of countries was that in some way or another a differentiation should be made between "handicrafts" and factory production. This was an idea which was obviously worthy of investigation as offering a rough guide to the degree of industrialization of different countries. The difficulty, however, was to define "handicrafts" in a way that would be both practicable and acceptable to all countries. The lower limit of size for inclusion in a Census of Production seemed to be the most useful indication of what would be an acceptable criterion. But when this was studied wide divergencies in national practice were at once apparent. Some countries included all establishments with more than ten employees, others only establishments with 20 or 25 and over. Yet other countries had an entirely different basis—the use of power-driven machinery, irrespective of size. The alternatives, therefore, seemed to be for the Statistical Commission to lay down some arbitrary line of demarcation, and ask all countries to accept it in their national censuses or to provide the necessary headings in the classification and leave each country to define "Handicrafts" in its own way. Neither of these was satisfactory, since the first represented a greater interference with national policies than could be accepted, and the second was likely to produce misleading results between one country and another—for example, if the lower limit for the 1935 Census of Production in the United Kingdom had been set at 25 instead of 10, over a quarter of a million workers in the "factory trades" would have been excluded.

48. Since the classification was intended for use for a variety of statistics—censuses of population, production or distribution, man-power surveys, unemployment statistics, etc.—it was independent of any definition of the gainfully-occupied population. This latter problem has, in fact, been under consideration not only by the Statistical Commission, but also by the Population Commission and the International Labour Office. But one aspect of the problem had to be considered—the treatment of domestic servants in farm households. The League of Nations Committee had recommended that in countries where they were not considered as occupied in agriculture they should at least be shown as a separate subdivision of the heading "Household and personal services." After a lengthy discussion it was decided not to make any such recommendation in the new classification, on the grounds that if such persons are mainly employed on farm work they should not be in this heading at all, but under "Agriculture," while if they are correctly classified as domestic workers there is no valid reason for segregating them from other domestics.

49. Perhaps the most difficult problem of all, and the one for which the solution adopted is the least satisfactory, was that of repair work. Here again national practices differed, some countries including repair with manufacture, and others treating it as a section of the service industries. Eventually it was decided to include clothing repairs with laundering and cleaning, and to include all other repair establishments with the corresponding manufacturing industry, separate headings being raised for the three most important categories, the repair of footwear, motor vehicles, and watches and clocks. In using statistics based on this classification, however, it will always be necessary to bear in mind that only establishments mainly engaged in repair work, can be classified in this way; in addition, there will always be a great deal of repair work allied to and inseparable from, retail distribution, and the proportion of the two types may vary considerably from one country to another.

50. As mentioned earlier, the classification drawn up by the League's Committee of Experts was never really tested in practice on any considerable scale. The new classification will meet a searching test when it is applied to the Population Censuses to be taken by most countries in or about 1950. From the experience then gained it should be possible to carry out the next revision on a much firmer basis. To speak of "the next revision" is not to take a pessimistic view of the present effort. Industry does not stand still, and that fact alone will make modifications essential sooner or later.

III. Comparability of the Two Classifications

In order that the United Kingdom classification could be brought into use in July 1948, when the new National Insurance scheme came into being, the classification had to be issued early in

February. At that time the final form of the International classification had not been settled, but the first draft was available, and this enabled some modifications to be made in the United Kingdom classification. In some cases small but identifiable activities were transferred from one heading to another; in other cases additional subdivisions were added to enable United Kingdom statistics to be rearranged in conformity with the international pattern. A few instances remained in which it was not possible to follow the International classification, either because the activities concerned were too insignificant in this country to be worth separate identification, or because it was not possible to subdivide some of the United Kingdom headings without producing statistics which would be so incomplete that they would be misleading. An example of the first type is hunting and trapping (for commercial purposes, not sport), which is important in countries with large stocks of wild animals, but negligible here; an example of the second type is the manufacture of tractors, which in some countries, notably the United States, is part of the agricultural machinery industry, but in this country is almost entirely within the motor industry. The effect of such minor differences, however, should not be exaggerated. In proportion to the size of the industrial groups affected they are usually negligible, and may be no more important than the errors which are inevitably introduced by classifying each establishment according to its major output.

Some of the groups in the International classification can only be obtained by using the optional subdivisions of the United Kingdom list. As a result it will be possible to rearrange statistics which will use all the subdivisions (such as those from the Population Census) more nearly in agreement with the International list than series (such as Ministry of Labour figures) for which only the Minimum List Headings are available. Appendix A to this paper shows the re-grouping of United Kingdom headings and subdivisions which will reproduce the International classification as closely as possible. Appendix B shows the corresponding re-grouping which will have to be adopted when figures are only available for Minimum List Headings.

APPENDIX A

REGROUPING OF UNITED KINGDOM FIGURES FOR INTERNATIONAL PURPOSES

	International group	United Kingdom headings or subdivisions	Differences between U.K. figures and the international groups
010	Agriculture and livestock production	1	..
021	Forestry	2	..
022	Logging		
030	Hunting, trapping and game propagation	..	Negligible; included in 010.
040	Fishing	3	..
110	Coal mining	10	Excludes open-cast working (strip-mining), which is included in Group 400.
121	Iron ore mining	11	..
122	Metal mining, except iron ore mining	19(1)	..
130	Crude petroleum and natural gas	19(3)	..
140	Stone quarrying, clay and sand pits	12, 13, 14	..
190	Non-metallic mining and quarrying, n.e.c.	19(2), 19(4)	Excludes peat digging and cutting, classified in 010.
201	Slaughtering, preparation and preserving of meat	153	..
202	Manufacture of dairy products	154	Includes sterilising and bottling of milk, except by retailers.
203	Canning and preserving of fruits and vegetables	157	..
204	,, ,, of fish and other sea foods	162(2)	Excludes canning, which is unimportant, and is included in 209.
205	Manufacture of grain milk products	150	..
206	,, of bakery products	151, 152	..

APPENDIX A—*cont.*

	International group	United Kingdom headings or subdivisions	Differences between U.K. figures and the international groups
207	Sugar factories and refineries . . .	155	..
208	Manufacture of cocoa, chocolate and sugar confectionery	156	..
209	,, of miscellaneous food preparations	162(1), (3)–(6)	..
211	Distilling, rectifying and blending of spirits .	168(1)	..
212	Wine industries	168(2)	..
213	Breweries and manufacturing of malt . .	163	..
214	Soft drinks and carbonated water industries .	168(3)	..
220	Tobacco manufactures	169	..
231	Spinning, weaving and finishing of textiles	110, 111, 112(1)–(3), 113–116, 119–121, 123	..
232	Knitting mills	118	..
233	Cordage, rope and twine industries . .	117	..
239	Manufacture of textiles, n.e.c. . . .	112(4) 129(2), 129(3), 191	..
241	,, of footwear, except rubber footwear	148	..
242	Repair of footwear	149	..
243	Manufacture of wearing apparel, except footwear	132, 140–143, 147	..
244	,, of made-up textile goods, except wearing apparel	122	Excludes curtains, slip-covers, pillows, etc., which are included in 260.
250	Manufactures of wood and cork, except manufacture of furniture . .	170, 173, 179	..
260	,, of furniture and fittings . .	99(1), 171, 172	Includes soft furnishings (curtains, slip-covers, pillows, etc.).
271	Pulp, paper and paperboard mills . .	180	..
272	Manufacture of articles of pulp, paper and paperboard	181–183	..
280	Printing, publishing and allied industries .	186, 189	..
291	Tanneries and leather finishing plants .	130	..
292	Manufacture of leather products, except footwear and other wearing apparel	131	..
300	,, of rubber products . .	190	..
311	Basic industrial chemicals, including fertilisers	31, 33	..
312	Vegetable and animal oils and fats . .	39(2) and (3)	..
319	Manufacture of miscellaneous chemical products	32, 34, 35, 39(4)	..
321	Petroleum refineries	36	..
322	Coke ovens	30	..
329	Manufacture of miscellaneous products of petroleum and coal	39(1)	Figures relate only to lubricating oils and greases not made in petroleum refineries. Other items are included in 339.
331	,, of structural clay products .	20	..

APPENDIX A—*cont.*

	International group	United Kingdom headings or subdivisions	Differences between U.K. figures and the international groups
332	Manufacture of glass and glass products	22, 23	..
333	,, of pottery, china and earthenware	21	..
334	,, of cement	24	
339	,, of non-metallic mineral pro-products, n.e.c.	29, 129(1)	See remarks to Group 329.
341	Iron and steel basic industries	40–44	..
342	Non-ferrous metal basic industries	49	..
350	Manufacture of metal products, except machinery and transport equipment	57, 58, 90–95, 99(2)–(8)	..
360	,, of machinery, except electrical machinery	52–56, 69	Excludes tractors and industrial trucks, which are in 383.
370	,, of electrical machinery, apparatus, appliances and supplies	70–75, 79	..
381	Shipbuilding and repairing	50, 51	..
382	Manufacture of railroad equipment	84–86	..
383	,, of motor vehicles	80(1)–(3), 83	Includes certain aircraft parts and accessories.
384	Repair of motor vehicles and cycles	81	..
385	Manufacture of bicycles	80(4)	..
386	,, of aircraft	82	Excludes certain parts and accessories.
389	,, of transport equipment, n.e.c.	89	..
391	,, of professional, scientific, measuring and controlling instruments	100(4), 100(5)	..
392	,, of photographic and optical goods	100(1)–(3), 199(2)	..
393	,, of watches and clocks	101(1)	..
394	Repair of watches and clocks	101(2)	..
395	Manufacture of jewellery and related articles	102	..
396	,, of musical instruments	103	..
399	Manufacturing industries, n.e.c.	192–194, 199(1) and (3)	..
400	Construction	200–202	..
511	Electric light and power	211	Includes Group 513.
512	Gas manufacture and distribution	210	..
513	Steam heat and power	..	Included in figures for Group 511.
521	Water supply	212	..
522	Sanitary services	..	Not separately identified. Included in figures for Group 810.
611	Wholesale trade	164, }	..
612	Retail trade	240–246 }	..
620	Banks and other financial institutions	250(2) and (3)	..
630	Insurance	250(1)	..
640	Real estate	250(4)	..
711	Railroad transport	220	Includes underground railways.

APPENDIX A—*cont.*

	International group	United Kingdom headings or subdivisions	Differences between U.K. figures and the international groups
712	Tramway and omnibus operators	221	Excludes underground railways. (*N.B.*—It may be possible to make an approximate adjustment for underground railways on information available from the London Transport Executive.)
713	Road passenger transport, except omnibus operators	222	..
714	„ transport, n.e.c.	223	..
715	Ocean transport	224	..
716	Water transport, except ocean transport	225, 226	..
717	Air transport	227	..
718	Services incidental to transport	238(1)	..
719	Transport, n.e.c.	238(2)	..
720	Storage and warehousing	239	..
730	Communication	228	..
810	Government services	260, 265	..
821	Educational services	271	..
822	Medical and other health services	273, 279(4)	..
823	Religious organisations	274	..
824	Welfare institutions	299(3)	..
825	Legal services	272	..
826	Business services	270, 279(1), (2), (3) and (6)	..
827	Trade associations and labour organisations	279(5)	..
829	Community service, n.e.c.	299(4)	..
831	Motion picture production, distribution and projection	195, 280(1)	..
832	Theatres and related services	280(2)	..
833	Recreation services, except theatres and motion pictures	281	..
841	Domestic service	290, 291	..
842	Restaurants, cafés, taverns and other eating and drinking places	285	..
843	Hotels, rooming houses, camps and other lodging places		
844	Laundries and laundry service, cleaning and dyeing	286, 287	..
845	Barber and beauty shops	288	..
846	Portrait and commercial photographic studios	299(2)	..
849	Personal service, n.e.c.	299(1), (5), (6)	..
900	Activities not adequately described	—	..

APPENDIX B

REGROUPING OF UNITED KINGDOM STATISTICS (MINIMUM LIST HEADINGS ONLY) TO AGREE AS CLOSELY AS POSSIBLE WITH THE INTERNATIONAL CLASSIFICATION

International group	United Kingdom minimum list heading.
010, 030	1
021, 022	2
040	3
110	10
121	11
122, 130, 190	19
140	12, 13, 14
201	153
202	154
203	157
205	150
206	151, 152
207	155
208	156
204, 209	162
211, 212, 214	168
213	163
220	169
231	110–116, 119–121, 123
232	118
233	117
239	129, 191
241	148
242	149
243	132, 140–143, 147
244	122
250	170, 173, 179
260	171, 172
271	180
272	181–183
280	186, 189
291	130
292	131
300	190
311	31, 33
312, 319, 329	32, 34, 35, 39
321	36
322	30
331	20
332	22, 23
333	21
334	24
339	29
341	40–44
342	49
350	57, 58, 90–95, 99
360	52–56, 69
370	70–75, 79
381	50, 51

APPENDIX B—*cont.*

International group	United Kingdom minimum list heading
382	84–86
383, 385	80, 83
384	81
386	82
389	89
391, 392	100
393, 394	101
395	102
396	103
399	192–194, 199
400	200–202
511, 513	211
512	210
521	212
611, 612	164, 240–246
620, 630, 640	250
711	220
712	221
713	222
714	223
715	224
716	225, 226
717	227
718, 719	238
720	239
730	228
522, 810	260, 265
821	271
822	273
823	274
825	272
826, 827	270, 279
824, 829, 846, 849	299
831, 832	195, 280
833	281
841	290, 291
842, 843	285
844	286, 287
845	288

The Sources and Nature of Statistical Information in Special Fields of Statistics

Statistics Relating to the Coal Mining Industry

By R. F. George

Statistics about coal mining fall broadly into four main classes: output, manpower, effort and earnings. In each group there are particular points to be watched by the careful user of the figures and these details are by no means always indicated in the published information. Some of these figures have been collected over a long period. The quantity of coal produced, for example, has been recorded annually for nearly a century, and the numbers of persons employed are available for over 75 years. It is not surprising that in series going back as far as these, continuity throughout the years has been broken from time to time. In the space of this memorandum it is not possible to follow every figure down the years, and explain each change introduced or to measure its probable effect. Those who use long-term trends are warned, however, to read the footnotes given at the bottom of the officially published figures and to avoid drawing fine distinctions between small movements. The statistics published at the present time are generally comparable for the past several years, and it is these current statistics with which we are mainly concerned for the present purpose.

(a) *Output*

Coal is either deep-mined or (since 1942) produced from opencast workings. Deep-mined coal is produced almost entirely from the mines, about 980, directly operated by the National Coal Board. There are, however, in addition, some 500 small mines operated under licence from the Board, which account for hardly more than 1 per cent. of the total output. The weekly figures of deep-mined coal production are derived from returns completed by all mines, both N.C.B. and licensed. In addition, the amount of coal obtained from opencast workings, which are the responsibility of the Ministry of Fuel and Power, is shown separately. The week for deep-mined coal begins with the descent of the afternoon shift on Saturdays, whereas the opencast week finishes on Wednesdays. Opencast coal, however, is statistically regarded as quite distinct from deep-mined coal, and the rest of this memorandum will deal with deep-mined coal only. Coal in the condition in which it is taken out of the mine is known as "raised and weighed." When it has gone through the various cleaning processes and the major impurities have been extracted, by hand or mechanically, it is known as "saleable coal." This is the figure which is published each week and, of course, represents somewhat less than the quantity raised and weighed. Not all the amount of saleable coal is available for domestic or industrial consumption, or for exports and bunkers. In order to arrive at the quantity available for commercial purposes, we have to deduct the coal consumed at the mine for its power requirements, and the amount taken by miners as free or concessionary coal. After allowing for these quantities, we are left with coal "commercially disposable." The following figures give an idea of the proportions represented by these classes for the year 1947 in respect of deep-mined coal:

	Quantity (mill. tons)	*Percentage*
Raised and weighed	199·7	100·0
Saleable	187·2	93·7
Colliery consumption	10·8	5·4
Free and concessionary	5·0	2·5
Commercially disposable	171·4	85·8

Finally, the quantity of coal held in stock at collieries and at landsale depots is recorded each week. Landsale depots are stocking points sometimes outside the colliery as such, where coal received from the colliery is held for general distribution.

It should be mentioned that alongside the tonnage produced, figures are also collected showing the estimated tonnage lost where manshifts are lost, other than for the reason of absenteeism. The

causes contributing to this lost tonnage include recognized holidays, disputes, accidents and breakdowns. The total quantity so estimated to have been lost in 1947 was nearly 13 million tons, and in 1948 over 10 million tons, in both instances the main cause being recognized holidays.

(b) *Equipment and Mechanization*

Annual returns are collected from all mines giving the number of coal-cutting machines and conveyors of various types in use and the quantity of coal cut and conveyed by these means. By relating these quantities to the total output of coal raised and weighed it is possible to indicate the progress of mechanization in the mines. The annual returns also give details of the number of coal-cleaning plants of various types in use and the output of mechanically cleaned coal.

These returns are supplemented by a census of certain types of equipment at June 30th each year, which provides a record of the number of electric motors installed, coal-breaking and crushing plants in use and the number of horses employed underground. Information relating to all these items of equipment is published in the *Statistical Digest of the Ministry of Fuel and Power.*

(c) *Manpower*

The gross manpower in the industry is termed "wage earners on colliery books," and is defined as those employed in, or in connection with, raising or handling coal (or other minerals which are got with coal), including mechanics and those employed on sidings at the pit, on private branch railways and tramways, and in washing coal on premises adjacent to and belonging to the mine, but excluding administrative and clerical staffs. This definition is not so precise as to indicate clearly who should be included and who should not. It does not, for example, define the position with regard to men employed in a central washery serving several mines but not situated at any one mine. Similarly men employed at a central workshop, serving several mines but actually an independent establishment, also give rise to difficulty. The ambiguities inherent in the definition given above are well recognized and an endeavour is being made at the present time to produce a definition at once adequate and practicable; this necessarily involves a detailed examination of a large number of particular cases covering every possibility, and will take some time to complete.

There is also a further difficulty. Men in the coal-mining industry sometimes leave their employment without giving notice of their intention or the reason for their departure, and there has been no uniformly established procedure for striking men off books after a given period of absence. It is possible for a man to remain on the books for many months, perhaps because of long-term injury, without reporting for work. The number of wage earners on books is therefore not necessarily a realistic figure of those available for work. The practice in this connection varies throughout the country and for this reason the published manpower statistics relate to what is known as the "standardized total." This figure consists of those on colliery books, less those who have been absent for more than six months, plus those already removed from colliery books who are not known to have left the colliery but whose absence has not exceeded six months.

In addition there is published each week the number of "effective" workers, which represents the "standardized total" less those who have been absent for the whole week, i.e. the number of wage earners who have made at least one appearance during the week.

Under the heading of manpower statistics we may include figures of recruitment and wastage. The former is analysed in some detail, distinguishing, for example, the number of new entrants to the industry under 18 and ex-miners from other industries. Wastage is similarly broken down into certain classes, e.g. deaths, retirements, compensation and long-term sickness cases and dismissals.

For some ten years up to and including 1947 an annual census has been taken to show the age distribution of the wage earners on colliery books in the following groups: 14 and under 16; 16 and under 18; 18 and under 20; 20 and over. Since 1944 the last group has been divided to distinguish those aged 65 and over. The average age has been calculated from these figures, with a 2 per cent. sample of insured men in the coal-mining industry taken by the Ministry of Labour on the basis of its mid-year count of Unemployment Insurance Books. For 1948 advantage has been taken of the Census of Production schedule to obtain a direct count of the

wage earners at a pay date early in December by a series of age intervals between 21 and 65, as well as for those under 21. The age distribution will therefore be available in rather greater detail than has hitherto been the case. The following table shows the number and proportion under and over 20 by place of work in 1938 and 1947.

Age Distribution of the Average Number of Wage Earners in 1938 *and* 1947

	1938				1947			
	Under 20		20 *and over*		*Under* 20		20 *and over*	
	Number	*Per cent. of total*	*Number*	*Per cent. of total*	*Number*	*Per cent. of total*	*Number*	*Per cent. of total*
Underground:								
Males	76,900	9·8	543,000	69·5	32,500	4·6	517,100	72·7
Surface:								
Males	32,000	4·1	127,500	16·3	21,000	2·9	139,100	19·5
Females	1,000	0·1	1,300	0·2	500	0·1	1,200	0·2
Total	109,900	14·0	671,800	86·0	54,000	7·6	657,400	92·4

In addition, details of the numbers recruited and lost to the industry are collected quarterly by age groups in two-year intervals up to 20; 20 and under 26; and in five-year intervals thereafter.

(d) *Shifts Worked*

The number of wage earners reported as belonging to an industry is only an approximation to the amount of effort put in. The work actually done is measured more precisely by the recording of the number of shifts worked. This record leads to the further measurements of absenteeism and productivity. The basic figures consist of the total number of manshifts worked each week, distinguishing those (*a*) at the coal face, (*b*) elsewhere below ground, and (*c*) on the surface. In addition, there is recorded the number of manshifts which were lost during the week and which could have been worked. These manshifts lost are required to be calculated from the "standardized" number of wage earners, and include all shifts lost from any cause other than stoppage at the mine. They are divided between shifts lost on account of sickness or for other acceptable reasons (involuntary absence) and those for which no satisfactory reason is given (voluntary absence), and are further split to distinguish shifts lost (*a*) at the coal face, (*b*) elsewhere below ground, and (*c*) on the surface.

For the purpose of shifts worked account is taken of the time actually worked. Thus a man working 8 hours instead of his 7½ hours is credited with $1\frac{1}{15}$ shifts. Similarly, a 9-hour shift worked instead of a basic 8½-hour shift is entered as $1\frac{1}{17}$ shifts. On the other hand, shifts lost can only be recorded as one shift for each non-attendance.

The total number of "possible" shifts consists of the addition of shifts worked (including the allowance for the length of time spent on the shift) and shifts lost. The latter expressed as a percentage of the former gives the measurement of absenteeism, and according to the detail used is calculated either for face workers or "overall," and sometimes distinguishes voluntary from involuntary absenteeism.

The average weekly number of shifts worked is calculated by dividing the total number of

Average Number of Shifts Worked per Week per Wage Earner on Colliery Books—3rd and 4th Quarters, 1947, *and 1st and 2nd Quarters,* 1948

	1947		1948	
	3rd Quarter	*4th Quarter*	*1st Quarter*	*2nd Quarter*
Face workers	3·98	4·51	4·44	4·54
Other underground workers	4·36	4·89	4·80	4·88
Surface workers	4·81	5·26	5·21	5·28
All wage-earners	4·31	4·82	4·74	4·83

shifts worked per week by the (standardized) number of men on books. The average number of shifts worked per man per week also reflects changes in attendance, but in this measure special allowance must be made for the occasions such as holidays or disputes when there is a stoppage at the mine.

(e) *Productivity*

By dividing the saleable tonnage of coal produced by the corresponding number of shifts worked we obtain the output per manshift, or O.M.S. as it is usually called. This figure is published in terms of either O.M.S. at the face or O.M.S. overall. The shift, however, is the basic unit of time for this purpose, and the real effort-value of the individual, it may be argued, is best considered in terms of output per man-year, since the longer period takes into account the number of shifts worked.

(f) *Accidents*

All fatal accidents and serious non-fatal accidents are required under the Coal Mines Act, 1911, to be reported to the Mines Inspectorate of the Ministry of Fuel and Power. Provisional figures are published quarterly in the *Board of Trade Journal*, and annual figures, analysed by major causes and expressed as rates per 100,000 manshifts worked, appear in the *Statistical Digest of the Ministry of Fuel and Power*.

In addition annual returns are collected by the Ministry, analysing in considerable detail all non-fatal accidents resulting in disablement for more than three days. Details of such accidents by causes, age of persons injured and nature of injury are also published in the Ministry's *Statistical Digest*.

(g) *Earnings*

Statistics of earnings in the industry are published twice a year in the *Ministry of Labour Gazette*, usually in April and October. This consists of a brief textual reference in the appropriate article, and shows average figures for the country as a whole. As indicated in section (j) below, average weekly and manshift earnings are published in the Quarterly Statistical Statement in some degree of detail. It is important to bear in mind that allowances in kind are a valuable supplement to the cash earnings. In 1948, for example, the latter for all workers was 157*s*. 1*d*. per week with allowances in kind representing a further 6*s*. 9*d*. These allowances consist very largely of free and concessionary coal, and to a lesser extent of house allowances. These, like cash earnings, vary from district to district according to local agreements.

The earnings for adult male workers are given as a footnote to the table relating to the country as a whole in the Quarterly Statistical Statement. Until the second quarter of 1948, the definition of an adult male worker depended on the interpretation recognized by local wages agreements, and varied from 18 years in some districts to 22 years in others. The figures given for the third quarter of 1948 and subsequently are calculated on a slightly different basis, and depend on a special census taken twice a year, relating specifically to all adult males of 21 and over.

A change of even greater importance was introduced in the figures published for the first quarter of 1949. Until then the classes of workers taken for the purpose of dividing into the wages bill in order to obtain average earnings lacked uniformity throughout the country, and over a period of nearly 30 years a number of variations had been locally introduced. Beginning with 1949, therefore, the procedure was placed on a uniform basis and many inconsistencies were removed. In the result average weekly earnings, including the value of allowances in kind, showed an increase on this account of some 3 per cent for all workers in the first quarter of 1949.

Average Weekly Earnings per Wage Earner (all ages), First Quarter 1949

	Underground workers		*Surface workers*		*All workers*	
	s.	*d.*	*s.*	*d.*	*s.*	*d.*
Average weekly earnings (including the value of allowances in kind):						
New basis	189	10	139	6	178	7
Old basis	184	1	136	2	173	5

The change had different effects in different Divisions; in the Northern Division the effect was an increase of over 9 per cent., while in the South-Western Division it was a decrease of 0·6 per cent. This interruption in the continuity of the series must not be overlooked by those using these official statistics of earnings over a series of years. In order to assist the interpretation of the figures the difference between the earnings calculated on the old and the new basis are given for each wages district in the Statement for the first quarter of 1949.

(h) *Consumption*

Statistics of the consumption of coal are necessarily derived from industrial consumers and other sources. The Ministry of Fuel and Power regularly collect figures of the quantities consumed for industrial and domestic purposes, distinguishing the public utilities and main industries. The figures complete the detail necessary for the balance-sheet of availability and consumption as published in the Ministry's Digest. The following figures, taken from that source, compress some of the detail given in the published table, and serve to indicate broadly the quantity of coal available and its disposal in 1947:

Availability	*Mill. tons*	*Consumption*	*Mill. tons*
Production of mined coal	187·2	Gas works	22·7
Decrease in colliery stocks	0·2	Electricity works	27·1
		Railways	14·6
Available mined coal	187·4	Coke ovens	19·8
Opencast disposals	10·4	Collieries	10·8
Imports	0·7	Industry	39·7
		Domestic (including boiler fuel)	31·6
		Miners' coal	5·0
		Other home consumption	11·2
		Total Great Britain	182·5
		Shipments to Northern Ireland	2·4
		Exports and foreign bunkers	5·5
		Total consumption and exports	190·4
		Increase in distributed stocks (held by principal consumers)	8·1
Total	198·5	Total	198·5

For a more preci.e definition of some of these items reference should be made to the Ministry's Digest.

(i) *Geographical Detail*

Statistics relating to the industry are usually published for the country as a whole together with details for the constituent geographical areas. There are several of these geographical patterns and, although in some cases the corresponding localities can be reconciled fairly easily, there are instances where they almost defy any attempt to compare one with another. The organization of the National Coal Board divides the country primarily into eight Divisions, which in turn are subdivided in the aggregate into nearly 50 Areas. Statistics conforming to these territories are, of course, collected for the purposes of the Board's administration. In addition, for the purpose of wages and earnings, there is a different set of localities known as Wages Districts, of which there are 20 in all. In a number of instances the Board's Divisions and Areas correspond directly to the appropriate Wages Districts, e.g. the Scottish Division, consisting of five areas,

corresponds exactly to the one wages district, Scotland. Similarly, four of the five areas in the North-Western Division represent the Lancashire and Cheshire wages district, while the fifth area in that Division is identical with the North Wales wages district. On the other hand, the wages districts in the East Midlands Division can only be reconciled with that Division's areas by manipulating the arrangement of a large number of individual mines.

A third geographical pattern is that used by the Ministry of Fuel and Power. This consists of nine Regions, counting Northern A and Northern B as two, and including Kent. These Regions comprise 25 districts, again including Kent as one. It may be added that Kent, consisting of only four collieries, presents no geographical difficulties at all; it is one of the Board's Divisions, it is not divided into areas, it is one wages district and, for the Ministry's purposes, one region. The Ministry's Regions agree almost exactly with the Board's Divisions, the only difference being one colliery included by the Ministry in their North Midland Region (i.e. the Board's East Midlands Division) and by the Board in their North-Eastern Division. This colliery, however, ceased production during 1948, and adjustment is now only necessary in respect of the relatively few numbers of workmen still employed. The Ministry's Districts are not always easy to reconcile with the other geographical classifications mentioned. Thus in the Ministry's North Midland Region, the four constituent districts are identical with the corresponding wages districts, but these can only be reconciled with the Board's areas by reference to the individual collieries involved. Again, the Ministry's North-Eastern Region (i.e. the Board's North-Eastern Division) consists of two districts, South Yorkshire and West Yorkshire; these together equal the one wages district of Yorkshire, but they cannot be made to agree with the Board's eight areas in that Division without juggling the constituent collieries. In some instances, e.g. the Board's Northern and North-Western Divisions, the areas making up the Divisions can be grouped conveniently to give the corresponding wages district and the Ministry's districts.

A further slightly different geographical grouping is laid down in the Mining Industry Act, 1920. The statistics published before the war in the Annual Report of the Secretary for Mines follow the boundaries of these districts. The statistics now published by the Ministry of Fuel and Power are broken down territorially into the Ministry's Regions and Districts, the former, as mentioned above, corresponding to the Board's Divisions. Certain statistics relating to the Board's areas are given in the Board's Annual Report.

(j) *The Quarterly Statistical Statement*

There has been published for nearly 30 years a paper entitled *Quarterly Statistical Statement of the Costs of Production, Proceeds and Profits of Collieries*. For the years 1940 to 1942 the figures were published on an annual basis only. This was formerly the responsibility of the Ministry of Fuel and Power, but has been continued by the Coal Board. Certain important changes were introduced with the statement for the first quarter of 1947, the first to be issued under nationalization, and particular care has to be taken in comparing these figures before and since nationalization. The disappearance of the Coal Charges Account and the effect of nationalization on royalties, rents and wayleaves formerly payable to the Coal Commission, have disturbed the comparability of the figures relating to costs, proceeds and profits. Furthermore, costs and proceeds when the statement was issued by the Ministry were expressed on the basis of "per ton commercially disposable," whereas the statement as issued by the National Coal Board uses "per ton saleable" as the basic unit. It is therefore recommended that particular attention should be paid to the notes contained in the first few statements as issued by the Board. In its present form, the figures relate to all mines worked by the Board, thus excluding licensed mines. They accordingly cover about 99 per cent. of the deep-mined output compared with about 97 per cent. before 1947. Since no account is taken of the interest and interim income payable by the Board, and since the figures refer to collieries only, the resulting profit or loss given in the statement is no measure of the Board's financial results as a whole.

The figures contained in this publication show for Wages Districts and Divisions the quarterly output of the commercially disposable tonnage, colliery consumption and free and concessionary coal. Costs of production distinguish (*a*) wages, (*b*) roof supports, general stores and repairs, (*c*) coal and power consumed, (*d*) other costs, including depreciation. Total proceeds and the resultant profit or loss complete the financial items. In addition, earnings per manshift worked

and average weekly earnings are given separately for underground, surface and all workers, distinguishing cash earnings and the value of allowances in kind.

(k) *Other Published Statistics*

There are few industries for which statistics are published as copiously or as frequently as for coal mining. Each week the Ministry of Fuel and Power issues a "press release" consisting of the figures of output, employment, productivity, etc., as collected by the Board for the deep mines, distinguishing regions and districts, and by the Ministry for opencast production. This detail, together with additional material collected from annual returns, is published for the year as a whole in the Ministry's Statistical Digest, which shows for the basic statistics the series running back for several years. A note of warning has already been sounded against using these series without careful reference to breaks in continuity.

The Board's Annual Report, usually issued in June or July, contains in the statistical appendix much detail showing for output, employment, productivity, etc., the weekly record throughout the year for each of the Board's Divisions. The Annual Report also contains a very detailed statement of the Board's Accounts; in the Report for 1948 no less than 90 pages were devoted to this record. The Board also publishes each month a statistical brochure entitled "Coal Figures," containing weekly averages of production, manpower and earnings, and monthly figures of exports. Comparative figures for recent periods are also given.

The salient statistics of the industry, necessarily in less geographical detail, are issued in the *C.S.O. Monthly Digest of Statistics*, and in the *Annual Abstract of Statistics*.

Acknowledgment is made to the National Coal Board for their courtesy in giving permission for the publication of this memorandum.

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In addition to the publications mentioned in the course of this note, the following may also be mentioned:

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Statistics for Economists.* By R. G. D. Allen. (Hutchinson's University Library, 13.) London: Hutchinson, 1949. vii + 216 pp. 7¼″. 7*s.* 6*d.*

In *Statistics for Economists* Professor Allen lucidly explains those few and elementary statistical methods, an understanding of which is almost indispensable for students of economics, even when they do not specialize in economic statistics. These methods include the construction of averages and index numbers and measures of dispersion and correlation, and of the study of time series for trend and seasonal and cyclic variation: they also include a knowledge of the kind of statistical information available, and of the shortcomings to be expected in it. Finally, they include a broad understanding of the purposes and problems involved in the collection of information by sample.

The majority of economists are intimidated by algebraic symbols, and Professor Allen has almost abolished algebra from his book by using, in its place, numerical examples and charts wherever possible. In explaining the meaning of a technical term he aims always at suggestive and homely phrases that will stick in the memory, rather than at pedantic accuracy. For this reason the book is not altogether suitable for the critical student with some training in mathematical rigour who is prepared to go deeper into the subject.

For example, a critical student would be puzzled by various points in the last chapter of the book on "Sampling and Significance," where the process of digesting the material for the non-mathematical reader is most formidable. "The further complication now arises that, with the substitution of s for σ in the standard error of the mean, the sampling distribution of the mean is no longer normal. It follows a distribution, known as the t-distribution . . ." (p. 174). This may vaguely suggest to the average student why he has to use tables of the t-distribution in testing the significance of a mean when the standard deviation of the population is unknown: but the sentence quoted is false, the truth being more complicated, namely, that although the sampling distribution of the mean $\bar{x}$ is normal, yet that of the ratio $\frac{\bar{x}-\xi}{s}\sqrt{n}$ follows the t-distribution. The good student will be confused by the text, although the average student may well be less confused by an inadequate text than by the more complicated truth.

On p. 173 we are asked, "Can it be asserted that the average rent paid by all Kensington working-class families occupying 4 rooms is higher than that paid by families with 3 rooms?" and the answer is found by using a two-tailed significance test. The very good student may be puzzled why a one-tailed test is not used.

The various methods given (Chapter VIII) for detecting seasonal variations all involve fitting a trend to the data as a preliminary. This is a tedious process, and the chapter would have benefited by the inclusion of one of the rough methods which avoids the fitting of a trend. The division of the table on seasonal corrections between pages 147 and 148 is little short of disaster.

As an arbitrary general rule for finding the quartiles and median, Professor Allen suggests (p. 81), "For n items of a variable, all specified and arranged in ascending order of magnitude, Q_1 is the $\left(\frac{n+1}{4}\right)$ th, M the $\left(\frac{n+1}{2}\right)$ th, and Q_3 the $\left(\frac{3(n+1)}{4}\right)$ th in the order." Although the point is of no practical importance, I am puzzled why he did not specify the $\left(\frac{n}{4}+\frac{1}{2}\right)$ th, $\left(\frac{n}{2}+\frac{1}{2}\right)$ th and the $\left(\frac{3n}{4}+\frac{1}{2}\right)$ th, which would have agreed exactly with the rule given later (p. 83), in the case of a uniform distribution: this latter specification satisfies the criterion that the interquartile range covers exactly $\frac{n}{2}$ data.

Apart from the comment on the printing on pp. 147–8, none of the points made above imply any serious criticism of the book's excellence for the purpose for which it is intended. It fills the need for an easy up-to-date book on statistical methods, specially adapted for the student of economics, who is not prepared to specialize in economic statistics. D. G. C.

2.—*Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: University of California Press, 1949. London: Cambridge University Press. viii + 501 pp. 10¼″. 42*s*.

This splendid volume is a record of papers read at a Symposium on Statistical Theory, held in the University of California in August, 1945, and January, 1946, and is edited by J. Neyman. It contains 29 papers, on a wide range of subjects, which are impossible to summarize in detail, and it is only possible to pick out the high lights.

Hotelling is represented with a paper on the teaching of statistics (followed by a general discussion), and also by a very valuable survey of various practical methods of matrix calculation. Pólya contributes two characteristically elegant notes, one dealing with inequalities for the normal distribution and the other with some properties of characteristic functions. Neyman has a long paper on the theory of the χ^2 test, and Hsu one on the limiting distributions of functions of sample means in multivariate analysis. Barankin has a paper on the Romanovsky-Bartlett-Scheffé test.

A number of smaller papers deal, in a general way, with biometric problems, but perhaps the most interesting part of the book consists of a series of survey articles on subjects of topical research interest. V. F. Lenzen outlines the method of applying statistical ideas in physics, and E. Brunswik has a long paper on psychological experimentation which is probably of more interest to the psychologist than the statistician. There are two very valuable papers on stochastic processes. The first of these, by Doob, surveys the theory of stationary processes with a continuous time and continuous variate, and their generalized harmonic analysis. This is particularly valuable in that it gives one of the few accounts of the subject which is clear and easy to read. W. Feller surveys the theory of evolutive processes in which the time is continuous, but the variate takes integral values. This subject is interesting both to biologists and physicists, and has been extensively studied in recent years by Arley, D. G. Kendall, Feller himself, and others. Feller's paper contains some new results, and although a good deal of work has been done since it was written, it does not give any impression of being dated. In another paper Wolfowitz gives a general discussion of non-parametric inference which contains some new results.

It is a pity this book was so long in publication, but every mathematical statistician will find it highly stimulating and suggestive. P. A. M.

3.—*Social Surveys*. By D. Caradog Jones. London: Hutchinson's University Library, 1949. 232 pp. 7¼″. 7*s*. 6*d*.

The development of social investigations in this country is noticeable not only for the immense increase in survey activity over the last twenty or so years, but also for the gradual change in the emphasis and character of social surveys, which has taken place during this period. The classical social surveys of Booth, Rowntree and Bowley were largely concerned with the problem of poverty; they consequently concentrated their attention on the working-class populations of their respective towns. To-day, largely as a result of the change in the priority of social problems, most social investigations are of different types. There are, for instance, the many sample surveys conducted by Government departments (pre-eminently the Social Survey division of the Central Office of Information), by semi-public and private organizations and by market and opinion research bodies; the regional social surveys which have become an integral part of town planning and reconstruction; and the numerous investigations, many of them covering the whole country, of special problems such as old age, family expenditure, social mobility, readership, travelling to work, etc., conducted by a large variety of organizations, and differing widely in their scope and their scientific and social value. It may seem inappropriate to group such different types of inquiries together under the single heading "social surveys" and, because of the special association of that term with the classical poverty surveys, "social investigations" may, perhaps, be preferable as an all-inclusive generic term.

Mr. Caradog Jones, who was himself Director of the Merseyside Survey, in fact devotes most of his book to social surveys in the more narrow, traditional sense. This makes for more homogeneous material, but inevitably leaves the picture slightly incomplete. His decision to exclude regional surveys—such as the Middlesbrough inquiry and many others—from the discussion seems particularly regrettable.

Mr. Jones traces the development of social surveys from the Domesday Book to the present-day Government Social Survey, with chapters on all the main poverty surveys: Booth's pioneering

study, Rowntree's York surveys, Bowley's Five Town Surveys, the New London survey and the Merseyside survey. There are also chapters on the Census, John Hilton's sampling experiments at the Ministry of Labour, Boyd-Orr's well-known nutrition study, the 1937–8 Family Expenditure inquiry, the Social Survey and a short chapter on methods.

The declared purpose of the book is "to serve as a simple textbook for students with no special knowledge of the subject" and, as such, it makes a welcome addition to the literature. Any more advanced student will, of course, want to study the readily available reports of the original surveys, and the extracts from and comments on these inquiries given in Mr. Jones's book should stimulate such interest.

As is seen from the list of chapters, the book is essentially historical in its treatment, and is therefore not so useful for those primarily interested in survey methodology. From the point of view of the student of statistical techniques in social investigations, it would have been useful if more space had been devoted to subjects such as sampling, questionnaire design and other aspects of methodological interest. C. A. M.

4.—*American Transportation in Prosperity and Depression.* By Thor Hultgren. (National Bureau of Economic Research, Studies in Business Cycles, No. 3.) New York, 1948. xxxiv + 397 pp. 9″.

If we came across this book without any marks of identification beyond its title and the author's name, we should recognize without difficulty that it was issued under the auspices of the National Bureau of Economic Research, and was intimately connected with the study of American business cycles so closely associated with the late Wesley Mitchell. In fact, it is the first of a group of monographs devoted to single industries as planned in the second work of this series—"Measuring Business Cycles" (1946), a joint production of the late Wesley C. Mitchell and Arthur F. Burns. The author, Mr. Thor Hultgren, has contributed three other papers on Transportation and the Business Cycle, being numbers 5, 13 and 15 of the National Bureau's Occasional Papers; much of the first two are incorporated into the full study now under review. It is to be noted from the dust cover that Mr. Hultgren is preparing an Occasional Paper on British Transport. This will be awaited with great interest in view of the very real differences, both institutional and operational, which exist between the transport systems of the two countries.

As is common with these studies of the American economy, the supply of data was comparatively good. In the words of Dr. Burns (Introduction, p. xiii), "The statistical records of *railroads* are unique in their excellence, abundance and time span" (reviewers' italics).

It is not practicable here to convey a large measure of detail contained in almost 400 pages, 151 tables and 133 charts of this book. Chapters 1 and 2 deal with the movement of goods and people, while Chapters 4 to 6 are concerned with the equipment necessary for the movements. Sandwiched between these is a short chapter on Composite Measurements of Traffic which it is felt would be better incorporated in Chapter 7 (Workers and their Performance), where such results have their first main use. The following three chapters deal with Fuel Economy, Prices and Wages, Cost and Profit, while Chapter 11 has to suffice for all the other forms of transport. In the final chapter the author surveys the future of Business Cycles and Transportation. The very detailed table of contents, together with a serviceable index and a note on the statistical sources, render the book easy of reference upon specific points.

Most of the data in this book come from the publications of the Interstate Commerce Commission. A further useful source is that of the Bureau of Railway Economics—a division of the Association of American Railroads. The author has been mainly concerned with steam railways, and outside this section of the industry it would appear that the available data are relatively sparse. The author states, "What we have to report about the other means of movement can be told in one chapter," which consists of twenty-two pages, albeit having already used 340 pages for the steam railways. This particular chapter (No. 11) is perhaps the most disappointing in the book. The lack of accurate information about the work done by road transport leaves a big gap in the economic statistics of most countries of the world. In this connection it is perhaps worth while noting an interesting and praiseworthy attempt by the Dutch to solve the difficulties of getting information partly by making estimates on the basis of samples, an example that might well be imitated in other countries. The only statistics available concerning freight traffic by road in the U.S.A. seem to suffer from serious defects. For example, only interstate carriers are included in one set, while another covers only traffic on rural (i.e. non-urban) roads. Mr. Hultgren points to neither of these, and it is no doubt very difficult to obtain estimates of road freight traffic in the United States which are both authoritative and complete.

The figures given in this book to illustrate the relative importance of the different forms of inland transport are not above suspicion. Table 138, on page 351, gives the number of lorries

registered (presumably "with current licences") in 1930 as being 3,486,000, but in a "note on the magnitude of the transport industry," Table 145, "estimated manpower available for transportation and other industries," gives the number of persons in "trucking and taxicab service" in that year as 434,786. This is odd. If the employment figure only relates to those persons operating vehicles for hire or reward, as seems possible, it gives no idea of the importance of road transport as a whole.

Attention should be drawn once more to the diametrical difference between the empirical analysis of business fluctuations, as adopted in this book (which also stands as a monument to the labours of the late Dr. Mitchell) and the work of the econometric school, e.g. Frisch, Haavelmo, Stone, Tinbergen and Tintner. This latter approach makes extensive use of modern statistical techniques, and it would appear that the only aspect in common is the generally unsatisfactory nature of annual data which affects the analysis of both. This is because the cyclical peaks and troughs occur at odd fractions of a year, and hence the analysis ought to be made with data recorded at intervals shorter than a year. The technique of the "reference cycle" inevitably introduces some element of subjective judgment in the counting of peaks and troughs. A further element of indecision is introduced by the practice of averaging the three observations at the peaks and troughs (footnote 20, page 31). Most of the cycle measures would be modified by the displacement of the turning-points involved. In particular the timing measures which are developed from these points are critically dependent upon the particular dating scheme used.

The technique of comparing fluctuations of one industry with generalized "reference cycles" instead of with specific series for other industries is open to criticism. From the point of view of employment and the national income the effect of the expenditure stream represented by railway investment depends partly on whether it led or lagged. It would be interesting to know, for instance, whether technological progress in the first part of the period covered by this book rendered investment in railways more buoyant than that in other industries of similar quantitative importance, and whether it was relatively sluggish in the later decades. Chapter 6 does not throw a great deal of light on these points. The author does suggest that orders placed for new equipment tended to fall off towards the close of expansions, but this is not the same thing as expenditure on construction. The causal significance of investment streams at turning-points depends partly on how early or late they turn compared with others, and the reference cycle technique tends to obscure the exact order of sequence.

In the chapter on "Costs and Profits" particularly interesting use is made of the concept of "traffic units" to show that costs and profits per traffic unit may exhibit a distinct pattern. This is because of the interaction between the short-run fixity (in physical terms) of certain inputs, changes in the nature of traffic in booms and slumps, changes in the prices of the factors of production which the railways buy, and the "stickiness" of the prices at which they sell. There is a sustained and interesting attempt to measure the relative importance of these influences, particularly at different stages of the cycle. The main conclusions arrived at may not be striking—that profits are higher at the end of an upswing than at the beginning, and lower at the end of a downswing than at the start—but it is important that the elements of the problem should have been thus sorted out. It may be questioned, however, whether the attempt to relate cyclical turning-points to profit per traffic unit is sound; investors and managers probably pay far more attention to profit per money unit of capital, i.e. total profit, and this will not necessarily behave in the same way. It is worth taking on extra business, even if it involves a fall in the average profit per unit, provided that the total number of units of traffic increases in a sufficient proportion.

Apart from some slight awkwardness of terminology (e.g. the use of the word "disappearance" on pages 348–9 instead of the more usual word "consumption"), there is one large problem upon which information may be sought in vain. It is the impact upon the operation and economics of steam railways of schemes for full or partial use of electric or diesel-electric traction. It is difficult to accept a position that such schemes in America had no effect upon the transportation business cycle, especially in view of the enormous capital sums involved and the sizeable quantities of special equipment required. The tabulations of the Bureau of Railway Economics mentioned above show the following figures for locomotives in service at the end of the year:

	1929	1939	1947
Steam	56,936	41,117	35,108
Electric or Diesel-Electric	623	1,353	6,593
Other	12	41	18
	57,571	42,511	41,719

Surely an increase from 1·1 per cent. in 1929 to 15·8 per cent. in 1947 for the electric or diesel-electric locomotives cannot have exerted no worthwhile or recordable reaction?

In conclusion, a British reviewer finds it difficult to avoid the feeling that works of this kind are slightly out-dated: the traditional trade cycle, characterized by a deficiency of effective demand, is not the most imminent of threats to the British economy. However appropriate for the United States experience the last chapter of Mr. Hultgren's book may be, the patterns of economic fluctuation are not universally uniform. At the end one is inclined to feel that the method of presentation goes little further than setting-up the task, and that the real analysis has yet to be made.

W. R. B. and K. F. G

5.—*Expectation in Economics.* By G. L. S. Shackle. Cambridge University Press, 1949. x + 146 pp. 8½″. 10*s.* 6*d.*

The title of this difficult but fascinating book is a little misleading, suggesting as it will to many readers an analysis of the part played by expectation in the dynamic movements of the economic system as a whole. Mr. Shackle's field is in fact much more restricted. He is largely concerned with the individual. He takes an "economic man" faced with the problem of deciding whether or not to embark on an "enterprise," and analyses this individual's mind in great detail, with particular reference to the part played by the element of uncertainty attaching to his expectations as to the outcome of the enterprise. The analysis results in a number of highly original conceptions, some of which may well become standard tools of economic thought when the academic world is familiar with them.

The author has devised an ingenious method whereby his potential enterpriser's mental reactions to a possible project may be represented in the form of a continuous three-dimensional mathematical function. This function can be displayed graphically, since the shape which it will take in normal circumstances can be discovered by an interesting process of introspection and intuition. The resultant ease with which the essentials of a complicated mental state can be assimilated is of great assistance to further analysis.

Having developed his conceptual technique, Mr. Shackle applies it to the formulation and proof of his basic theorem (see below), and then proceeds to erect the first storeys of what may one day become an edifice. He applies his conceptions to the analysis of a number of economic problems, such as the effect on the pace of investment of "expected clarification of expectations," and the choice of assets to be held for speculative gain. The analysis is always interesting in its very novelty, though one feels that at times the real world is very far away. It is too early, at this stage in the development of the author's conceptions and technique, to evaluate the ultimate contribution which this book may make to economic theory. For practical purposes the whole thing may be much ado about nothing, or an important missing link in the theory of the trade cycle. This, however, one can say with confidence: whatever its practical implications may or may not be, this book has sufficient justification in the pleasure it will give to many as an elegant example of the immaterial beauties of logical deduction. The pleasure given will be none the less if the reader finds himself disagreeing violently with some of the author's conclusions.

Controversy there will certainly be, and it is likely to centre on the author's conception of "focus outcomes." Much of the book is based on the thesis that for all practical purposes an enterpriser's view of any given project's attractions is compounded of two elements only, a "focus loss" and a "focus gain." Out of all the possible gains in which the project might result, one alone will occupy the enterpriser's effective attention to the exclusion (after due processes of thought) of all others, and similarly with the losses. If the argument used gains acceptance, it is hard to see in it anything but a death-blow to the use of the integration based on a sham frequency distribution as a means of analysing the effect of uncertainty on economic judgments. Mr. Shackle denies the process of mental integration absolutely, it being a corollary of his basic thesis that the effects of various hypothetical losses and gains which could result from an enterprise, each qualified by a greater or a lesser belief in the possibility of its occurring, are *not additive.* The enterpriser's attention being focused exclusively on two elements in the whole range of possible outcomes, changes in his view as to the possibility of other outcomes occurring do not alter the attractiveness of the enterprise as a whole, unless they are of such magnitude that one of them is thereby raised to such importance in the enterpriser's mind that it displaces one of the existing focus outcomes entirely.

Now, I do not believe that this is realistic. I believe that the human mind has its own peculiar way of performing what amounts to an approximate *integration* of the various possible outcomes of an enterprise, each modified by the enterpriser's view of its likelihood. I prefer Mr. Shackle's analysis of the mental ingredients of this integration to that displayed by the sham frequency-distribution, but I still believe that the final result is an integration, and not what amounts to a

mental obsession with the two maxima amongst the ingredients. Mr. Shackle's model is, in many cases, a fair approximation to the enterpriser's state of mind, but he goes further than just saying this, for he denies that there is any state of mental integration at all by which the outside chance is given *some* weight in the prospective enterpriser's calculations. In order to show that this is not realistic it would seem that one has only to take two competing projects, whose prospects appear exactly similar to the potential enterpriser over the whole range of possible profits and losses, and consider the effect of adding to or subtracting from some of the elements which are dismissed as ineffective. It appears indisputable that the curtailment or extension of some of the outside chances of heavy loss or substantial profit in one of these projects, the other remaining unaltered, will result in that project now presenting a clear advantage or disadvantage relative to the other. A similar process of reasoning can be applied to the effect of slightly altering the potential surprise (or "chance of occurrence," in more familiar language) attaching to gain and loss elements within the two "focus outcomes," with similar results. In a sense, therefore, there appears to be no doubt as to the additive nature of the elements portraying the project's uncertain outcome.

Can one nevertheless accept the author's analysis, subject to the qualification that considerations, additional to his "focus outcomes," may be relevant where the necessity arises of choosing between two projects which are of exactly equal attraction when considered individually in isolation? It seems that any such qualification would be the thin end of a dangerous wedge, for one would then be led to ask how the individual would react to the problem of choosing between two projects of *nearly equal* attraction according to his "focus outcome" model. Readers may be left to try and hammer in the wedge for themselves from this starting point. In the real world, this case of competing projects with only marginal differences is of very frequent occurrence.

It is only fair, having indicated dissatisfaction with the author's conclusions, to attempt to indicate the point in his argument at which error could have crept in. This, I believe, occurs when he considers the "inner range" of possible outcomes to an enterprise, being those outcomes the occurrence of any one of which would not surprise the enterpriser in the least. Mr. Shackle logically concludes that "when he contemplates this inner sub-set of outcomes, each of which carries no potential surprise, the individual will concentrate his attention on the *best* and *worst* hypotheses in this range." For example, if a man considering a project confidently expects that the result will lie somewhere between a gain of 10 per cent. and a loss of 2 per cent., and attaches varying degrees of disbelief to the possibility of gains exceeding 10 per cent. and losses exceeding 2 per cent., it is true that these two figures will monopolize his attention. In my view, however, it is not true that if he is only considering this inner range he will *evaluate* the project mentally as one balancing a possible gain of 10 per cent. against a possible loss of 2 per cent.: he will "integrate" intuitively and on no mathematically determinate pattern, and evaluate the project as one balancing a "safe estimated profitable outcome" of (say) 5 per cent. against a "possible loss of *2 per cent.*" If he is a cautious Scot, or a committee, his "integration" may be 4 per cent., or even 3 per cent., for the profit element, but still 2 per cent. for the loss element. In other words, while he may react like Mr. Shackle's economic man with regard to losses within the inner range, I do not believe he does so with regard to profits. This is perhaps illogical, but I believe it is how the human mind works in the economic sphere. Your big industrialist and investor is sometimes a gambler, but usually he is a committee. He works, not on a system of belief, but of deliberate pessimism, rejecting the highest values in his "inner range." But while the apparent outcome of this pessimism must be satisfactory, he is certainly sensitive to the *whole range* of possible outcomes more favourable than this. Whether this effect is marginal or important is largely a matter of temperament and circumstance.

If Mr. Shackle's psychological analysis does not meet with whole-hearted acceptance, it does not follow that either his general method or the majority of his conclusions are invalidated. His method is applicable, and his conclusions will stand, in many cases where an enterpriser's view of a project can be summarized in the form a higher gain element counterbalanced by a lower gain element or a loss element. Whether these elements are reached by a process of mental integration or of selection of appropriate maxima does not always matter. Thus, the author's suggestion as to the method of taxation best calculated to preserve the incentive to enterprise is still perfectly sound, so far as it goes. The study of its further implications should provide an interesting exercise in academic circles, even though the chances of its achieving the status of an extra worry for the Chancellor of the Exchequer are remote. J. R. C.

STATISTICAL NOTES

(1) British Official Statistics

A useful change in the presentation of official agricultural statistics has been the issue by the Agricultural Departments of combined figures for the United Kingdom, bringing together information collected separately for England and Wales, Scotland, and Northern Ireland. The first volume covered acreage and production of crops and number of live stock during the war years, and the second volume, now available (*Agricultural Statistics, United Kingdom*, Part II, Stationery Office, price 1*s*.), deals with the *output and utilization of farm produce* in 1939–40 to 1945–46. Information of this type was available for Great Britain in pre-war times, but the activities of the Ministry of Food and other forms of control have made it possible to give more comprehensive and more reliable particulars than was formerly possible, especially as regards the production of meat, milk and milk products, eggs, poultry, wool, etc.

The main object of the Report is the calculation of the gross and net agricultural output of the United Kingdom. The gross output is briefly that part of the total agricultural production which is sold in any one year for consumption by the non-farming community. Part, however, of this gross output is the finished product resulting from the use of imported feeding stuffs, seeds and live animals by British farmers, so that to ascertain the true or net output of the land of the country the cost of these overseas materials must first be deducted. These figures provide a means of measuring the changes which have been taking place in recent years in British agriculture, and also show how far home farming is dependent on these imported goods. The net output figure is of special interest because no similar estimate has hitherto been available.

The following table gives a few leading figures for two war years in comparison with a pre-war average (1936–7 to 1938–9):—

Estimated Value of Agricultural Output (U.K.) at 1945–46 *Prices* (£ *million*).

	Pre-war average	1943–44	1945–46
Gross output	591·7	622·3	615·1
Less imported feeding stuffs	113·2	16·5	30·1
Less other requisites and charges	44 0	49·4	48·2
Net output	434·5	556·4	536·8
Index	100	128·1	123·5

It will be seen that valued at constant prices the gross output rose from a pre-war average of £591·7 million to £622·3 million in the peak year 1943-4, an increase of just over 5 per cent., while the net output rose from £434·5 million to £556·4 million, or by 28 per cent. In pre-war years about one-fifth of the gross was atrributable to imported feeding stuffs, but these were heavily reduced during the war, and in 1943–4 they only amounted to about 2½ per cent. Their replacement by home-grown feeding stuffs (including grass) constituted one of the main war-time accomplishments of farmers. It naturally resulted in an appreciable change in the structure of the industry: live stock and live stock products which accounted pre-war for 71 per cent. gross output represented only 49 per cent. in 1943–4; on the other hand, the share of farm crops sold off the farm for human consumption rose from 15 to 29 per cent.

Fertilizers are not included among the imported materials used for agriculture, though a table is given showing that in the year 1945–46 the estimated expenditure by farmers on artificial fertilizers of all kinds was £28,321,000 as against £8,443,000 in 1938–39. These figures include both imported and home-produced materials (such as sulphate of ammonia), and also presumably manufacturing and other costs. The Report makes no attempt to measure the contribution which home production made to the country's food consumption.

The Ministry of Labour and National Service has prepared a summary of the results of the inquiry into the weekly expenditure of working-class households in 1937–38. The document has not been published, but copies have been supplied to University and other research libraries. The supply is strictly limited, but the Statistics Department of the Ministry is prepared to consider requests for copies from research workers who have a serious need for the information.

The summary contains the general results of the inquiry, which were published in the *Ministry of Labour Gazette* at the end of 1940 and early in 1941. It also contains analyses by Regions and expenditure groups, which have not hitherto been available. Copies of the inquiry form and of the instructions to investigators are appended to the report.

(2) OTHER STATISTICS

I.L.O. Year Book of Labour Statistics.—The tenth issue of the Year Book of Labour Statistics, published by the International Labour Office, presents a summary of many of the statistics relating to labour in some sixty countries. It provides figures for the years 1947 and 1948, in continuation, in most cases, of a time series extending, where possible, back to 1929. The publication serves two purposes. It provides a comprehensive survey of the information available in the various countries. It is also designed to stimulate the production of more adequate statistics which are internationally comparable, by indicating the wide divergences which exist in the available statistics as well as the gaps which exist in the different countries. Most of the figures have been drawn from official publications, but were submitted before publication to the statistical authorities concerned. The text and headings to the tables are given in English, French and Spanish.

The Year Book is divided into chapters dealing with the various subjects. These include total and economically active population, employment and unemployment, hours, wages and labour income, cost of living and retail prices, family living studies, social security, industrial injuries, industrial disputes and migration. Appendices relate to production, wholesale prices and exchange rates. The tables in each chapter are preceded by text describing the nature of the figures and defining their scope and, what is most useful, the limitations and qualifications to be attached to the figures in order to avoid pitfalls in drawing inferences. In the chapter relating to employment, for example, the sources of information used in the different countries are described, e.g. social insurance statistics, labour force sample surveys, trade union fund statistics, employment exchange statistics and "estimates." Attention is drawn to the non-comparability which may arise from the fact that the basis of the information varies in different countries.

In the tables on wages care is taken to indicate which figures relate to rates of wages and which to earnings. Attention is directed to the different sources used as the basis for estimates of total wages and salaries.

In the chapter relating to cost of living and retail prices, index numbers in the time series are given together with average retail prices of food and coal in 1946 and 1947, and indices of international comparisons of food costs. The latter figures are calculated by the I.L.O. on the basis of information supplied specifically for this purpose. The Office does not attempt a calculation of relative purchasing power by relating these indices to the level of wage rates, but the figures given in the chapter on wages enable interesting calculations to be made on a restricted basis.

The figures relating to costs of family living in the different countries reveal the deficiency of the information on this subject. Although as full information as possible is given, it is shown that the differences in the composition of the figures are so numerous that little comparison is possible. The statistics relating to industrial injuries and migration also show that much has still to be done in these fields to achieve international comparisons.

The Year Book for the first time includes statistics relating to costs of social security, obtained largely by means of questionnaires addressed to the countries. This is the first attempt that has been made to assemble statistics on this subject, and the results illustrate the difficulty of presenting comparable figures based on widely diverse systems of administration and finance.

OBITUARY

CONRAD ALEXANDER VERRIJN STUART

Professor C. A. Verrijn Stuart, who died on October 26th, 1948, was a statistician of international repute for over half a century and an Honorary Fellow of the Society since 1904. He was born in December, 1865, and after graduating in Law and Political Science at Amsterdam, was appointed Director of the Central Statistical Bureau of the Netherlands at the early age of 33. He held this office until 1906, when he became Professor of Political Economy at the University of Groningen. He was appointed to the corresponding chair in the University of Utrecht in 1917, and occupied that post for some seventeen years until he retired in 1934.

He was an active member of the International Statistical Institute for several years before he became Secretary-General in 1907, and he achieved the record of attending every one of its biennial sessions between 1899 and 1938, in most of them playing a very prominent part. When Methorst succeeded him as Secretary-General in 1911, Verrijn Stuart continued for almost a further thirty years to take a lively interest in the Institute's affairs. Throughout his working life he served his Government in an advisory capacity, and exercised a major influence on the development of statistics in the Netherlands.

It will be recalled that Verrijn Stuart opened the discussion on the paper read by Sir Josiah Stamp at the Centenary Meeting of the Society in 1934. In making this contribution Verrijn Stuart was the only one of the distinguished foreign statisticians who spoke in English, and those who were present will not forget his complete command of the English language.

He was a prolific writer, largely but not exclusively on demography. His principal publication was, perhaps, his *Introduction to Statistics*, of which Volume I appeared in 1910, Volume II in 1913, and Volume III (in two parts) in 1915 and 1917. He wrote many papers on a very wide range of subjects with, as suggested above, some emphasis on the demography of the Netherlands. He was also much interested in monetary and fiscal questions.

He was, of course, well known to those Fellows of the Society who are long-standing members of the International Statistical Institute, and by repute to a much wider circle. His very attractive appearance, perfectly matched by a charming personality, gave him an air of commanding distinction which made a deep impression on all those who came in contact with him.

The Society will recognize in his death the loss of one who exercised in the past generation a very considerable influence in international statistics. R. F. G.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Advancement of Science—

July 1949—The economics of strategic dispersal: *Brinley Thomas.* Mobility in the location of industry in Great Britain: *W. Smith.* The administration of large scale enterprises: *J. C. Picton.*

British Journal of Psychology (Statistical Section)—

July 1949—Item difficulty as the measuring device in objective mental tests: *A. E. Peel.* Factor analysis of assessments for army recruits: *C. Banks.* Factor analysis by Lawley's method of maximum likelihood: *W. G. Emmett.* Alternative methods of factor analysis and their relations to Pearson's method of "Principal axes": *C. Burt.*

British Journal of Social Medicine—

April 1949—Milk-borne infections in Great Britain: *Sir William Savage.* A study on birth-weight and intelligence: *C. Asher* and *J. A. Fraser Roberts.* Control charts for the standard mortality ratio: *G. Herdan.* Social medicine: *R. C. Browne, I. F. Beck* and *R. I. McCallum.* Mortality amongst babies from injury at birth: *H. T. Russell* and *I. Sutherland.*

Economica—

August 1949—The economic theory of index numbers: *R. G. D. Allen.* Rehabilitation of time dimension of investment in macrodynamic analysis: *S. C. Tsiang.* Period analysis and inflation: *R. Turvey.* The national insurance funds: *A. T. Peacock.* Evolution of the exchange control: *A. H. Smith.* Independent applications of national accounts: *J. Sandee.* The economy of Europe: *D. J. Morgan.*

Economic Journal—

September 1949—The British Commonwealth and European economic co-operation: *D. J. Morgan.* Changes in level and diversity of employment in regions of Great Britain 1939–47: *C. E. V. Leser.* Accountants, economists and the valuation of fixed assets: *T. H. Silcock.* Involuntary unemployment and the Keynesian supply function: *D. Patinkin.* Equal pay for equal work: *E. H. Phelps Brown.*

Heredity—

August 1949—The manifold effect of selection (Part II): *K. Mather* and *B. J. Harrison.* The inheritance of the MNS blood groups: a second series of families: *R. R. Race, R. Sanger, S. D. Lawler* and *D. Bertinshaw.* The estimation of linkage with different viability: I, Note on the test of significance for differential viability in frequency data from a complete three-point test: *R. A. Fisher.* II, The use of the product formula for the estimation of linkage in intercrosses when differential viability is present: *N. T. J. Bailey.* III, A method of allowing for differential viability in estimating linkage from backcross matings in coupling only or repulsion only: *N. T. J. Bailey.* A preliminary linkage test with *agouti* and *undulated* mice: *R. A. Fisher.* A dominant mutant mosaic house mouse: *N. R. Bhat.*

Oxford University Institute of Statistics, Bulletin—

July and August 1949—A national income matrix: *D. Seers* and *P. F. D. Wallis.* The movement of labour in 1948 (Part II): *C. A. R. Crossland.* Those "frightfully high" profits: *T. Barna.* Should sterling be devalued?: *T. Balogh.*

September 1949—Income distribution in 1938 and 1947: *D. Seers.* The regional distribution of juvenile labour: *R. Godson.* Some international aspects of strikes: *K. Forchheimer.*

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June 1949—The demographic history of the Northern European countries in the eighteenth century: *H. Gille.* Population statistics and population registration in Norway. Part 3. Population censuses: *K. Ofstad.* Mortality in New Zealand and England and Wales: *E. Lesof.* The population of Libya: *Chia-Lin Pan.*

AUSTRALIA—

Economic Record—

June 1949—The consumer in a planned economy: *F. R. E. Mauldon.* The composition of personal income: *H. P. Brown.* Australian rural population changes: *J. Gentilli.* Robot economics: *K. Singer.* New Zealand official estimates of national income: *W. Rosenberg.*

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Bulletin de l'Institut de Recherches Économiques et Sociales—

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CZECHOSLOVAKIA—

Statistický Obzor—

Vol. XXIX, Part 2—Třicet let československé statisticky: *F. Fajfr.* Diskriminační analysa a hodnocení zkoušek schopností: *O. Fischer.* Vliv některych faktorů na denní rytmus klinickécho začátku porodu: *J. Malek.* (Text in Czech—English summaries.)

FRANCE—

Revue d'Économie Politique—

May-June 1949—Utilité et bien-être: *A. Landry.* Progrès technique agricole et dimensions de l'exploitation: *P. Fromont.* Quelques aspects des politiques économiques des Etats andins: *L. Baudin.* Le plan Marshall et le plan Truman peuvent-ils éviter une "crise de surproduction"?: *R. Florin.* Épargne réelle et épargne monétaire: *G. Fain.* Les travaux publics et le chômage conjectural: *M. Bouniatian.*

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July-September 1949—Migration et balance des comptes: *J. Bourgeois-Pichat.* Structure de la population et besoins de logements: *L. Henry.* Le "faux problème" de la population mondiale: *A. Sauvy.* Attitudes devant la vie et devant la mort du 17e au 19e siècle: *P. Aries.* Le conflit entre les mesures masculine et féminine de la reproduction: *P. H. Karmel.* Exemple d'évolution d'une société rurale en "milieu répulsif": Asnières-sous-Bois (Yonne): *J. Tricart.* Quelques précisions sur le problème sucre-alcool: *L. Tabah.*

GERMANY—

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Vol. 62, Part 2—Über die Neuberechnung von Indexziffern der Lebenshaltungskosten in Deutschland: *O. Anderson.* An evaluation of post-war monetary reforms: *J. Pedersen.* Finanzpolitik und Zins: *K. Philip.* Zum Problem des abnehmenden Kapitalertragszuwachses: *H. Meinhold.* Die Einkommenselastizität des Mineraldüngerverbrauchs: *H-H. Herlemann.*

HOLLAND—

Statistica—

Vol. 3, No. 3—Over de spreiding bij het droge-stofonderzoek van brood: *L. van der Burg.* Een verband tussen zonnervlekken en extreme winters: *S. W. Visser.* Het verband tussen werkelijke verdeling, foutenverdeling en waarneembare verdeling: *H. C. Picard* and *Gent.* De evolutie van het gebit de orang oetan vergeleken met de evolutie van het gebit van de mens: *D. A. Hooijer.*

SWITZERLAND—

Schweizerische Zeitschrift für Volkswirtschaft und Statistik—

August-October 1949—Loi probabilitaire complètement formulée et familles d'intervalles de confiance: *L. Féraud.* Bildungswert der Statistik: *H. Schorer.* Die verheiratete berufstätige Frau in der Schweiz: *A. Koller.* Sterile und kinderarme Ehen in der Schweiz: *C. Brüschweiler.* Des pertes infligées à l'âge de la procréation par certaines causes de décès en Suisse: *L. Hersch.* Überprüfung der Sterbeziffern: *O. H. Jenny.* Internationale Vergleichbarkeit der Todesursachenstatistik: *W. Ott.* Early Swiss mortality tables: *W. Bickel.* Grundlagen der Schweizerischen Volksterbetafeln 1931/41 und 1939/44: *W. Wegmüller.* Nouvelles tables suisses de mortalité: *E. Marchand.* The calculation of reserves by means of samples: *E. Zwinggi.* Emploi d'une méthode statistique pour étudier

les dépenses d'une famille: *A. Weber* and *A. Linder*. Zur Frage des regionalen Vergleichs der Konzentration der Einkommen natürlicher Personen: *W. Kull*. Möglichkeiten und Grenzen internationaler Zahlenvergleiche in der Industrie- und Gewerbestatistik: *A. Schwarz*. Verbrauchsausgaben und Volkseinkommen: *U. Zwingli*. Die Veränderung de Produktivität der schweizerischen Landwirtschaft im Lichte der amtlichen Statistik: *W. Bäggli*. Wohnungszählungen in der Schweiz: *F. Bachmann*.

INTERNATIONAL—

International Labour Review—

August 1949—Post-war trends in social security: Medical care, I.
September 1949—Post-war trends in social security: Medical care, II.

Revue de l'Institut International de Statistique—

Vol. 17, No. 1/2—Furtherance of statistical education: Report to the XXVIth Session of the International Statistical Institute. (Whole number.)

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part I, 1949, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom

Agriculture and Fisheries, Ministry of. Factors in the marketing of home-produced apples in England and Wales. (Economic Series 50.) London, H.M.S.O., 1949. iv, 68 pp. 9½″. 1*s.* 3*d.*

Air Ministry, Meteorological Office

Meteorology of airfields, by C. S. Durst. 1949. [2], 87 pp. 9½″. 2*s.*

Some correlation coefficients between certain upper air data, by Sir Gilbert Walker. (Professional Notes, 97.) London, H.M.S.O., 1949. 7 pp. 9½″. 2*d.*

Colonial Office

Colonial annual reports, 1947: St. Lucia. 1949. 52 pp., map. 8½″. 2*s.*

Colonial annual reports, 1948: British Solomon Islands. 36 pp., map. 8½″. 2*s.* Brunei. 54 pp. 9½″. 4*s.* The Gambia. 48 pp., map. 8½″. 2*s.* New Hebrides (Anglo-French Condominium). 35 pp., map. 8½″. 1*s.* 6*d.* Seychelles. 28 pp., map. 8½″. 2*s.* Somaliland Protectorate. 40 pp., map. 8½″. 2*s.* Tonga. 24 pp., map. 8½″. 1*s.* 3*d.* London, H.M.S.O., 1949.

The Colonial territories (1948–1949). London, H.M.S.O., 1949. Cmd. 7715. x, 136 pp. 9½″. 2*s.* 6*d.*

Report by His Majesty's Government...to the General Assembly of the United Nations on the administration of Tanganyika, 1948. vi, 322 pp., map. 9½″. 7*s.* 6*d.*...Togoland...1948. vii, 240 pp., map. 9½″. 6*s.* 6*d.* London, H.M.S.O., 1949.

Report on tobacco, with particular reference to the prospects of increased production in Central and East Africa, by S. S. Murray. (Colonial Research Publications, 4.) London, H.M.S.O., 1949. 99 pp. 9½″. 2*s.*

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Fuel and Power, Ministry of. Report of the Mineral Development Committee. London, H.M.S.O., 1949. Cmd. 7732. iv, 106 pp. 9½″. 2*s.*

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Royal Commission on the Press, 1947–49. Report...London, H.M.S.O., 1949. Cmd. 7700. v, 363 pp. 9½″. 6*s.*

Scientific and Industrial Research, Department of. The investigation of atmospheric pollution: a report on observations in 5 years ended March 31st, 1944...London, H.M.S.O., 1949. vi, 125 pp. 9½″. 2*s.* 6*d.*

Scotland, Department of Agriculture for. Agriculture in Scotland: the report of the Department of Agriculture for Scotland for the ten years 1939–1948. Edinburgh, H.M.S.O., 1949. Cmd. 7717. viii, 120 pp. 9½″. 2*s.* 6*d.*

Scottish Home Department. Report on the fisheries of Scotland, 1939–1948. Edinburgh, H.M.S.O., 1949. Cmd. 7726. 113 pp. 9½″. 2*s.*

Supply, Ministry of. Directorate of Royal Ordnance Factories—Explosives. Industrial experimentation, fourth edition, by K. A. Brownlee. London, H.M.S.O., 1949. 194 pp. 9½″. 3*s.* 6*d.*

Trade, Board of

Report of the Committee on Resale Price Maintenance. London, H.M.S.O., 1949. Cmd. 7696. vi, 122 pp. 9½″. 2*s.* 6*d.*

Overseas Economic Surveys: Finland, Nov., 1948. vi, 100 pp. 1949. 2*s.* Peru, Jan., 1949. vi, 85 pp. 1949. 1*s.* 6*d.* Portugal, Nov., 1948. v, 135 pp. 1949. 2*s.* 6*d.* London, H.M.S.O., 1949. 9½″.

Treasury. Report of the Departmental Committee on Taxation and Overseas Minerals. London, H.M.S.O., 1949. Cmd. 7728. 28 pp. 9½″. 6*d*.

(b) Other National and International Publications

Amsterdam

Bureau van Statistiek der Gemeente. Amsterdam gedurende den tweeden wereldoorlog: statistisch jaarboek der gemeente Amsterdam...30ste jaargang, 1940–1944. Amsterdam, 1949. ix, 320 pp. 10″.

Austria

K. K. Handelsministerium

Übersicht der Waaren-Ein und Ausfuhr des allgemeinen österreichischen Zollverbandes... 1858, 1859, 1860. Vienna, 1859–61, 3 vols. in 1. 9¾″.

Mittheilungen aus dem Gebiete der Statistik: Herausgegeben von der Direction der Administrativen Statistik im K. K. Handelsministerium. 3, 1854. 4, 1855. 5, 1856. 6, 1857. 7, 1858. 8, 1860. Vienna, 6 vols. 9¾″. (Presented by the Society of Antiquaries.)

Colombia

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Census of Cyprus, 1946. Tables of occupation (main occupation). 1948. [37 fols.] 8½″ × 13″. 3*s*. Tables of literacy, educational standard and knowledge of English, with report. 1948. [18 fols.] 8½″ × 13″. 2*s*. Report on housing in the six district towns...[13 fols.] 13¼″. 1*s*. Nicosia, 1948.

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Methods of family living studies: report prepared for the Seventh International Conference of Labour Statisticians. (Studies and Reports, N.S. 17.) 1949. [4], 63 pp. 9½″. 2*s*.

Wages and payroll statistics: report prepared for the Seventh International Conference of Labour Statisticians. (Studies and Reports, N.S. 16.) 1949. [4], 180 pp. 9½″. 5*s*.

Year book of labour statistics, 1947–48, tenth issue...1949. [2], xv, 303 pp. 12″. 15*s*. Geneva, 1949. (London, Staples Press.)

International Monetary Fund

Balance of payments yearbook, 1938, 1946, 1947. Washington, 1949. vii, 383 pp. 11″.

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Wartime prices, part 1: August, 1939 to Pearl Harbor, by John M. Blair and Melvill J. Ulmer...(Bulletin No. 749). Washington, 1944. vi, 272 pp. 9″. 35*c*.

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VARIATIONS IN WORKING CLASS FAMILY EXPENDITURE

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Foreword

WITH a few exceptions, family budget enquiries in the past have not usually provided separate data for families of different size. It has not previously been possible, therefore, at any rate for this country, to study the effects of variations in family size on the composition of family expenditure.

The present study was made possible through the courtesy of the Ministry of Labour, who generously supplied sample figures from the working-class family budget enquiry of 1937–8. The Ministry of Labour were also kind enough to grant permission for publication, but responsibility for all statements and conclusions rests, of course, entirely with the author. It should be explained that a large part of the work was carried out while I was a member of the Oxford University Institute of Statistics; and that Part II consists of a special analysis made on behalf of the Royal Commission on Population in 1946, when it was privately circulated. A great deal of computational work was involved and most of it was done by Miss J. Morris, of the Oxford Institute of Statistics, and Mrs. J. E. Manley, of the Central Statistical Office.

The data which formed the basis of this study relate to working class households consisting of two adults (one man and one woman) and 0, 1, 2 or 3 children under 14 years of age. Separate figures of expenditure were available for a large number of items, cross-classified by the size of family and the level of total expenditure. The sample consisted of 124 families in London and 704 in the rest of Great Britain, but the number in London, when cross-classified in this way, was too small to provide reliable averages, so that most of Part I and the whole of the analysis

in Part II are confined to households in the rest of Great Britain; for the same reason, families with 3 children were omitted from Part II, and the figures given in Part I relating to these families can only be accepted with reserve. A fuller description of the data will be found in Appendix III, together with most of the figures which were directly employed.

In Part I a study is made of the variations in family expenditure in relation to (i) the number of children in the family, and (ii) the level of total expenditure. For families of a given size, the relations between the amounts spent on particular items, such as food or clothing, and total family expenditure are described by means of regression lines. It is then possible to classify different items as either "necessities" or "luxuries," these terms being used in a technical sense, and to make estimates of their income elasticities of demand.

Part II is devoted to the much more hazardous task of trying to estimate that portion of total expenditure which can be attributed to the presence in a family of one or two children. The whole of this part of the work is in the nature of an experiment, and with the existing data it is impossible to show more than the approximate orders of magnitude involved. The results—particularly those relating to the subdivision of the total expenditure on each child—must be interpreted with caution, and with due regard for all the qualifications that accompany the figures.

It should, perhaps, also be emphasized that both parts of the analysis refer to working class families of a given type, and are based on the conditions prevailing in 1937–38. The results cannot be extended to other types of family, to different conditions, or beyond the range of income covered by the data.

PART I.—THE GENERAL PATTERN OF FAMILY EXPENDITURE

Butcher, baker, candlestick-maker,
Blood, and bread, and taper,
Meat, and wheat, and light,
Along with Jones the draper
The wife finds these in the little shops
On the right of the undertaker.
—DOROTHY WELLESLEY.

1. *Variations in Total Family Expenditure*

The family budgets collected by the Ministry of Labour in 1937–38 showed, in great detail, how much was spent by working class families on "meat and wheat and light," as well as on numerous other things. But, to begin with, we shall confine our attention to the figures of total expenditure, with particular reference to the following questions:

(1) How much do families of a given size spend, on the average, and how much does total expenditure vary from one family to another, or from one district to another?

(2) Do larger families for the most part spend more, or less, than smaller families*—in other words, how do the answers to (1) compare for families of different size?

Some idea of the variations between different regions† of the country can be obtained from the figures of average family expenditure, given in Table 1. A good many of these averages, including all those for families with 3 children, are based on relatively few families, and may not, therefore, be reliable. The broad differences nevertheless stand out. Average expenditure was clearly highest in London; it was also relatively high in South-West England and lowest in Wales and Northern England.

Families in London are bound to have different habits of expenditure from those in the rest of Great Britain; expenditure on rent and travelling, for instance, are considerably higher in London than elsewhere. Throughout this study, therefore, the London families are kept separate from those in the rest of Great Britain.

* The answer to this question depends partly, of course, on whether they have more to spend. But as no comparable figures of income are available, the discussion is confined to the recorded figures of expenditure.

† The regions are those used by the Ministry of Labour.

TABLE 1

Average Weekly Expenditure in Different Regions

	Families with no child		*Families with 1 child*		*Families with 2 children*		*Families with 3 children*	
	Number of families	*Average expenditure*	*Number of families*	*Average expenditure*	*Number of families*	*Average expenditure*	*Number of families*	*Average expenditure*
		s. d.		s. d.		s. d.		s. d.
Scotland	34	75 1	18	77 0	19	82 9	18	67 2
North-East	48	69 5	28	69 5	18	87 2	4	53 8
North-West	35	75 3	27	78 3	25	77 1	10	69 2
Northern	22	65 3	20	71 2	11	65 9	4	77 3
Midland	53	75 6	37	81 4	21	76 11	10	75 7
Wales	19	63 10	10	70 6	12	70 5	2	63 5
South-East	37	66 3	29	75 9	18	64 2	2	62 10
South-West	36	78 2	23	74 4	14	76 2	5	77 0
London	46	85 0	40	78 8	20	85 8	14	83 4

The main features of the variations in total expenditure of families in the rest of Great Britain are summarized in Table 2. The full distribution, with the families grouped in ranges of 10 shillings, is shown in Table 3 and Diagram 1.

TABLE 2

*Weekly Expenditure of Families in Great Britain, excluding London: Mean, Median and Quartiles**

	Mean	*Lower Quartile*	*Median*	*Upper Quartile*
	s. d.	s. d.	s. d.	s. d.
Families with no child	71 11	54 7	66 8	83 3
,, ,, 1 child	75 5	60 4	71 11	85 8
,, ,, 2 children	75 11	59 11	70 8	86 7
,, ,, 3 ,,	69 5	50 0	68 9	84 7

* Median and Quartiles were calculated from the data grouped in ranges of 10 shillings, as in Table 3.

One might expect to find that, as the size of the family increases, total expenditure also increases, since the incomes of families with children must be sufficient to support the children. It is not surprising, therefore, that the total expenditure of families with 1 or 2 children should have been higher, on the average, than that of families with no children, although there appears to have been little difference between the average expenditure of families with 1 child and that of families with 2 children. What is remarkable is that families with 3 children apparently had a smaller average expenditure than families with 1 or 2 children, and smaller even than families with no child. Even more remarkable is the fact that a quarter of the families with 3 children were able to subsist on less than 50 shillings a week.

In Diagram 1 the distributions of families with 1, 2 and 3 children are each compared with the distribution of families which had no child. It can be seen that the distributions of families with 0, 1 or 2 children are similar in shape, the only real difference being that families with 1 or 2 children had a consistently higher total expenditure than families without children. There appears to have been a tendency for families with 3 children to bunch themselves together at the two ends of the distribution; but, as the numbers in each expenditure group are relatively small (see Table 3), no significance should be attached to the bi-modal appearance of this distribution.

The distribution by total expenditure of families in London is given in Table 4. The quartiles have not been calculated for these families, since the number in each size group is not large enough to yield reliable estimates.

So far, we have confined our attention to the figures of total expenditure. But more interesting problems arise when we consider the variations in the composition of family expenditure.

TABLE 3

Distribution of Families by Total Expenditure. Great Britain, excluding London

	Total expenditure									*Total number of families*
	Under 30*s*.	30–40*s*.	40–50*s*.	50–60*s*.	60–70*s*.	70–80*s*.	80–90*s*.	90–100*s*.	*Over* 100*s*.	
Families with no child										
Number . . .	3	11	33	53	64	38	33	16	33	284
Percentage of total .	*1·1*	*3·9*	*11·6*	*18·7*	*22·5*	*13·4*	*11·6*	*5·6*	*11·6*	*100·0*
Families with 1 child										
Number . . .	—	6	8	33	42	40	26	14	23	192
Percentage of total .	—	*3·1*	*4·2*	*17·2*	*21·9*	*20·8*	*13·5*	*7·3*	*12·0*	*100·0*
Families with 2 children										
Number . . .	—	4	8	23	33	24	17	12	17	138
Percentage of total .	—	*2·9*	*5·8*	*16·7*	*23·9*	*17·4*	*12·3*	*8·7*	*12·3*	*100·0*
Families with 3 children										
Number . . .	—	1	13	7	8	6	13	2	5	55
Percentage of total .	—	*1·8*	*23·7*	*12·7*	*14·5*	*10·9*	*23·7*	*3·6*	*9·1*	*100·0*
All families										
Number . . .	3	22	62	116	147	108	89	44	78	669

TABLE 4

Distribution of London Families by Total Expenditure

	Total expenditure								*Total number of families*
	30–40*s*.	40–50*s*.	50–60*s*.	60–70*s*.	70–80*s*.	80–90*s*.	90–100*s*.	*Over* 100*s*.	
Families with no child	1	1	0	7	13	6	7	11	46
,, ,, 1 child	..	1	6	10	6	6	4	7	40
,, ,, 2 children	..	..	1	3	5	4	4	3	20
,, ,, 3 ,, ...	..	..	2	3	3	3	1	2	14
All families	1	2	9	23	27	19	16	23	120

2. *Analysis of Family Expenditure by Means of Regression Lines: Introduction*

It has been known for some time that, among families of a given type, there is usually a fairly simple relation between expenditure on a particular item, such as food, and total family expenditure. Suppose that we have a homogeneous group of families and that figures of expenditure on a number of different items are given, classified according to the level of total expenditure. Let E_r denote the amount spent on the r^{th} item and let Y denote total expenditure. It has been found* that the relation between E_r and Y can often be described, with reasonable accuracy, by a linear equation of the form:

$$E_r = a_0 + a_1 Y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Such relations are naturally influenced by the size and composition of the family, but separate data have not always been available for families of different size. A method which has frequently been used is to express all the figures in terms of expenditure per equivalent number of adults. It has been pointed out, however, that there are serious objections to this method, and that it does not usually succeed in eliminating the effects of varying family composition.† Fortunately, in the present instance, separate figures have been made available for families of different size, so that the relations can be studied for each size of family. When the results for families of varying size are compared, it turns out that there are important differences, which cannot be accounted for in any simple way.

* *Cf.* "Family Expenditure," by R. G. D. Allen and A. L. Bowley (1935).
† "Expenditure Patterns of Families of Different Types," by R. G. D. Allen; one of the *Studies in Mathematical Economics and Econometrics*, published in memory of Henry Schultz (Chicago, 1942).

The households included in the present analysis were homogeneous in the sense that they were all working class households, containing two adults (one man and one woman), of whom only one was a wage or salary earner,‡ and varying numbers of children, but no other person. As expenditure on rent and travelling was rather higher in London than elsewhere, the London households were considered separately from those in the rest of Great Britain. The whole of Sections 2 to 6 are devoted to households outside London, and those in London are briefly discussed in Section 7.

When the figures for any given set of families, containing either 0, 1 or 2 children, were examined, it was found that the relations between E_r and Y could not, in general, be represented by simple linear equations, of the form (1). As can be seen from Diagrams 2 to 10, there is definite evidence, in some cases (e.g. food), of a curvilinear relationship; while in other cases (e.g. rent) the points are rather widely dispersed, and it is probably not possible to say definitely that the relationship is non-linear. If the data for different sized families were merged together, the curvilinear relationship would tend to disappear.

TABLE 5

Values of Regression Coefficients

	Number of children	b_0	b_1	$b_2 \times 10^3$
Food	0	30·4	·4479	−·13157
	1	−89·7	·7356	−·26303
	2	−53·0	·6814	−·23354
	3	−22·9	·5929	−·12864
Rent	0	−22·9	·2460	−·07273
	1	101·0	−·0283	·05312
	2	−42·1	·2936	−·12237
	3	77·4	−·0425	·09480
Fuel and light	0	24·7	·0655	−·01532
	1	32·2	·0256	·01808
	2	0·6	·0998	−·02320
	3	29·8	·0448	−·00734
Clothing	0	−26·3	·1370	−·02554
	1	−67·0	·2271	−·06814
	2	−90·0	·2680	−·08128
	3	38·7	−·0135	·07229
Sundries	0	−6·0	·1044	·24479
	1	23·6	·0397	·26017
	2	184·4	−·3443	·46109
	3	−123·4	·4183	−·03054
Meat, bacon and fish	0	11·9	·1368	−·04486
	1	−24·9	·1959	−·06739
	2	−31·6	·2110	−·07680
	3	−62·6	·2936	−·11337
Dairy products	0	18·3	·1112	−·03720
	1	−18·3	·2014	−·07361
	2	−4·6	·1833	−·05898
	3	16·3	·1062	·00992
Fruit and vegetables (excluding potatoes)	0	−11·4	·0563	−·00544
	1	−29·8	·1061	−·03425
	2	−33·8	·1101	−·03774
	3	−30·1	·0828	−·01076
Tobacco and drink	0	−14·6	·0790	−·01578
	1	−12·6	·0580	·00142
	2	−67·5	·1915	−·07785
	3	−54·4	·1788	−·08071

‡ With the exceptions mentioned in Appendix III.

For reasons which will soon become clear, the analysis is simplified if a similar type of equation is assumed to hold good for all items of expenditure. It was decided, therefore, to represent the relation between E_r and Y in all cases by equations of the form:

$$E_r = b_0 + b_1 Y + b_2 Y^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When equations of this type are fitted for all the different items, the values obtained for the regression coefficients must satisfy the condition $\sum_r E_r = Y$, since the sum of the amounts spent on all items must equal total expenditure. The condition implies that $\sum_r b_0 = 0$, $\sum_r b_1 = 1$ and $\sum_r b_2 = 0$. It is interesting to notice that, as shown in Appendix I, the ordinary principle of least squares does, in fact, satisfy these conditions.

Regression coefficients, based in all cases on equations of the form (2), were computed for five main groups of items—food, rent,* fuel and light, clothing and sundries—which together accounted for all household expenditure, as well as for four subdivisions of these groups which appeared to have particular interest. The method of least squares was employed, the average figures in each range of expenditure being weighted by the number of families in the range. The regression coefficients are collected together in Table 5, and the resulting equations are compared with the observations in Diagrams 2 to 10.

Equations were obtained for each of the four sets of families, containing 0, 1, 2 and 3 children respectively, but since the number of families included in the sample becomes smaller as the number of children increases,† the regression equations are less reliable for the larger families. In particular, the number of families with 3 children (only 61, divided between six expenditure groups) was insufficient to provide accurate results, and all the estimates relating to these families are subject, therefore, to very wide margins of error. The regression lines for 3-children families could not, for the same reason, be expected to fit the observations at all closely.

Bearing this in mind, it can be seen from the diagrams that, in the cases of food, clothing and sundries, the observed points correspond very closely to the equations. In the case of fuel and light there is some departure from the regression equations, and in the case of rent the observed points are rather widely scattered. As regards the smaller groups, the correspondence is moderately good for the three food groups (meat, bacon and fish; dairy products; and fruit and vegetables), though perhaps not quite so good for tobacco and drink. It should be remembered that figures of expenditure on drink and tobacco, given in family budgets, are generally very unreliable.

3. *Some General Comparisons*

With the help of the regression equations, we can proceed to compare the distribution of expenditure shown by families having different numbers of children and the same total expenditure; as well as the effects, on families of different size, of changes in the level of income (which can be assumed to be correlated with total expenditure).

The reason for basing these comparisons on the theoretical values derived from the regression lines needs, perhaps, a few words of explanation. In the first place, although the data for different sized families are available for the same ranges of expenditure, there is some variation in the average figure of total expenditure within each range. Secondly, it is assumed that, in the population from which the sample is drawn, the relationship between expenditure on a given item and total expenditure could be described by means of equations of the form (2). The regression coefficients obtained from the sample are estimates, subject to ordinary sample errors, of the corresponding parameters of the population. Thus the value of E_r corresponding to any given value of Y, obtained from the regression equation, is an estimate of the value which would be found in the population. And it takes into account not only the figure in a given range of expenditure, but also the relationship between the figures in different ranges.

Table 6 shows, for each size of family, the estimated expenditure on various groups of items at three different levels of total expenditure, viz. 50*s.* a week, 72*s.* 8*d.* a week, which is the average

* The figures for rent, used in this analysis, are the average payments of those families which were renting their dwellings.

† See Appendix III.

for families with no children,‡ and 100*s*. a week. It should be remembered that the estimates for rent and all the estimates for 3-children families are subject to wide margins of error.

TABLE 6

Distribution of Weekly Expenditure at Different Levels of Total Expenditure, Estimated from Regression Lines

	Families with no children		*Families with 1 child*		*Families with 2 children*		*Families with 3 children**	
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
Food	21	0	21	5	22	8	23	11
Rent	8	3	8	7	7	6	7	2
Fuel and light	4	10	4	6	4	4	4	6
Clothing	3	11	3	9	3	6	4	9
Sundries	12	0	11	9	12	0	9	8
Included above:								
Meat, bacon and fish	6	6	5	8	5	7	6	1
Dairy products	6	0	6	4	7	0	7	0
Fruit and vegetables†	1	8	1	10	1	7	1	4
Tobacco and drink	2	3	1	10	1	7	2	0
Total	50	0	50	0	50	0	50	0
Food	26	9	29	4	30	4	33	1
Rent	11	4	9	9	10	1	9	4
Fuel and light	5	10	5	8	5	10	5	3
Clothing	6	2	6	7	6	10	6	10
Sundries	22	7	21	4	19	7	18	2
Included above:								
Meat, bacon and fish	8	1	7	11	7	10	8	11
Dairy products	7	3	8	5	9	3	9	9
Fruit and vegetables†	2	10	3	1	2	10	2	10
Tobacco and drink	3	6	3	3	3	4	3	4
Total	72	8	72	8	72	8	72	8
Food	31	6	34	6	35	8	41	11
Rent	14	0	11	11	11	2	13	7
Fuel and light	6	9	7	5	7	3	6	1
Clothing	8	5	9	0	9	7	10	7
Sundries	39	4	37	2	36	4	27	10
Included above:								
Meat, bacon and fish	9	4	9	5	9	3	10	6
Dairy products	8	2	9	9	10	11	13	2
Fruit and vegetables†	4	0	4	0	3	8	4	6
Tobacco and drink	4	10	4	11	4	2	3	8
Total	100	0	100	0	100	0	100	0

* The estimates for families with 3 children, being based on only 58 families, are subject to wide margins of error.

† Excluding potatoes.

At the level of 50*s*. a week it can be seen that, as the number of children increases, more is spent on food, but only slightly less on each of the other main groups. As might be expected, the amount spent on dairy products increases with the number of children, but the amount spent on the meat group seems to decline, while expenditure on fruit and vegetables shows very little change. The amount spent on drink and tobacco shows a small reduction.

At a total expenditure of 72*s*. 8*d*. a week, the amount spent on food shows a substantial rise as the size of family increases, expenditure on fuel and light shows little change, that on clothing

‡ The difference between this figure and that shown in Table 2 is explained by the omission from the regression analysis of the three families whose total expenditure was less than 30*s*. a week (see Appendix III).

shows only a slight increase, while expenditure on sundries shows a marked decline. We find, again, that dairy products show a marked increase, expenditure on fruit and vegetables remains about the same, while the consumption of meat appears to go up only when the number of children increases to three. At this level of expenditure, there is only a very small reduction in the amount spent on tobacco and drink as the number of children increases.

Similar tendencies are shown at a level of total expenditure of 100*s.* a week. A good deal more is spent on food, slightly more on fuel and light, more on clothing and rather less on sundries, as the size of family increases. There is little change in the amounts spent on fruit and vegetables, or meat, except that the addition of a third child appears to cause an increase in the consumption of meat; while expenditure on dairy products now shows a very substantial increase. Expenditure on drink and tobacco shows some decline only when the number of children increases from one to two or more.

TABLE 7

Percentage Distribution of Expenditure, Estimated from Regression Lines

	Families with no children	*Families with 1 child*	*Families with 2 children*	*Families with 3 children**
	Total Expenditure = 50s. per week			
Food	42·0	42·8	45·3	47·8
Rent	16·4	17·2	15·0	14·3
Fuel and light	9·7	9·0	8·7	9·0
Clothing	7·8	7·5	6·9	9·4
Sundries	24·1	23·5	24·1	19·5
Included above:				
Meat, bacon and fish	13·0	11·4	11·2	12·1
Dairy products	11·9	12·7	14·0	13·9
Fruit and vegetables†	3·4	3·6	3·1	2·6
Tobacco and drink	4·5	3·6	3·2	4·0
Total	100·0	100·0	100·0	100·0
	Total Expenditure = 72s. 8d. per week			
Food	36·8	40·3	41·7	45·4
Rent	15·6	13·4	13·8	12·9
Fuel and light	8·0	7·8	8·0	7·3
Clothing	8·5	9·1	9·4	9·4
Sundries	31·1	29·4	27·1	25·0
Included above:				
Meat, bacon and fish	11·1	10·9	10·8	12·3
Dairy products	10·0	11·6	12·7	13·4
Fruit and vegetables†	3·8	4·2	3·9	3·9
Tobacco and drink	4·8	4·5	4·6	4·6
Total	100·0	100·0	100·0	100·0
	Total Expenditure = 100s. per week			
Food	31·5	34·5	35·7	42·0
Rent	14·0	12·0	11·2	13·6
Fuel and light	6·8	7·4	7·2	6·1
Clothing	8·4	9·0	9·6	10·6
Sundries	39·3	37·1	36·3	27·7
Included above:				
Meat, bacon and fish	9·3	9·4	9·3	10·5
Dairy products	8·2	9·8	10·9	13·2
Fruit and vegetables†	4·0	4·0	3·7	4·5
Tobacco and drink	4·8	4·9	4·2	3·7
Total	100·0	100·0	100·0	100·0

* The estimates for families with 3 children, being based on only 58 families, are subject to wide margins of error.

† Excluding potatoes.

To facilitate the comparison between different levels of income, the same figures are shown in Table 7 in the form of percentages of total expenditure. The proportions spent on food, rent and fuel and light are seen to fall as total expenditure rises, while the proportion spent on the sundries group increases. In general, the proportion spent on clothing also increases with rising total expenditure, except in the higher ranges of income for families with few, or no, children. As regards the smaller groups, the proportions spent on meat and dairy products both fall as income increases, but in the case of dairy products the decline becomes less as the number of children increases. Similarly, the proportion spent on fruit and vegetables generally seems to increase, and the proportion devoted to drink and tobacco also shows a slight tendency to increase with the level of income. The changes in proportionate expenditure as income rises can, in fact, be used to indicate the relative urgency of different commodities.

4. "*Necessities*" and "*Luxuries*"

Those commodities on which the proportionate expenditure diminishes with rising total expenditure may be defined as "necessities." Those commodities on which the proportionate expenditure increases with rising total expenditure may be defined as "luxuries." For example, at very low incomes a large proportion of expenditure is bound to be devoted to those items, such as food or rent, which are absolutely necessary, leaving very little for items which are less urgently needed; while, as income increases, a gradually increasing proportion can be spent on the less necessary items. There is some correspondence, therefore, between the definitions of "necessities" and "luxuries," given above, and the common-sense meaning of these terms, but the use of this classification should not be misunderstood. It is certainly not implied that expenditure on items which are classified as luxuries is, in any sense, unnecessary or wasteful. Indeed, whatever the range of incomes considered, it is obvious that if the proportion spent on some things declines with rising incomes, the proportion spent on other things must increase at the same time.

Referring back to formula (2), the proportionate expenditure on a particular item is given by

$$\frac{E_r}{Y} = \frac{b_0}{Y} + b_1 + b_2 Y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

whence

$$\frac{d}{dY}\left(\frac{E_r}{Y}\right) = b_2 - \frac{b_0}{Y^2} = \theta, \text{ say.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Positive values of θ define luxuries and negative values define necessities, while the arithmetical values of θ indicate varying degrees of urgency. Since θ is a function of Y, a given commodity may be a necessity in one range of income and a luxury in another range.

Table 8 shows the values of θ corresponding to three different levels of total expenditure (the same as were used in the previous Tables). It can be seen that food, rent and fuel and light, as might be expected, are all "necessities" and, of these, food is the most urgent; fuel and light seem to become less necessary, however, as income increases. Clothing is a "luxury" at low incomes, but becomes more necessary as income increases. The sundries group is also, in general, a luxury, though some items included in this group may be necessities.

The meat group is usually a necessity, but it is interesting to notice that for families with very small incomes and two or three children—in other words, for those families which are worst off—it becomes a luxury. Dairy products are also a necessary item. Fruit and vegetables behave as a luxury at low incomes, but at the higher levels of income they appear to become necessary. As regards tobacco and drink, it must be remembered that the basic figures are not very trustworthy; at low incomes this group is evidently a luxury, but as incomes increase it appears to become more necessary. This conclusion is not, in fact, unreasonable, since the proportionate expenditure on drink and tobacco is unlikely to increase indefinitely with rising incomes.

The next problem is to compare the relative urgency of a particular group of items among families of different size. Suppose we first consider the different values of θ for given values of Y, bearing in mind that small differences in θ probably have no significance. No obvious similarity can be noticed between the value of θ for different sized families, at given levels of income. What we do notice is that the change in θ as the number of children increases, when Y is fixed, is in the

same direction, in several cases, as the change in θ for families of a given size, when Y is reduced. This tendency, though not consistent, is apparent as regards clothing, fruit and vegetables, tobacco and drink, and also, except for families with no children, in the cases of food and the smaller meat group.

TABLE 8

Values of θ *(Multiplied by* 10^3*)**

Negative value = necessity. Positive value = luxury.

	Number of children in family	$Y = 600$	$Y = 872$	$Y = 1{,}200$	*Values corresponding to* $x_c = 44\cdot7$
Food	0	−·22	−·17	−·15	−·18
	1	−·01	−·15	−·20	−·17
	2	−·09	−·16	−·20	−·19
	3	−·07	−·10	−·11	(−·11)
Rent	0	−·01	−·04	−·06	−·03
	1	−·23	−·08	−·02	−·06
	2	−·01	−·07	−·09	−·09
	3	−·12	−·01	+·04	(+·05)
Fuel and light	0	−·08	−·05	−·03	−·06
	1	−·07	−·02	−·00	−·02
	2	−·02	−·02	·02	−·02
	3	−·09	−·05	−·03	(−·03)
Clothing	0	+·05	+·01	−·01	+·02
	1	+·12	+·02	−·02	+·00
	2	+·17	+·04	−·02	−·00
	3	−·04	+ 02	+·05	(+·05)
Sundries	0	+·26	+·25	+·25	+·26
	1	+·19	+·23	+·24	+·23
	2	·05	+·22	+ 33	+·30
	3	+·31	+·13	+ 06	(+·04)
Meat, bacon and fish	0	−·08	−·06	·05	−·07
	1	+·00	−·03	−·05	− 04
	2	+·01	−·04	− 05	−·05
	3	+·06	−·03	−·07	(−·08)
Dairy products	0	−·09	−·06	−·05	−·07
	1	−·02	−·05	−·06	−·05
	2	−·05	−·05	−·06	−·05
	3	−·04	−·01	−·00	(+·00)
Fruit and vegetables (excluding potatoes)	0	+·03	+·01	+·00	+·01
	1	+·05	+·00	−·01	−·00
	2	+·06	+·01	−·01	−·01
	3	+·07	+·03	+·01	(+·01)
Tobacco and drink	0	+·02	+·00	−·01	+·01
	1	+·04	+·02	+·01	+·02
	2	+·11	+·01	− 03	−·02
	3	+·07	−·01	−·04	(−·05)

* The estimates for families with 3 children, being based on only 58 families, are subject to wide margins of error; this is particularly so of the figures corresponding to a given value of x_c, shown in the last column, which are therefore placed in brackets.

Such a tendency is not, in fact, difficult to explain. For a given size of family, the standard of living is closely related to the level of income (or to total expenditure). But for any given level of income (or total expenditure) the standard of living is bound to fall as the number of children increases. A given sum of money obviously goes further when there are fewer people

on which to spend it. Thus the standard of living enjoyed by a family with, say, two children and a given total expenditure would be similar to the standard of living enjoyed, with a smaller total expenditure, by families with one child. And the degree to which one commodity is more (or less) urgently required than another is likely to be more closely related to the standard of living than to the level of income.

The problem of comparing the standards of living of families of different size is treated at length in Part II, the results of which must here, to some extent, be anticipated. It is possible, as shown in Part II, to select a group of items, the expenditure on which can be used to provide an approximate indication of the standard of living. After examination of numerous different selections, it is found that expenditure on adults' clothing, which is denoted by x_c, provides the best index for this purpose; and a functional relationship is established between x_c and Y, so that, for each size of family, we can determine the value of Y corresponding to a given standard of living. For present purposes, the standard of living defined by the mean value of x_c ($\bar{x}_c = 44 \cdot 7$) is taken as the basis of comparison, and the values of Y, corresponding to this standard, can be determined for each size of family. Hence it was possible to calculate values of θ, all corresponding to the same standard of living, and these are shown in the last column of Table 4. The values of θ for families with 3 children, which are shown in brackets, are very rough indeed, since the families of this size were too few to be included in the analysis of Part II.

We would not, of course, expect to find that the values of θ, at a given standard of living, were identical for any one item. The values of θ shown in the last column of Table 8 do, however, show much greater similarity than the values corresponding to a given total expenditure.

It would be risky to draw any conclusions from the small differences between these values of θ. But, if we compare families having 0, 1 or 2 children, we notice that in two of the main groups—rent and clothing—the values of θ show a consistent movement in one direction: in both these cases the (algebraic) values of θ become smaller as the size of family increases. If any inference were possible, this would suggest that rent and clothing, at the standard of living referred to, become more necessary as the size of family increases.

5. *Income Elasticities of Demand*

We have seen that, as income, or total expenditure, increases, the amount spent on a particular commodity generally increases, the proportional change being sometimes greater and sometimes less than the proportional change in total expenditure. The income elasticity of demand is defined as the ratio of the proportional change in expenditure on a given commodity to the proportional change in income. For present purposes, since income figures are not available, we shall identify income with total expenditure. Writing η for the income elasticity of demand, and making use of formula (2), we define

$$\eta = \frac{Y}{E_r}\frac{dE_r}{dY} = \frac{Y}{E_r}(b_1 + 2b_2Y) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Since η is a function of Y, we may expect to obtain different values at different levels of total expenditure.

We notice, immediately, that there is a close connection between η and θ. For

$$\theta = \frac{d}{dY}\left(\frac{E_r}{Y}\right) = \frac{1}{Y}\frac{dE_r}{dY} - \frac{E_r}{Y^2}$$
$$= \frac{E_r}{Y^2}(\eta - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence when $\eta = 1$, $\theta = 0$. When $\eta > 1$, θ is positive; and when $\eta < 1$, θ is negative.

The estimated values of η corresponding to three different levels of total expenditure (those used above) are given in Table 9. The values of η show somewhat similar characteristics to those of θ. As regards the five main groups, we find that the values of η for food, rent and fuel and light are generally lower than unity, while those for clothing and sundries are usually greater than unity. We notice also that in the case of food, as well as in the case of clothing, there is a tendency for η to fall as income increases, while the sundries group shows the opposite tendency. (It must be remembered that the estimates for 3-children families are less reliable than the others.)

Turning to the smaller food groups, the income elasticity of demand is generally less than unity for meat and dairy products and generally greater than unity for fruit and vegetables. The income elasticity of demand for fruit and vegetables is particularly high in the case of families with small incomes. For each of these food groups, as well as for the whole food group, the value of η shows a tendency to fall as total expenditure increases; or, at a given level of total expenditure, as the number of children declines. In other words, η becomes less in each of these cases as the standard of living increases.

TABLE 9

*Estimated Values of η (Income Elasticity of Demand)**

	Number of children in family	$Y = 600$	$Y = 872$	$Y = 1{,}200$	*Values corresponding to* $x_c = 44{\cdot}7$
Food	0	·69	·59	·42	·64
	1	·98	·69	·30	59
	2	·89	·66	·34	·48
	3	·92	81	·68	(·64)
Rent	0	·97	·76	·51	·85
	1	·21	·48	·83	·58
	2	·98	·58	·00	·26
	3	·50	·95	1·36	(1·45)
Fuel and light	0	·48	·48	42	·49
	1	·52	·73	·93	·79
	2	·83	·74	·61	·66
	3	·40	·44	·45	(·44)
Clothing	0	1·37	1·09	·90	1·19
	1	1·95	1·19	·71	1·05
	2	2·46	1·34	·76	·98
	3	·78	1·20	1·52	(1·58)
Sundries	0	1·65	1·71	1·76	1·69
	1	1·50	1·68	1·79	1·72
	2	·87	1·71	2·10	2·00
	3	1·96	1·46	1·24	(1·20)
Meat, bacon and fish	0	·64	·53	·31	·58
	1	1·01	·72	·36	·63
	2	1·06	·71	·29	·47
	3	1·30	·78	·20	(·00)
Dairy products	0	·56	46	·27	·51
	1	·89	·63	·25	·54
	2	·80	·63	·38	·49
	3	·85	·92	·99	(1·00)
Fruit and vegetables (excluding potatoes)	0	1·46	1·22	1·07	1·30
	1	1·81	1·10	·59	·95
	2	2·08	1·15	·53	·78
	3	2·67	1·65	1·27	(1·20)
Tobacco and drink	0	1·33	1·06	·86	1·15
	1	1·65	1·35	1·25	1·31
	2	3·05	1·20	·11	·56
	3	2·06	·82	−·41	(−·93)

* The estimates for families with 3 children, being based on only 58 families, are subject to wide margins of error; this is particularly so of the figures corresponding to a given value of x_c, shown in the last column, which are therefore placed in brackets.

The values of η for tobacco and drink must be regarded as unreliable, since the figures given for expenditure on these items cannot usually be trusted.† As might be expected, there is a noticeable tendency, in this group as well, for the value of η to fall as the standard of living rises.

† A negative value of η for drink and tobacco is obviously absurd.

When families of different size are compared, we should expect, for the reasons given in the previous section, to find greater similarity in the values of η if the comparison is based on a given standard of living, rather than on a given level of income (or total expenditure). At any given standard of living we should not, however, expect the values of η to show the same degree of similarity as the values of θ. In fact (6) may be rewritten in the form:

$$\eta = 1 + \frac{Y^2}{E_r}\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now suppose we assume that θ is practically constant at any given standard of living. Then we should usually expect to find that, if θ is negative, the values of η, corresponding to a given standard of living, become smaller with an increase in the size of family; and that, if θ is positive, the values of η corresponding to a given standard of living become larger as the size of family increases.† The reason for this is that the proportional change in Y^2, as the size of family increases and the standard of living remains constant, would usually be greater than the proportional change in E_r. But the foregoing argument would not apply if the values of θ, corresponding to a given standard of living, were to change when the size of family changes.

The values of η corresponding to the standard of living defined by $x_c = 44{\cdot}7$ are given in the last column of Table 9. When these values are compared, it can be seen that, for food and rent, η becomes smaller when the size of family increases, as we should expect from the values of θ. Similarly, in the case of sundries, θ being positive, the values of η at this standard of living increase with the size of family. In the case of clothing, on the other hand, θ declines as the size of family increases and η shows a similar movement.

We have already noticed that, when the comparison is based on a given total expenditure, the values of η for food as a whole, as well as for the smaller food groups, show a tendency to increase with the size of family. But when the comparison is based on a given standard of living, the values of η for food, as well as for rent and clothing, show a general tendency to fall as the size of family increases. For sundries, on the other hand, the values of η corresponding to a given standard of living show a tendency to increase with the size of family.

We may conclude, therefore, that at a given standard of living families with several children have a higher income elasticity of demand for sundries and lower income elasticities of demand for food, rent and clothing than families with few or no children. In other words, at a given standard of living, food, rent and clothing become more necessary, and sundries become more of a luxury, as the size of family increases.

Another method of comparison which suggests itself is to measure the elasticities of demand with respect to variations in the standard of living (as opposed to variations in income). Suppose we denote by λ the elasticity of demand with respect to variations in x_c, which is assumed to represent the standard of living. Then

$$\lambda = \frac{x_c}{E_r} \cdot \frac{dE_r}{dx_c} = \frac{x_c}{E_r}\frac{dE_r}{dY} \cdot \frac{dY}{dx_c} \, .$$

Now let $\mu = \frac{x_c}{Y} \cdot \frac{dY}{dx_c}$, so that μ may be termed the elasticity of income with respect to changes in the standard of living. Then

$$\lambda = \mu\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For a given standard of living, μ increases as the size of family increases: the larger the family, the larger the proportionate change in income (or total expenditure) required to bring about a given proportionate change in the standard of living. It follows that, if the values of η at a given standard of living decline as the size of family increases, the corresponding reduction in λ would be less pronounced; and if the values of η at a given standard of living increase with the size of family, the corresponding increase in λ would be more pronounced. The values of λ for the five main groups of items, at the standard of living corresponding to $x_c = 44{\cdot}7$, are tabulated below.

† We assume that η is positive. In other words, we exclude from consideration the case of "inferior" goods.

TABLE 10

Values of λ *Corresponding to* $x_c = 44{\cdot}7$

	Families with no children	*Families with 1 child*	*Families with 2 children*
Food	·47	·47	·37
Rent	·62	·46	·21
Fuel and light	·36	·62	·52
Clothing	·87	·83	·76
Sundries	1·24	1·36	1·55

It can be seen that variations in λ and η, as the size of family changes, have the expected relations to one another (*cf.* Table 9). We still find, however, that in the cases of food, rent and clothing the elasticity of demand with respect to the standard of living generally seems to decline as the size of family increases; and for sundries it increases with the size of family. As the standard of living increases from a given level, larger families show a greater proportionate increase in expenditure on sundries, and a smaller proportionate increase in expenditure on food and clothing, than smaller families.

6. *Food, Rent and Income*

Hitherto our attention has been focussed on the variations in total family expenditure and the accompanying changes in expenditure on particular groups of items. We have been able to compare the changes in expenditure on one group of items, such as food, with the simultaneous movements in another group, such as rent, as income (or total expenditure) increases. We have not, so far, considered the relation between variations in expenditure on food and variations in rent at a given level of income. Yet this is a problem of some practical importance. The level of rent may change from one day to the next, but it usually takes some time for a family's income to change.

Suppose that two households have the same total expenditure, but that one of them is obliged to pay a high rent while the other has a relatively low rent. How does the difference in rent affect the relative amounts spent on food? We would naturally expect the household with a low rent to spend more on food, simply because it has more money to spare. But does it, in fact, spend on food as much as, or more than, we would for this reason expect?

To answer this question, we consider the correlation between the deviations of rent and food expenditure, in individual families, from the values given by the regression equations, discussed above (see Table 5). Since the computational work involved in this type of calculation is very heavy, the comparison is confined to food and rent and to families with one child.* It would also be interesting to compare other items of expenditure—food with drink and tobacco, for example—at given levels of income; and to see whether different results were obtained for families of different size. But with the resources and time at our disposal, it has not been possible to make these additional calculations.

Let f denote the amount spent by an individual family on food and r the amount spent on rent at a given level of total expenditure, denoted as before by Y.

Let δ_f denote the difference between the individual family's expenditure on food and the figure given by the regression equation of food on total expenditure. Similarly let δ_r denote the difference between the same family's expenditure on rent and the amount given by the regression equation of rent on total expenditure, the regression equations being those shown in Table 5.

Let σ_{δ_f} and σ_{δ_r} denote the standard deviations of δ_f and δ_r respectively.

Let p_f, p_r and p_x denote the average proportions of a given total expenditure devoted to food, rent and all other items respectively, so that:

$$p_f + p_r + p_x = 1.$$

Let $r_{fr.Y}$ denote the correlation coefficient between δ_f and δ_r; that is, the partial correlation coefficient between food and rent with respect to variations in total expenditure. What is the "expected" value of this correlation coefficient?

* Only those families which were renting their houses are included in the calculation.

A family which spends less on rent than the amount given by the regression equation of rent on total expenditure can be regarded as having a "surplus" available for distribution among other items; and this "surplus" may be either positive (negative δ_r) or negative (positive δ_r). Suppose that, on the average, this "surplus," whether positive or negative, is distributed in proportion to the average expenditure on other items. On this assumption, the regression coefficient of δ_f on δ_r (assuming linear correlation) would be given by:

$$r_{fr.Y}\frac{\sigma_{\delta_f}}{\sigma_{\delta_r}} = -\frac{p_f}{p_f + p_x}.$$

With a similar assumption about the average variations in rent which accompany given variations in food expenditure, the regression coefficient of δ_r on δ_f would be given by:

$$r_{fr.Y}\frac{\sigma_{\delta_r}}{\sigma_{\delta_f}} = -\frac{p_r}{p_r + p_x}.$$

Hence we should find, on the assumptions mentioned, that:

$$r_{fr.Y} = -\sqrt{\frac{p_f}{1 - p_r} \cdot \frac{p_r}{1 - p_f}}.$$

Thus the "expected" correlation depends on the average proportions of total expenditure devoted to the items in question; and these proportions, as we have seen, vary with the level of total expenditure. The families were therefore arranged in groups, according to total expenditure, and separate calculations were made for each group, with the following results:

TABLE 11

Partial Correlation of Expenditure on Food and Rent, with Respect to Variations in Total Expenditure, in Families with 1 Child

Total expenditure per week	*Number of families*	$-\sqrt{\frac{p_f}{1-p_r} \cdot \frac{p_r}{1-p_f}}$	$r_{fr.Y}$
Under 50*s*.	14	−·444	−·114
50–60*s*.	28	−·382	−·276
60–70*s*.	35	−·341	−·497
70–80*s*.	32	−·333	−·166
80–90*s*.	21	−·312	−·241
Over 90*s*.	24	−·248	−·211

In most cases, the partial correlation between food and rent is numerically less than the "expected" correlation. At a given level of total expenditure, the amount spent on food is relatively inelastic (food behaves as a necessity) with respect to variations in rent; and *vice versa*. This seems to be true, at least, of the group of families considered. And whereas the "expected" correlation between food and rent declines numerically as total expenditure increases, the observed correlation shows no such tendency.

These results invite comparison with those previously obtained. We may wish to know, for example, which has a greater effect on food expenditure—a change in rent, total expenditure remaining constant, or a comparable change in income. In order to ensure comparability, the change in total expenditure on all items other than rent must be the same in both cases.

The effect on food expenditure of a change in rent while total expenditure remains constant is measured by the regression coefficient of δ_f on δ_r, that is, by

$$r_{fr.Y}\frac{\sigma_{\delta_f}}{\sigma_{\delta_r}}.$$

Let F denote expenditure on food, as given by the regression equation of food on total expenditure. Similarly let R denote expenditure on rent, as given by the regression equation of rent on total expenditure. Thus F and R are both functions of Y. The effect on food expenditure

of a change in total expenditure sufficient to allow a similar change in the amount spent on all items other than rent is measured by

$$\frac{dF}{d(Y-R)} = \frac{dF}{dY} \div \left(1 - \frac{dR}{dY}\right).$$

Values of this expression, at different levels of total expenditure, can easily be derived from the regression coefficients listed in Table 5.

The results of this comparison are shown in columns (2) and (3) of Table 12. The figures in columns (4) and (5) of the same table show the effects on rent of a change in food expenditure, total expenditure remaining constant, and of a comparable change in total expenditure; the former being measured by $r_{fr.Y}\frac{\sigma_{\delta_r}}{\sigma_{\delta_f}}$ and the latter by $\frac{dR}{d(Y-F)}$.

The values of $r_{fr.Y}\frac{\sigma_{\delta_f}}{\sigma_{\delta_r}}$ and of $r_{fr.Y}\frac{\sigma_{\delta_r}}{\sigma_{\delta_f}}$ relate to families falling within the specified ranges of total expenditure; the values of $\frac{dF}{d(Y-R)}$ and of $\frac{dR}{d(Y-F)}$ relate to the mid-points of each range, except for the two outside groups, where they relate to a total weekly expenditure of 40*s*. and 100*s*. respectively. The Table relates, as before, to families with one child which were renting their houses.

TABLE 12

Associated Variations in Expenditure on Food, Rent and Total Expenditure among Families with 1 Child

(1) Total expenditure per week	(2) $-r_{fr.Y}\frac{\sigma_{\delta f}}{\sigma_{\delta r}}$	(3) $\frac{dF}{d(Y-R)}$	(4) $-r_{fr.Y}\frac{\sigma_{\delta r}}{\sigma_{\delta f}}$	(5) $\frac{dR}{d(Y-F)}$
Under 50*s*.	·073	·494	·179	·044
50–60*s*.	·248	·405	·308	·068
60–70*s*.	·768	·344	·321	·081
70–80*s*.	·199	·281	·139	·091
80–90*s*.	·245	·216	·237	·100
Over 90*s*.	·216	·116	·207	·111

It appears that for most of the families a change in income, or total expenditure, has a greater effect on the amount spent on food than a change in rent while income remains constant; but that, as income increases, the effect of a change in income declines, and eventually becomes less than the effect of a change in rent. We have, indeed, already noticed that the income elasticity of demand for food declines as income increases (see Table 9).

On the other hand, it appears that the amount spent on rent is more likely to be influenced by a change in the amount spent on food, income remaining constant, than by a change in income,* a conclusion which it is not easy to explain. The number of families included in this calculation is in any case too small to enable us to attach much importance to these results.

7. *Households in London*

The sample on which the present study is based included 124 households in the Greater London area. When subdivided by the size of family and the level of total expenditure, the number was found to be too small to justify the full analysis which has been made of the households in the rest of Great Britain. Nevertheless, by comparing the figures for London with those for the rest of the country, it is possible to draw a few tentative conclusions. We shall confine our attention to families with no children (numbering 46) and families with one child (numbering 44).

When the figures of expenditure, shown by the London households, were plotted on diagrams, the points for each size of family did not lie on any clearly defined lines. We would not, in this

* The apparent increase in the effect of a change in income, as we move up the scale, should be disregarded, in view of the low correlation between income and rent (*cf.* Diagram 3).

case, have been justified in assuming a curvilinear relationship between E_r and Y, and we assumed, instead, a linear relationship of the form (1). Nor did it seem worth while computing regression coefficients by the method of least squares. It was decided, therefore, to draw freehand lines of the form:

$$E_r = a_0 + a_1 Y$$

for each of the five main groups of expenditure. It needs to be emphasized that the regression coefficients and the derived functions, for the London households, are subject to very wide margins of error.

Table 13 shows the percentages of total expenditure, derived from the regression lines, and the values of θ and η corresponding to a total expenditure of 72*s*. 8*d*. These figures may be compared with those relating to households in the rest of Great Britain at the same level of total expenditure.

TABLE 13

Percentages of Total Expenditure and Values of θ *and* η *Corresponding to a Total Expenditure of* 72*s*. 8*d*. *Households in Greater London Area*

	Number of children	*Percentage of total expenditure*	$\theta \times 10^3$	η
Food	0	33·8	−·07	·79
	1	36·8	−·03	·92
Rent	0	19·7	−·06	·74
	1	19·7	−·15	·35
Fuel and light	0	8·3	+·02	1·18
	1	7·9	−·08	·14
Clothing	0	7·7	−·00	·96
	1	6·9	+·04	1·47
Sundries	0	30·5	+·23	1·64
	1	28·7	+·22	1·68

As is to be expected, the percentage expenditure on rent is a good deal higher in London than in the rest of Great Britain. The proportionate expenditure on each of the other groups is less in London, except for fuel and light, where the proportions are about the same.

We find, as before, that food, rent and fuel and light are all "necessary" items, while the sundries group is a "luxury"; but food seems to be less necessary in London than elsewhere, despite the fact that a given income represents a lower standard of living in London. Clothing appears to be on the border-line between these two categories; and it is amusing to notice that clothing seems to be more necessary in London than elsewhere.

The income elasticity of demand for food is higher in London, but otherwise the values of η do not show any marked differences, when compared with households in the rest of Great Britain.

PART II.—EXPENDITURE ON CHILDREN IN WORKING-CLASS FAMILIES

"I wish you wouldn't squeeze so," said the Dormouse, who was sitting next to her. "I can hardly breathe."

"I can't help it," said Alice very meekly: "I'm growing."

"You've no right to grow *here*," said the Dormouse.

—*Alice's Adventures in Wonderland.*

1. *Introduction*

1.1. Little attention has hitherto been given to the problem of distinguishing, for families with children, expenditure on the children from expenditure on the rest of the family. Some items, of course, are consumed jointly by adults and children (e.g. most meals), and in such cases

it would clearly be very difficult, if not impossible, to find out exactly how much is consumed by each person. But a family budget enquiry in which householders were carefully instructed to keep separate accounts (so far as possible) for adults and children, and to give full details of all meals consumed by each member of the family, should enable a reasonably good estimate to be made of expenditure which is due to the children. Until such a special enquiry is organized, it will not be possible to obtain any really accurate information on this problem.

1.2. But in the absence of any direct information, it seems worth while examining the possibility of obtaining, by indirect methods, at least a rough idea of the total amount spent on children in working-class families. An attempt to arrive at an estimate is made in the following pages. The method, of which a fuller description is given below, is based on regression analysis, and makes use of the relationships which have been found to obtain, among families which are broadly similar in type, between expenditure on separate items and total household expenditure.

2. *The Methods Employed*

2.1. Let us assume that we have figures of household expenditure for families which are broadly similar in type, but which have different numbers of children. Clearly the amount spent on any particular purpose, e.g. on children, will be closely related to the standard of living which the family enjoys. A family with a large income will, generally speaking, spend more on its children than a family of the same size which has only a small income. Consequently, if a comparison is to be made between families of different size with a view to estimating the amount spent on children, we must ensure that the families have approximately the same standard of living. The problem reduces itself, therefore, to that of finding some method of determining when the standard of living of different sized families can be regarded as equivalent.

To put the problem in a slightly different way: suppose that two families, the first consisting of husband and wife, the second consisting of husband, wife and one child, being otherwise of similar type, are known to enjoy approximately the same standards of living. The difference between their total household expenditure provides an estimate, for that particular standard of living, of the amount spent on the child by the second family.

2.2. With the available data, it is very difficult to find any means by which the standard of living of a given family can be clearly and unambiguously defined. An approximate solution to this problem may, however, be sought along the following lines.*

Several studies of household budgets have shown that, for a group of families with similar needs and similar scales of preference, there is usually a regular relationship between (i) the amount spent on a particular item, and (ii) the level of income, or of total family expenditure.† We consider a group of such families in which the number of adults is the same and their needs are assumed to be similar, but the number of children varies. Let us assume that each family consists of husband and wife plus 0, 1, 2 or more children. From the list of items for which separate figures of expenditure are given, we try to select all those which do not appear to depend, in any way, on the number of children in the family, and which may be assumed, therefore, to be entirely for the use of the husband and wife. The expenditure, which may be denoted by X, on this set of items is taken as defining the standard of living. The total amount spent on all items combined is denoted by Y, and the subscripts 0, 1, 2 . . . are used to indicate the number of children in the family. It was found that, for families having the same number of children, Y could generally be represented by a parabolic function of X, so that equations could be obtained of the form:

$$Y_0 = a_0 + b_0X + c_0X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$Y_1 = a_1 + b_1X + c_1X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$Y_2 = a_2 + b_2X + c_2X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (2) shows, for families with one child, the relationship between total expenditure (Y_1) and expenditure on those items (X) which are taken to define the standard of living. Equation (1)

* The basic idea in this method was first suggested by E. Rothbarth in his "Note on a Method of Determining Equivalent Income for Families of Different Composition," published as Appendix IV to *Wartime Pattern of Saving and Spending*, by Charles Madge, O.U.P., 1943.

† *Cf.* Part I of the present work.

shows the corresponding relationship for families having no children. From the data which were available (see Appendix III), it was possible to obtain such equations for families consisting of husband, wife and 0, 1 or 2 children, but not for larger families.

Using y_1 to denote the amount spent on the first child we have:

$$y_1 = Y_1 - Y_0 = a_1 - a_0 + (b_1 - b_0) X + (c_1 - c_0) X^2 \quad . \quad . \quad . \quad (4)$$

Using y_2 to denote the amount spent on the second child, we have:

$$y_2 = Y_2 - Y_1 = a_2 - a_1 + (b_2 - b_1) X + (c_2 - c_1) X^2 \quad . \quad . \quad . \quad (5)$$

These equations show the functional connection between expenditure on the first or second child and expenditure on those items which depend on the standard of living. It was possible to obtain these relationships only for the first and second children.

2.3. Perhaps the main weakness in this method is the underlying assumption that the tastes and needs of adult members of a household do not vary as the number of children increases. The mother and father of two children, spending the same on themselves as a childless couple, may prefer to distribute this expenditure in a different way, while still enjoying approximately the same standard of living. But without a much more thorough knowledge of family spending habits than we have at present, it is impossible to make any allowance for differences of this kind.

2.4. So far we have given only a brief description, in very broad terms, of the general method of attack. In practice, a number of further problems arose which must now be discussed.

The most difficult problem was that of selecting the list of items (X) which might be assumed to be purchased entirely for the benefit of the adult members of the household, and which could be used to define the standard of living. The procedure adopted was as follows: a large number of different selections were made and, for each selection, the corresponding values of X and Y, families being distinguished according to the number of children, were plotted on a graph. A set of freehand lines were then drawn, showing the approximate relationships between X and Y, for families of different size. From these lines, it was possible to ascertain roughly the form of the connection between Y_0 and X, Y_1 and X, etc.; and hence between y_1, or y_2, and X.

Many of the items which were shown separately in the family budgets could reasonably be assumed to have been purchased entirely (or almost entirely) for the use of the husband and wife. It was found, however, that all the larger selections of items gave quite unplausibly low figures for y_1 and y_2. There are, in fact, reasons for believing, as explained in section 2.5 below, that if X contains *any* items which are consumed partly by the children, the resulting estimates of the amounts spent on children are likely to be too small. The conclusion reached, after a large number of graphs were drawn, was that, in order to avoid obtaining quite unreasonable answers, the items to be included in X would have to be selected from among the following: adults' clothing; tobacco and cigarettes; drink; "other expenditure" (i.e. expenditure not included under separate headings, of which there were about 90). The results obtained by using different selections from this list are illustrated in the Diagrams shown below.

2.5. While it was possible to reject some of the selections on the grounds that the conclusions obtained were quite unreasonable, it was clearly desirable to find an objective method of testing and comparing the validity of the different results. The data made it possible, fortunately, to introduce a rough check on most of the results.

From the family budgets separate figures were given, under the heading of clothing, for weekly expenditure on the following items:

I. Men's clothing.	IV. Repairs to clothing.
II. Women's clothing.	V. Boots and shoes.
III. Children's clothing.	VI. Boot and shoe repairs.

These figures (see Appendix III below) were based on average purchases over a period of 12 months.

Suppose we denote by K_0, K_1 and K_2 the sum of expenditure on items I, II and III, for families with 0, 1 and 2 children respectively. For any given set of items included in X, we obtain the

corresponding values of X and K_0, X and K_1, etc., which can be plotted on a graph. We can then estimate regression lines of the form:

$$K_0 = l_0 + m_0X + n_0X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$K_1 = l_1 + m_1X + n_1X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$K_2 = l_2 + m_2X + n_2X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Writing k_0 and k_1 for the estimated expenditure on clothing for the first and second child respectively, we have:

$$k_1 = K_1 - K_0 = l_1 - l_0 + (m_1 - m_0) X + (n_1 - n_0) X^2 . \quad . \quad . \quad (9)$$

$$k_2 = K_2 - K_1 = l_2 - l_1 + (m_2 - m_1) X + (n_2 - n_1) X^2 . \quad . \quad . \quad (10)$$

These equations enable us to estimate k_1 and k_2 for different values of X. The actual figures of expenditure on clothing for the first and second child can be obtained as the differences (for given values of X) between the total amounts spent on children's clothing in families having 0, 1 or 2 children. By comparing estimates and actual figures, we can form some conclusion about the validity of our assumptions.

This test is applicable so long as X includes expenditure on a number of items. But if, for example, X consists only, or to a large extent, of expenditure on adults' clothing, the test becomes tautologous and cannot be applied. Where it seemed appropriate, the test was applied and the results are shown below. Precise agreement could not, of course, be expected, because, apart from difficulties inherent in the method, the figures of expenditure on children's clothing are so small that even minor errors might easily produce relatively large discrepancies. In all cases the estimated expenditure on children's clothing was smaller than the true figures; and we must now examine why this should be so.

2.6. Suppose that, in making the selection of commodities which are taken to define the standard of living (X), some items are included which should not have been included; which, in fact, are purchased on behalf of the children. This will not affect the relation between X and Y_0, which refers to families with no children.* But, as regards the relation between Y_1 (total expenditure in families with one child) and X, the effect will be that for a given value of Y_1, the value of X will be higher than it should be; and, consequently, for a given value of X the value of Y_1 will be lower than it should be (since it is assumed that, for a given size of family, X and Y increase together, which is generally the case). The same, of course, will be true of the relation between Y_2 (total expenditure in families with two children) and X; and, with two children for whom the goods may be purchased, the error is likely to be greater.

It follows that, for a given value of X, the value of y_1 ($= Y_1 - Y_0$), and also of y_2 ($= Y_2 - Y_1$), are likely to be too small, if X includes any items which are purchased, in whole or in part, on behalf of the children.

This would explain why the different selections of items to be included in X give results, in terms of y_1 and y_2, which, judged by the test described in 2.4, are always too low.

2.7. There is another difficulty which is closely linked with the problem we have just discussed. It might be thought that we could avoid the possibility of including in X items which are actually purchased for the children by considering only a very small group of items (or even one item alone) which are definitely known to be purchased entirely on behalf of adults. But the difficulty here is twofold. On the one hand, as we reduce the number of items, the figures of expenditure become smaller, so that irregularities become more prominent and we can no longer expect to find a simple relation between X and Y. On the other hand, if tobacco and beer, which can be assumed to be purchased entirely for adults,† form a large proportion of the items included in X, we are unable again to expect a regular relationship between X and Y, since figures of expenditure on beer and tobacco, given in family budgets, are notoriously unreliable. And there is, in

* Except in so far as purchases may be made for children in other families.

† Children are defined as persons under 14 (see Appendix III).

fact, substantial evidence that expenditure on these items was seriously understated in the Ministry of Labour family budgets.*

For these reasons, the regression equations are bound to become less reliable as the number of items included in X is reduced.

2.8. A further problem which arose in the practical application of the method was connected with the form of equations (4) and (5), describing respectively the relations between y_1 and X, and y_2 and X. These two equations are derived from equations (1), (2) and (3), as explained in 2.2 above. Now, if equations (1), (2) and (3) are obtained by the ordinary method of fitting regression lines, there is no guarantee that equations (4) and (5) will be such that y_1 and y_2 increase as X increases $\left(\frac{dy_1}{dX} \text{ and } \frac{dy_2}{dX} \text{ positive}\right)$ for the whole range covered by the data. In fact, when equations (1), (2) and (3) were obtained in the ordinary way, it was frequently found that equations (4) and (5) showed negative values of $\frac{dy_1}{dX}$ and $\frac{dy_2}{dX}$ over a substantial part of the range. Such results are not acceptable, and can only be explained by weaknesses in the method, or the data, since we should expect the (absolute) amount spent on children to increase as the standard of living increases.

2.9. It seemed possible that such anomalous results might be avoided if we assumed that y_1 (or y_2) and X were connected by a linear function, instead of a parabolic function, as in (4) or (5). This would be the case if the coefficient of X^2 were the same in equations (1), (2) and (3). Alternative results were therefore obtained, based on equations of the form:

$$Y_0 = \alpha_0 + \beta_0 X + \gamma X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$Y_1 = \alpha_1 + \beta_1 X + \gamma X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$Y_2 = \alpha_2 + \beta_2 X + \gamma X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The seven parameters in these equations were obtained by a simple modification of the usual procedure of fitting regression lines, as explained in Appendix III.

From equations (11), (12) and (13) we obtain the following expressions for y_1 and y_2:

$$y_1 = Y_1 - Y_0 = \alpha_1 - \alpha_0 + (\beta_1 - \beta_0)X \quad . \quad . \quad . \quad . \quad (14)$$

$$y_2 = Y_2 - Y_1 = \alpha_2 - \alpha_1 + (\beta_2 - \beta_1)X \quad . \quad . \quad . \quad . \quad (15)$$

These give y_1 and y_2 as linear functions of X and, in accordance with the argument of 2.8, we should expect both $(\beta_1 - \beta_0)$ and $(\beta_2 - \beta_1)$ to be positive.

Equations of the type (1), (2) and (3) will be referred to as "individual" regression lines, while equations of the type (11), (12) and (13) will be referred to as "combined" regression lines. The latter type can, of course, also be used when applying the test described in 2.5.

2.10. A further point requires to be mentioned here. The data to which the regression lines were fitted consist of mean values of X for given values of Y. But, in fitting the equations, the sums of squares of deviations were always minimized in the direction of Y, so that there is here a certain methodological inconsistency.† Since the figures of total expenditure (Y) shown in the family budgets are more meaningful, as well as more reliable, than the figures relating to a small group of items (X), strictly speaking the regression equations should have been obtained by minimizing the sums of squares of deviations in X. The reasons for preferring the former procedure were that it enabled expressions for y_1 and y_2 to be more easily derived; and, in addition, the procedure could be modified, as described in 2.9, without discarding the parabolic form of equation between Y and X. In practice, the point is hardly likely to be important, especially if there is a high correlation between Y and the mean values of X.

* As may be seen by comparison with figures of national expenditure.

† It would, of course, have been quite wrong to use the mean values of Y for given values of X in the regression analysis; for expenditure on a small group of items is much more liable to be affected by random influences—it is likely, for instance, to fluctuate more from one week to another—than total expenditure. Variations in Y, therefore, have more meaning than variations in X.

2.11. The method described above enables us to obtain equations in which y_1 or y_2 (expenditure on the first or second child) is expressed as a function of X, which is taken to define the standard of living. From these equations we can estimate, for a particular value of X, the corresponding value of y_1 and y_2. But, since the method employed is essentially experimental and since the results of applying the test described in 2.5 were usually not very satisfactory, we cannot expect to reach a precise solution to our original problem. Probably the most that we can hope to obtain are very approximate estimates of the amounts spent on children corresponding, say, to the average standard of living of the families concerned. The procedure which is used to obtain equations such as (14) and (15) should merely be regarded, therefore, as a device for utilizing all the available information in order to obtain single estimates of y_1 and y_2, corresponding to the mean value of X. Any attempt to go further than this, and to consider the variations in expenditure as X changes, must rest on very uncertain foundations.

2.12. So far, we have only discussed the problem of estimating the total amount spent on children, without considering how this total is made up. Exactly the same method of analysis can, of course, be applied, in order to obtain estimates relating to expenditure on particular groups of items. It should be remembered, however, that if the total expenditure devoted to children, under any heading, is small, the results obtained by this method are not likely to be at all reliable.

Another method of determining the composition of the total expenditure on each child, once this is estimated, would be to make use of the results obtained in Part I. Regression lines were obtained, showing, for each size of family, the relation between expenditure on particular items and total household expenditure, of the form:

$$t_0 = f_0(Y_0),\ t_1 = f_1(Y_1),\ \text{etc.},$$

where t_0 denotes the amount spent on a particular item in families with no children, t_1 the amount spent on the same item in families with one child, and so on. If these equations are combined with equations (11), (12) and (13), we can obtain $t_0, t_1, \ldots$ as functions of X and hence obtain the values of $t_0, t_1, \ldots$ corresponding to given values of X. The differences, $t_1 - t_0$, and $t_2 - t_1$, would then provide estimates of the amounts spent, on behalf of the first and second children respectively, on the particular item.

2.13. We have already given several reasons which would explain why the results, which are summarized in the next section, are not very conclusive; but there are other complicating factors, in addition to those already mentioned.

The data which were used consist of average weekly figures of expenditure in groups of families, containing two adults (man and woman) plus 0, 1 or 2 children. We have assumed that each of these groups of families has roughly similar needs, although within each size group there may, in fact, be considerable variations in needs. Such variations may be due to differences in the sex and age of the children, to differences in geographical location, or simply to differences in habits and tastes.

In addition, there may be families which are accustomed to supplying some of their own requirements. Families with allotments, for example, are able to produce some of their own food. Clothing is occasionally made up at home, so that only the materials have to be purchased, while larger families can often economize by allowing clothes to be passed on from one member to another. It did not seem possible to make any allowance for factors of this kind, which may, however, partly account for some of the peculiarities revealed in the results.

Finally, there are fluctuations which are ordinarily associated with the process of sampling. But, in view of the numerous other difficulties, any attempt to estimate sampling errors would, I think, be out of place.

3. *The Results Obtained*

Estimates of Total Expenditure on Children

3.1. As explained in Section 2, the results were found to depend very largely on the choice of items which were assumed to define the standard of living. More than a dozen different selections were made, and the corresponding graphs were drawn to illustrate the relations between Y (total expenditure) and X (expenditure on the particular set of items). Some of these were discarded,

since they gave quite unreasonable results. Diagrams 11, 12 and 13 show the relations obtained, when X includes the following different selections of items:

x_a = weekly expenditure on men's and women's clothes; tobacco (including cigarettes), drink, and sundries (items not separately specified);
x_b = weekly expenditure on men's and women's clothes and tobacco;
x_c = weekly expenditure on men's and women's clothes.

Throughout this survey, unless otherwise stated, the variables are all expressed in terms of weekly expenditure, in pence, and all the figures relate to households in Great Britain, excluding London.

3.2. *Results obtained with the use of x_a.*—Diagram 11 illustrates the relations between Y and x_a for the three sets of families, comprising 2 adults and 0, 1 or 2 children. The following equations are obtained for the "individual" regression lines (see 2.2) between Y and x_a:

$$Y_0 = 201 \cdot 8 + 6 \cdot 5426 x_a - \cdot 0070925 x_a^2 \qquad \text{(i)}$$

$$Y_1 = 232 \cdot 2 + 7 \cdot 4438 x_a - \cdot 0096226 x_a^2 \qquad \text{(ii)}$$

$$Y_2 = 316 \cdot 6 + 6 \cdot 6271 x_a - \cdot 0060407 x_a^2 \qquad \text{(iii)}$$

Hence we obtain the following equations for y_1 and y_2:

$$y_1 = 30 \cdot 4 + \cdot 9012 x_a - \cdot 0025301 x_a^2 \qquad \text{(iv)}$$

$$y_2 = 84 \cdot 4 - \cdot 8167 x_a + \cdot 0035819 x_a^2 \qquad \text{(v)}$$

The values of y_1 and y_2 corresponding to the mean value of x_a in the families considered ($\bar{x}_a$ = 115·3) are:

$$y_1 = 100 \cdot 7, \quad y_2 = 37 \cdot 9.$$

The "combined" regression lines (see 2.9) obtained from the same data are as follows:

$$Y_0 = 192 \cdot 4 + 6 \cdot 6844 x_a - \cdot 0074934 x_a^2 \qquad \text{(vi)}$$

$$Y_1 = 268 \cdot 0 + 6 \cdot 8260 x_a - \cdot 0074934 x_a^2 \qquad \text{(vii)}$$

$$Y_2 = 294 \cdot 3 + 7 \cdot 0352 x_a - \cdot 0074934 x_a^2 \qquad \text{(viii)}$$

from which:

$$y_1 = 75 \cdot 6 + \cdot 1416 x_a \qquad \text{(ix)}$$

$$y_2 = 26 \cdot 3 + \cdot 2092 x_a \qquad \text{(x)}$$

and the values of y_1 and y_2 when $x_a = \bar{x}_a = 115 \cdot 3$ are:

$$y_1 = 91 \cdot 9, \quad y_2 = 50 \cdot 4.$$

Equations (vi), (vii) and (viii) are shown in Diagram 11. Equations (iv) and (v) are not entirely acceptable, since in each case they show y falling, as x_a increases, over a part of the range.

The "individual" and "combined" regression lines both lead, however, to similar estimates at the mean value of x_a: about 8 shillings for the first child and about 4 shillings for the second child.

3.3. These results can now be tested by the method of 2.5. Let K_0, K_1, K_2 (as before) denote expenditure on adults' and children's clothing (excluding repairs and footwear) in families having 0, 1 and 2 children respectively. The equations obtained for the "individual" regression lines are:

$$K_0 = 1 \cdot 1 + \cdot 5288 x_a - \cdot 0006845 x_a^2 \qquad \text{(xi)}$$

$$K_1 = -15 \cdot 3 + \cdot 9437 x_a - \cdot 0021616 x_a^2 \qquad \text{(xii)}$$

$$K_2 = -10 \cdot 8 + \cdot 9068 x_a - \cdot 0019378 x_a^2 \qquad \text{(xiii)}$$

Hence we obtain the following equations for expenditure on children's clothes (excluding repairs and footwear):

$$k_1 = -16{\cdot}4 + {\cdot}4149x_a - {\cdot}0014770x_a^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(xiv)}$$

$$k_2 = 4{\cdot}5 - {\cdot}0369x_a + {\cdot}0002238x_a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xv)}$$

For the "combined" regression lines we obtain:

$$K_0 = -9{\cdot}8 + {\cdot}6942x_a - {\cdot}0011519x_a^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(xvi)}$$

$$K_1 = 1{\cdot}7 + {\cdot}6507x_a - {\cdot}0011519x_a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xvii)}$$

$$K_2 = 1{\cdot}2 + {\cdot}6861x_a - {\cdot}0011519x_a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xviii)}$$

from which:

$$k_1 = 11{\cdot}5 - {\cdot}0435x_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xix)}$$

$$k_2 = -0{\cdot}5 + {\cdot}0354x_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xx)}$$

The values given by these two sets of equations are compared with the actual values of expenditure on children's clothing in Diagram 14. The comparison is made first for k_1, then for $k_1 + k_2$, since the figures relating to k_2 are very small.

The data for each size of family were available for various ranges of total expenditure. In order to obtain the actual figures of expenditure on children's clothing, it was necessary to find the differences between the amounts shown, in the same ranges of income, for different sized families; these differences then had to be adjusted so as to correspond to the same values of x_a (since, for a given level of total expenditure, the value of x_a varies with the size of the family).

It can be seen from Diagram 14 that the values of k_1 and k_2 given by the equations are too small. It follows that the values of y_1 and y_2 given by equations (iv), (v) and (ix), (x) are probably also too small.

3.4. *Results obtained with the use of* x_b.—The relations between Y and x_b (expenditure on adults' clothing and tobacco) for the three sizes of family are shown in Diagram 12. There appears to be less correlation than in the case of x_a, and for this reason it was not worth estimating the "individual" regression lines. The equations obtained for the "combined" regression lines are as follows:

$$Y_0 = 216{\cdot}1 + 5{\cdot}4200x_b + {\cdot}0284651x_b^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(xxi)}$$

$$Y_1 = 279{\cdot}9 + 6{\cdot}2349x_b + {\cdot}0284651x_b^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(xxii)}$$

$$Y_2 = 231{\cdot}4 + 8{\cdot}3536x_b + {\cdot}0284651x_b^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(xxiii)}$$

from which:

$$y_1 = 63{\cdot}8 + {\cdot}8149x_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xxiv)}$$

$$y_2 = -48{\cdot}5 + 2{\cdot}1187x_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xxv)}$$

The coefficient of x_b is much greater in equation (xxv) than in equation (xxiv), but neither figure is reliable (*cf.* the remarks in 2.11).

Corresponding to the mean value of x_b ($\bar{x}_b = 75{\cdot}5$) we have

$$y_1 = 125{\cdot}3, \quad y_2 = 111{\cdot}5.$$

These results are more plausible than those obtained from the use of x_a (see 3.2).

3.5. As the values of x_b are dominated by expenditure on adult's clothing, in this case no definite conclusions can be expected from applying the test of 2.5.

A diagram was drawn showing the relations between K and x_b for the three sets of families. It was not worth estimating regression equations, but freehand lines were drawn through the various points, describing the relations between K_0 and x_b, K_1 and x_b, and K_2 and x_b.

From these freehand lines we were able to obtain the approximate relations between k_1 and x_b, and between k_2 and x_b, which are shown by the lines in Diagram 15. When these estimates are compared with the actual values of k_1 and k_2, which are represented by the dots in the same Diagram, it is found, as indeed we should expect (see 2.5), that they agree more closely than those

obtained with the use of x_a (compare Diagrams 14 and 15). The estimates are still too low, however, and it appears that they should be raised by roughly 25 per cent. With this correction the values of y_1 and y_2, corresponding to the mean value of x_b, would appear to be about 13*s*. and 11*s*. 6*d*. respectively.

3.6. *Results obtained with the use of x_c.*—The results so far obtained, as well as those derived from other selections of items (not shown here), indicate that the estimates can be improved by reducing the number of items which are included in X. As a final example, we shall consider the results obtained when X includes only adult's clothing, expenditure on which is denoted by x_c.

Diagram 13 shows the relations between Y and x_c for the three sizes of family. The correlation is evidently much smaller than in either of the previous cases studied. This is to be expected, since the amount which is common to both Y and X becomes smaller as X is reduced. Further, the relations between the three sets of values do not indicate a regular connection between $y_1 (= Y_1 - Y_0)$, or $y_2 (= Y_2 - Y_1)$, and x_c. It is clear, therefore, that any estimates based on the present data must be very approximate.

The equations obtained in this case, using the method of "combined" regression lines, are as follows:

$$Y_0 = 293 \cdot 2 + 8 \cdot 2864 x_c + \cdot 0466506 x_c^2 \quad \text{(xxvi)}$$

$$Y_1 = 294 \cdot 3 + 12 \cdot 8158 x_c + \cdot 0466506 x_c^2 \quad \text{(xxvii)}$$

$$Y_2 = 328 \cdot 7 + 14 \cdot 4217 x_c + \cdot 0466506 x_c^2 \quad \text{(xxviii)}$$

Diagram 13 shows that, especially in the higher ranges of expenditure, these regression lines do not give a very close fit. The resulting equations for y_1 and y_2 are:

$$y_1 = 1 \cdot 1 + 4 \cdot 5294 x_c \quad \text{(xxix)}$$

$$y_2 = 34 \cdot 4 + 1 \cdot 6059 x_c \quad \text{(xxx)}$$

Corresponding to the mean value of x_c ($\bar{x}_c = 44 \cdot 7$), the values of y_1 and y_2 given by the last equations are:

$$y_1 = 203 \cdot 6, \quad y_2 = 106 \cdot 2$$

or approximately 17 shillings* and 9 shillings respectively.

For reasons which have already been mentioned, these results cannot be tested by the method of 2.5. Comparing the final estimates with those based on x_b, after the latter are corrected for bias (*cf.* 3.5), it is seen that the individual estimates for y_1 and y_2 do not agree very closely, although the two estimates for the sum of $y_1 + y_2$ are very similar. It should be remembered that y_1 and y_2 are not independent; an error in y_1 is likely to produce an error, in the opposite direction, in y_2, so that the estimated expenditure on the two children combined is more reliable than the separate estimates for each child.

3.7. As the points in Diagram 13 appeared to lie on S-shaped curves, equations of the third degree in x_c were also fitted, keeping the coefficients of both x_c^2 and x_c^3 the same for the three sizes of family. But since the correspondence with the observations was only slightly improved, these equations are not given here.

It will also be noticed that, in most of the Diagrams, the point corresponding to the highest range of expenditure falls rather far away from the regression lines. It may be that, at this level, the distribution of expenditure no longer conforms to the pattern revealed by families with smaller incomes and, accordingly, another complete set of calculations were made omitting the highest range of expenditure in each case. Since the results obtained were very similar, on the whole, to those given above, they are not worth quoting in detail.

3.8. *Summary.*—It has already been emphasized that, with the data at our disposal, it is not possible to obtain results having any sort of precision. The most that can be claimed for the foregoing analysis is that it gives a rough idea of the probable orders of magnitude involved.

* An approximate estimate of the amount spent on the first child by families in London gives an almost identical figure. To obtain the same standard of living, the total expenditure required in London appears to have been about 10 per cent. higher than in the rest of the country.

At the average standard of living of the families considered, the amount spent on the first child appears to be of the order of 16 shillings a week, and the amount spent on the second child of the order of 10 shillings a week. But the ratio between these two figures is very uncertain.

The corresponding figure of total expenditure is about 89 shillings a week in families with 2 children. It appears, therefore, that in families with 2 children, the average expenditure on a child is about 40 per cent. of the average expenditure on an adult.

3.9. The meaning of these results requires, perhaps, a further word of explanation. We have attempted to estimate, for a given standard of living, the differences between the average weekly expenditure—

(*a*) of families having no children and families having 1 child (y_1);
(*b*) of families having 1 child and families having 2 children (y_2).

To avoid prolixity, these have been termed estimates of the amounts spent on the first and second child respectively. But it should be realized that y_2 does not refer to a particular child, and that no allowance has been made for differences in age* or sex. In fact, the average age of a single child is probably somewhere between the average age of the elder of two children and the average age of the younger child. This should be borne in mind whenever reference is made to expenditure on the "second" child. The estimates naturally take account also of any economies which may result from extra numbers (*cf.* 2.13).

Estimated Expenditure on Children's Food and Clothing

3.10. Estimates of the amounts spent, on behalf of children, on particular groups of items are bound to be even less reliable than estimates of total expenditure. Bearing in mind that the conclusions are likely to be rather uncertain, it was thought that it might, nevertheless, be of some interest to obtain estimates for a few of the main groups of items, particularly food and clothing. One reason for doing this is that, if we know the proportions spent on the main items, the previous estimates, which are expressed in terms of pre-war prices, can then be converted into terms of current prices.

3.11. Two methods are available. In the first method, to be termed *Method A*, exactly the same procedure that was applied to total household expenditure is applied to expenditure on particular items. In the second method, which will be termed *Method B*, we first estimate from equations (xxvi), (xxvii) and (xxviii) values of Y_0, Y_1 and Y_2 corresponding to the mean value of x_c. These values of Y_0, Y_1 and Y_2 are then substituted in the regression equations connecting expenditure on particular items with total expenditure, which were obtained in Part I. In this way we can estimate the amount spent on a particular item, corresponding to a given standard of living (defined by x_c), for each size of family. The differences provide estimates of the amounts spent on the first or second child. The accuracy of the results will vary for different items, depending on how well the regression lines fit the observations.

These two methods are not independent, since they are both based, one directly and the other indirectly, on the use of x_c.

3.12. *Method A*.—Let F_0, F_1 and F_2 denote the total expenditure on food in families having 0, 1 and 2 children respectively.

Let f_1 and f_2 denote expenditure on food which is attributed to the first and second children respectively.

Let A_0, A_1 and A_2 denote total expenditure on clothing and footwear (including repairs) in families with 0, 1 and 2 children respectively.

Let a_1 and a_2 denote expenditure on clothing and footwear (including repairs) which is attributed to the first and second children respectively.

For the estimates relating to food and clothing, we shall give the results obtained by the method of "combined" regression lines, based on equations of the type (11), (12) and (13), using first x_b and then x_c.

* An attempt was made to ascertain to what extent expenditure on children varied with the age of the child. But when the families were subdivided according to the age of the children, the numbers in each "cell" were too small to be of any use.

3.13. *Results obtained with the use of x_b.*

For food expenditure we obtain the following equations:

$$F_0 = 85 \cdot 1 + 3 \cdot 5643 x_b - \cdot 0090627 x_b^2 \quad \text{(xxxi)}$$

$$F_1 = 127 \cdot 6 + 3 \cdot 6669 x_b - \cdot 0090627 x_b^2 \quad \text{(xxxii)}$$

$$F_2 = 155 \cdot 1 + 3 \cdot 7189 x_b - \cdot 0090627 x_b^2 \quad \text{(xxxiii)}$$

from which:

$$f_1 = 42 \cdot 5 + \cdot 1026 x_b \quad \text{(xxxiv)}$$

$$f_2 = 27 \cdot 5 + \cdot 0520 x_b \quad \text{(xxxv)}$$

For clothing, the coefficient of x_b^2 was found to be negligible and the equations obtained are as follows:

$$A_0 = -3 \cdot 8 + \cdot 9173 x_b \quad \text{(xxxvi)}$$

$$A_1 = 3 \cdot 4 + 1 \cdot 0143 x_b \quad \text{(xxxvii)}$$

$$A_2 = -2 \cdot 9 + 1 \cdot 2690 x_b \quad \text{(xxxviii)}$$

from which:

$$a_1 = 7 \cdot 2 + \cdot 0970 x_b \quad \text{(xxxix)}$$

$$a_2 = -6 \cdot 3 + \cdot 2547 x_b \quad \text{(xl)}$$

Corresponding to the average value of x_b ($\bar{x}_b = 75 \cdot 5$), we have:

$$f_1 = 50 \cdot 2 \qquad a_1 = 14 \cdot 5$$

$$f_2 = 31 \cdot 4 \qquad a_2 = 12 \cdot 9$$

If these estimates are increased, as before (*cf.* 3.5), by 25 per cent., the results are approximately as follows:

$$f_1 = 5\tfrac{1}{4} \text{ shillings} \qquad a_1 = 1\tfrac{1}{2} \text{ shillings}$$

$$f_2 = 3\tfrac{1}{4} \text{ shillings} \qquad a_2 = 1\tfrac{1}{4} \text{ shillings}$$

It can be seen from Diagrams 16 and 17 that the above equations do not, especially in the case of food, fit the observations very well. There is naturally a closer correspondence in the case of clothing, since x_b includes expenditure on adults' clothing.

3.14. *Results obtained with the use of x_c.*

The equations obtained for food expenditure are as follows:

$$F_0 = 102 \cdot 3 + 5 \cdot 4337 x_c - \cdot 0241594 x_c^2 \quad \text{(xli)}$$

$$F_1 = 155 \cdot 4 + 5 \cdot 6966 x_c - \cdot 0241594 x_c^2 \quad \text{(xlii)}$$

$$F_2 = 196 \cdot 7 + 5 \cdot 4586 x_c - \cdot 0241594 x_c^2 \quad \text{(xliii)}$$

from which:

$$f_1 = 53 \cdot 1 + \cdot 2629 x_c \quad \text{(xliv)}$$

$$f_2 = 41 \cdot 3 - \cdot 2380 x_c \quad \text{(xlv)}$$

For clothing, the following equations are obtained:

$$A_0 = 2 \cdot 3 + 1 \cdot 4412 x_c - \cdot 0019268 x_c^2 \quad \text{(xlvi)}$$

$$A_1 = 7 \cdot 9 + 1 \cdot 8000 x_c - \cdot 0019268 x_c^2 \quad \text{(xlvii)}$$

$$A_2 = 11 \cdot 7 + 1 \cdot 9964 x_c - \cdot 0019268 x_c^2 \quad \text{(xlviii)}$$

and hence:

$$a_1 = 5 \cdot 6 + \cdot 3588 x_c \quad \text{(xlix)}$$

$$a_2 = 3 \cdot 8 + \cdot 1964 x_c \quad \text{(l)}$$

The foregoing equations are compared with the actual values in Diagrams 18 and 19. Corresponding to the average value of x_c ($\bar{x}_c = 44\cdot7$) we have:

$$f_1 = 64\cdot9 \qquad a_1 = 21\cdot6$$

$$f_2 = 30\cdot7 \qquad a_2 = 12\cdot6$$

As in the case of the results relating to total expenditure, the estimates based on x_c are, on the whole, higher than those based on x_b. When the latter are corrected for bias, the two sets of estimates agree fairly well.

3.15. It is not, of course, claimed that any of the above equations provides an adequate description of the relation between expenditure on children's food, or clothing, and the standard of living. They are given here for purposes of illustration, and are used only in order to obtain estimates corresponding to the average standard of living of the families considered. In particular, the negative value of the coefficient of x_c in equation (xlv) is not acceptable.

3.16. *Method B.*—The values of Y obtained by substituting $x_c = \bar{x}_c = 44\cdot7$ in equations (xxvi), (xxvii) and (xxviii) are as follows:

$$Y_0 = 756\cdot8 \qquad Y_1 = 960\cdot4 \qquad Y_2 = 1066\cdot6.$$

Substituting these values in the regression equations, of which the coefficients are given in Table 5 (Part I, p. 363), we obtain the following estimates for the amounts spent on children's food and clothing:

$$f_1 = 80\cdot1 \qquad a_1 = 25\cdot6$$

$$f_2 = 34\cdot0 \qquad a_2 = 15\cdot1$$

where f_1, f_2, a_1, a_2 denote the same quantities as before.

The regression equations connecting food and clothing with total expenditure fit the observations fairly well for families with 0, 1 and 2 children, the fit being slightly better in the case of food.

Comparing these results with those based on Method A with the use of x_c (see 3.14), we notice that Method B gives higher estimates in all cases; but the estimate of food expenditure on the first child is the only case where the difference is substantial.

3.17. *Summary.*—In considering these estimates, it should be remembered that an error in one direction in f_1 (for instance) would produce an error in the opposite direction in f_2; and that the estimated expenditure on the two children together is always more reliable than the separate estimates for each child. It seems unlikely, in fact, that the amounts spent on food for the first and second children would have the ratio indicated by the "B" estimates. All we can probably say is that, in families with two children, the average amount spent on food for each child appears to be between 4 shillings and 5 shillings, at the average standard of living of the families considered—that is, approximately a third of the total expenditure on each child; and that the average expenditure on clothing appears to be about one-eighth of the total expenditure on each child.

The value of F_2 corresponding to the mean value of x_c ($\bar{x}_c = 44\cdot7$) is found from equation (xliii) to be 32*s*. 9*d*. The results imply that, at this level, the average amount spent on food for a child is about 35–40 per cent. of the average amount spent on food for an adult.

Estimated Expenditure on Other Items

3.18. It is possible, finally, to make estimates for a few of the remaining groups of items, but it should be remembered that, where the total expenditure is small, the estimates are bound to have a wide margin of error. The following additional groups of items were considered; rent, fuel and light, household goods, medical expenses and voluntary insurance.

For each of the first two children estimates were made, corresponding to the mean value of x_c, by means of both the Methods A and B and the full results are summarized in Table 14 below. For Method A, since it was not worth estimating the various regression equations, the figures were plotted on graphs and freehand lines were drawn as closely as possible through the points relating to each set of families. The points were found to be scattered rather widely about the

lines, thus emphasizing that no precision can be claimed for any of these results. The estimated amounts spent on behalf of each of the first two children, corresponding to the average value of x_c, were then read off the diagrams.

TABLE 14

Estimated Average Weekly Expenditure on Children

	Method A			Method B		
	1st child	2nd child	Total, 1st and 2nd children	1st child	2nd child	Total, 1st and 2nd children
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
Food	5 5	2 7	8 0	6 8	2 10	9 6
Clothing	1 10	1 1	2 11	2 2	1 3	3 5
Rent	1 0	10	2 0	1	9	10
Fuel and light	10	6	1 4	8	7	1 3
Household goods*	1 8	1 2	2 10	1 2	1 6	2 8
Medical†	7	3	10	7	3	10
Voluntary insurance‡	7	4	11	10	2	1 0
Remaining items	5 1	2 1	7 0	4 10	1 6	6 4
Total	17 0	8 10	25 10	17 0	8 10	25 10

* Furniture, carpets, drapery, pottery, brushes, ironmongery, etc.
† Medicines, doctor, dentist, hospital funds, etc.
‡ Excluding National Insurance, Trade Unions and Friendly Societies.

The B estimates for rent and fuel and light were based on the regression equations previously estimated (see Table 5, p. 363). For the remaining cases the amounts spent on each group were plotted against total expenditure, freehand lines were drawn through the observations and the required estimates were then read off the diagrams.

In order to allow for the different proportions of families which owned, or were buying, their own houses, the rent figures were adjusted, each figure being divided by the proportion of families which were renting their houses.* Rent was a case in which the correlations were rather low, and the estimates are therefore associated with very wide margins of error.

3.19. It has already been emphasized that the estimates obtained for the combined expenditure on the first and second children are more reliable than the separate estimates for each child. Except in the cases of rent and food, the estimates of total expenditure on both children, obtained by the two methods, agree remarkably well. Although the two methods are not completely independent, these results enable us to have more confidence than we might otherwise have had in the general method here employed.

Estimates in Terms of 1946 *Prices*

3.20. All the foregoing estimates are expressed in terms of prices ruling in 1937–38, at the time of the Ministry of Labour family budget enquiry. These estimates may now be converted into terms of recent prices. For this purpose we shall use the price indices relating to 1946, which are obtainable from the White Paper on National Income and Expenditure (Cmd. 7099).

When the price indices for food, clothing, rent, fuel and light and other items are weighted by the proportions spent for both children together on each of these groups, the general price index shows an increase between 1938 and 1946 of 46 per cent. if the A estimates are used, and 48 per cent. if the B estimates are used.

Hence, using the estimates given in 3.6, the equivalent expenditure on two children, in terms of 1946 prices, would be in the region of 38*s*. a week, at the average standard of living of the families concerned. This estimate corresponds to a total expenditure, in families having two children, of about £7 a week.

* Very similar results were obtained when the rent figures were left unadjusted. The figures of total expenditure, used in the earlier sections of Part II, included the unadjusted figures for rent. Both the unadjusted (*a*) and the adjusted (*b*) figures are given in Appendix III.

4. *Summary and Main Conclusions*

4.1. From the Ministry of Labour working-class family budget enquiry of 1937–38, detailed figures of expenditure were obtained for families consisting of two adults (one man and one woman) and varying numbers of children, the data being classified according to total expenditure. The figures which were found to be suitable for analysis were drawn from about 650 families of industrial workers in Great Britain (excluding London), having not more than two children. From the various items for which separate figures of expenditure were given several different selections were made of those commodities, the consumption of which seemed to be closely related to the standard of living and independent of the number of children. A study was then made of the relationships between the average figures of expenditure on these commodities and total household expenditure, for three groups of families, containing either 0, 1 or 2 children (under 14 years of age). At any given standard of living total household expenditure must increase with the number of children. By comparing the relationships obtained for the different sized families, it was possible to arrive at approximate estimates, corresponding to a given standard of living, of that part of total expenditure which could be explained by the presence of one or two children. Estimates were also made of the average expenditure, attributable to the children, on various groups of items, such as food and clothing.

4.2. The results were found to depend, to a large extent, on the particular group of commodities, expenditure on which was assumed to define the standard of living. Because of the difficulties inherent in the problem and the experimental character of the methods employed, several different estimates were made, on various assumptions.

The most acceptable estimates, which can, however, only be regarded as approximate, were those obtained when the standard of living was defined by expenditure on adults' clothing (x_c). At the average standard of living, so defined, total household expenditure in families having two children amounted to about 89*s*. a week, and it was estimated that about 26*s*. a week was spent on the two children. The amount spent on the first child appeared to be rather more than on the second child, but separate estimates for each child could not be made with any confidence.

In terms of 1946 prices, the equivalent expenditure on two children would be about 38s. a week out of a total family expenditure of about £7 a week.

These estimates are subject to numerous qualifications, which have been mentioned in previous sections. In particular, no allowance could be made for differences in the age or sex of the children, and the results should be interpreted in the light of the remarks in 3.9.

APPENDIX I—REGRESSION LINES FOR FAMILY EXPENDITURE

Consider a group of n families, all having the same number of children. Let Y denote total expenditure and let E_r denote the amount spent on the r^{th} item by a given family. Let Σ denote summation over all different families and let $\sum_r$ denote summation over all different items.

We assume that the relation between E_r and Y is of the form:

$$E_r = a_r + b_rY + c_rY^2$$

Since $\sum_r E_r = Y$, we must have:

$$\frac{d}{dY}\sum_r E_r = \sum_r b_r + 2Y\sum_r c_r = 1,$$

and

$$\frac{d^2}{dY^2}\sum_r E_r = 2\sum_r c_r = 0.$$

In other words, the values of a_r, b_r and c_r must be such that the following conditions are satisfied:

$$\sum_r c_r = 0, \quad \sum_r b_r = 1 \text{ and } \sum_r a_r = 0.$$

Suppose that the regression coefficients are determined by the ordinary condition of least squares, namely that

$$\Sigma(E_r - a_r - b_r Y - c_r Y^2)^2$$

should be a minimum.

This condition leads to the following equations:

$$\Sigma(E_r - a_r - b_r Y - c_r Y^2) = 0,$$

$$\Sigma Y(E_r - a_r - b_r Y - c_r Y^2) = 0,$$

$$\text{and} \quad \Sigma Y^2(E_r - a_r - b_r Y - c_r Y^2) = 0.$$

We have 3 such equations for any given value of r. Combining the equations for all different values of r, we have:

$$\Sigma(Y - \Sigma_r a_r - Y \Sigma_r b_r - Y^2 \Sigma_r c_r) = 0,$$

$$\Sigma(Y^2 - Y \Sigma_r a_r - Y^2 \Sigma_r b_r - Y^3 \Sigma_r c_r) = 0,$$

$$\text{and} \quad \Sigma(Y^3 - Y^2 \Sigma_r a_r - Y^3 \Sigma_r b_r - Y^4 \Sigma_r c_r) = 0.$$

Writing

$$\Delta = \begin{vmatrix} n & \Sigma Y & \Sigma Y^2 \\ \Sigma Y & \Sigma Y^2 & \Sigma Y^3 \\ \Sigma Y^2 & \Sigma Y^3 & \Sigma Y^4 \end{vmatrix}$$

we obtain

$$\Sigma_r a_r \,.\, \Delta = \begin{vmatrix} \Sigma Y & \Sigma Y & \Sigma Y^2 \\ \Sigma Y^2 & \Sigma Y^2 & \Sigma Y^3 \\ \Sigma Y^3 & \Sigma Y^3 & \Sigma Y^4 \end{vmatrix} = 0,$$

$$\Sigma_r b_r = \frac{\Delta}{\Delta} = 1,$$

and

$$\Sigma_r c_r \,.\, \Delta = \begin{vmatrix} n & \Sigma Y & \Sigma Y \\ \Sigma Y & \Sigma Y^2 & \Sigma Y^2 \\ \Sigma Y^2 & \Sigma Y^3 & \Sigma Y^3 \end{vmatrix} = 0.$$

Hence the values of a_r, b_r and c_r, obtained by the method of least squares, satisfy the required conditions.

APPENDIX II—METHOD OF FITTING "COMBINED" REGRESSION LINES

The method of fitting "combined" regression lines, referred to in 2.8, is as follows:

Let summation extending over families with no children be denoted by Σ_0, over families with one child by Σ_1, and over families with two children by Σ_2. Let the values of Y, X and X^2, measured from their respective means in each set of families, be denoted by ζ, τ and ξ. For

example, $\sum_{0}\tau$ refers to values of X for families with no children, measured from the mean value in these families. The regression equations can then be written in the form:

$$\zeta_0 = \beta_0\tau + \gamma\xi$$

$$\zeta_1 = \beta_1\tau + \gamma\xi$$

$$\zeta_2 = \beta_2\tau + \gamma\xi$$

To obtain the four parameters, β_0, β_1, β_2 and γ, in these equations, we impose the usual condition that

$$\sum_{0}(\zeta_0 - \beta_0\tau - \gamma\xi)^2 + \sum_{1}(\zeta_1 - \beta_1\tau - \gamma\xi)^2 + \sum_{2}(\zeta_2 - \beta_2\tau - \gamma\xi)^2$$

should be a minimum. By partial differentiation, we obtain:

$$\sum_{0}\tau\zeta_0 - \beta_0\sum_{0}\tau^2 - \gamma\sum_{0}\tau\xi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$\sum_{1}\tau\zeta_1 - \beta_1\sum_{1}\tau^2 - \gamma\sum_{1}\tau\xi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$\sum_{2}\tau\zeta_2 - \beta_2\sum_{2}\tau^2 - \gamma\sum_{2}\tau\xi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

$$\sum_{0}\xi\zeta_0 + \sum_{1}\xi\zeta_1 + \sum_{2}\xi\zeta_2 - \beta_0\sum_{0}\xi\tau - \beta_1\sum_{1}\xi\tau - \beta_2\sum_{2}\xi\tau - \gamma(\sum_{0}\xi^2 + \sum_{1}\xi^2 + \sum_{2}\xi^2) = 0 \quad . \quad (d)$$

From equations (a), (b) and (c) we have:

$$\beta_r = \frac{\sum_{r}\tau\zeta_r - \gamma\sum_{r}\tau\xi}{\sum_{r}\tau^2} \qquad (r = 0, 1, 2)$$

and hence:

$$\beta_r \,.\, \sum_{r}\xi\tau = \frac{\sum_{r}\tau\zeta_r - \gamma\sum_{r}\tau\xi}{\sum_{r}\tau^2} \,.\, \sum_{r}\xi\tau \qquad (r = 0, 1, 2)$$

Equation (d) now becomes:

$$\sum_{0}\xi\zeta_0 + \sum_{1}\xi\zeta_1 + \sum_{2}\xi\zeta_2 - \left(\frac{\sum_{0}\tau\zeta_0 \,.\, \sum_{0}\xi\tau}{\sum_{0}\tau^2} + \frac{\sum_{1}\tau\zeta_1 \,.\, \sum_{1}\xi\tau}{\sum_{1}\tau^2} + \frac{\sum_{2}\tau\zeta_2 \,.\, \sum_{2}\xi\tau}{\sum_{2}\tau^2}\right)$$

$$= \gamma\left[(\sum_{0}\xi^2 + \sum_{1}\xi^2 + \sum_{2}\xi^2) - \left\{\frac{(\sum_{0}\xi\tau)^2}{\sum_{0}\tau^2} + \frac{(\sum_{1}\xi\tau)^2}{\sum_{1}\tau^2} + \frac{(\sum_{2}\xi\tau)^2}{\sum_{2}\tau^2}\right\}\right].$$

This equation gives a figure for γ which is then substituted in the expressions for β_r.

APPENDIX III—DESCRIPTION OF THE DATA

In 1937–38 an inquiry was held by the Ministry of Labour into working-class family expenditure in the United Kingdom. Budgets were collected for each of 4 weeks, at quarterly intervals. A detailed account of the inquiry, together with the main results, was given in the *Ministry of Labour Gazette* for December, 1940, and January-February, 1941.

In order to obtain separate figures for families of different size, the Ministry of Labour made a special analysis of the budgets supplied by about 800 households, the heads of which were engaged in an industrial or commercial occupation. These households were drawn from about 2000 which had furnished weekly records of clothing expenditure for a whole year; each household included two adults (one man and one woman) and 0, 1, 2 or 3 children under 14 years, but no other person. Budgets supplied by households in which the number of children varied during the year, or which were living in houses provided by the employers, free of rent, were excluded. For present purposes, the figures relating to the four weeks of the inquiry were averaged.

In the large majority of households only one person was earning a wage or salary. Although it was believed that all households in which both adults were earning money had been deliberately excluded from the sample, it came to light, at a late stage in the work, that some of these households—namely those containing children—had not been excluded. It is unlikely, however, that their inclusion would affect the results of the regression analysis, where variations in income are taken into account. And all such families were excluded from the calculations described in Section 1 of Part I, where they would have influenced the comparative figures of average expenditure of families of different size.

Where the number of households in a particular expenditure group was too small to provide reliable averages, particulars of such households were omitted from the data originally supplied by the Ministry of Labour, and they were therefore excluded from the regression analysis in Parts I and II. They have, however, been included in the calculations described in Section 1 of Part I.

In view of the relatively high expenditure on rent and travelling in London, the budgets supplied by households living in London were tabulated separately from those living in other parts of the country. The families included in the regression analysis were distributed as shown below:

	London	*Rest of Great Britain*
2 adults (man and woman) and no other person	46	281
2 ,, (,, ,,) ,, 1 child under 14	44	222
2 ,, (,, ,,) ,, 2 children under 14	20	143
2 ,, (,, ,,) ,, 3 ,, ,, 14	14	58
Total	124	704

For each group, detailed figures of expenditure on various items (numbering about 90) were given for different levels of total expenditure, classified in ranges of ten shillings (30*s*. to 40*s*., 40*s*. to 50*s*., . . . 90*s*. to 100*s*., 100*s*. and over). It was found, however, that the number of families in London and the number of 3-children families in the rest of Great Britain were too small to provide reliable averages when the data were cross-classified in this way. The main analysis was consequently restricted to households in the rest of Great Britain, and families with 3 children were included only in Part I.

The following tables contain all the data relating to these families which were directly employed in the present analysis.

TABLE I.—*Average Expenditure (in pence) of Households containing 2 Adults and No Children, in Great Britain, excluding London*

Range of Total Expenditure	30–40*s.*	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	*Over* 100*s.*
Number of Families	11	33	53	64	38	33	16	33
FOOD								
Meat, bacon and fish	53	72	87	94	100	99	103	118
Dairy products	59	69	78	80	83	98	103	100
Fruit and vegetables	8	20	24	29	34	39	49	62
Other food	65	75	86	101	109	112	102	130
Total	185	236	275	304	326	348	357	410
RENT								
(*a*) Average payment on rent or purchase of dwelling, all families	73	92	102	123	128	177	159	169
(*b*) Average payment on rent by families renting dwellings	73	95	106	123	128	171	163	181
FUEL AND LIGHT	49	54	64	67	70	72	84	89
CLOTHING								
(x_c) Men's and women's apparel	16	25	39	48	55	62	65	98
Children's apparel	0	0¾	0½	1½	1½	1	1	1¼
Repairs and footwear	9	12	15	18	20	21	24	26
Total	25	38	54	67	77	84	90	125
SUNDRIES								
Tobacco and drink	12	24	33	37	49	41	59	70
Household goods*	9	8	11	22	31	48	63	200
Medical†	3	5	12	14	14	29	24	61
Voluntary insurance‡	5	11	15	25	23	42	42	75
Rest	51	80	100	135	175	174	250	328
Total	80	128	171	233	292	334	438	734
TOTAL EXPENDITURE								
Including rent (*b*)	412	551	670	794	893	1,009	1,132	1,539
SELECTED GROUPS								
(x_a) Adults' clothing, tobacco, beer and miscellaneous§	31	56	82	98	130	131	186	298
(x_b) Adults' clothing and tobacco	27	48	64	76	87	96	103	140

* Furniture, carpets, drapery, pottery, brushes, ironmongery, etc.
† Medicine, doctor, dentist, hospital funds, etc.
‡ Excluding National Insurance, Trade Unions and Friendly Societies.
§ Items not separately specified in the family budgets.

TABLE II.—*Average Expenditure (in pence) of Households containing 2 Adults and 1 Child, in Great Britain, excluding London*

Range of Total Expenditure	30–40*s.*	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	*Over* 100*s.*
Number of Families	6	10	37	47	46	32	18	26
FOOD								
Meat, bacon and fish	41	63	71	96	94	101	105	119
Dairy products	54	62	82	98	106	104	110	120
Fruit and vegetables	11	18	25	32	38	42	47	52
Other food	71	86	93	118	120	130	128	136
Total	177	229	271	344	358	377	390	427
RENT								
(*a*) Average payment on rent or purchase of dwelling, all families	95	109	118	97	126	145	156	162
(*b*) Average payment on rent by families renting dwellings	95	109	111	99	121	132	136	174
FUEL AND LIGHT	47	48	57	65	70	73	84	110
CLOTHING								
(x_c) Men's and women's apparel	14	19	24	33	43	55	60	61
Children's apparel	5½	5½	11	13¼	15½	17	19¾	25½
Repairs and footwear	10	12	16	22	22	24	26	31
Total	29	37	51	68	80	96	105	118
SUNDRIES								
Tobacco and drink	10	21	28	35	35	45	66	75
Household goods*	17	14	9	20	37	33	32	103
Medical†	7	7	10	11	18	25	26	54
Voluntary insurance‡	12	21	20	22	27	33	29	91
Rest	32	66	99	122	149	188	240	329
Total	78	129	166	210	266	324	393	652
TOTAL EXPENDITURE Including rent (*b*)	426	552	656	786	895	1,002	1,108	1,481
SELECTED GROUPS								
(x_a) Adults' clothing, tobacco, beer and miscellaneous§	28	45	66	80	103	127	151	240
(x_b) Adults' clothing and tobacco	23	39	49	63	72	91	109	110

* Furniture, carpets, drapery, pottery, brushes, ironmongery, etc.
† Medicine, doctor, dentist, hospital funds, etc.
‡ Excluding National Insurance, Trade Unions and Friendly Societies.
§ Items not separately specified in the family budgets.

TABLE III.—*Average Expenditure (in pence) of Households containing 2 Adults and 2 Children, in Great Britain, excluding London*

	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	*Over* 100*s.*
Range of Total Expenditure	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	*Over* 100*s.*
Number of Families	8	23	35	26	20	13	18
FOOD							
Meat, bacon and fish	71	73	80	99	105	109	111
Dairy products	83	86	102	117	120	120	139
Fruit and vegetables	19	22	27	35	39	44	46
Other food	94	111	116	129	129	148	147
Total	267	292	325	380	393	421	443
RENT							
(*a*) Average payment on rent or purchase of dwelling, all families	74	105	120	121	132	155	135
(*b*) Average payment on rent by families renting dwellings	74	102	116	116	118	152	120
FUEL AND LIGHT	49	54	66	72	74	85	99
CLOTHING							
(x_c) Men's and women's apparel	16	20	28	33	49	53	62
Children's apparel	8¾	11¾	16¾	22¼	24¾	27½	32¾
Repairs and footwear	15	16	23	24	29	28	34
Total	40	48	68	79	103	109	129
SUNDRIES							
Tobacco and drink	16	24	36	36	47	54	43
Household goods*	17	15	19	27	36	73	223
Medical†	10	8	12	14	28	20	43
Voluntary insurance‡	18	22	19	26	30	34	73
Rest	71	88	110	137	177	191	346
Total	132	157	196	240	318	372	728
TOTAL EXPENDITURE							
Including rent (*b*)	562	653	771	887	1,006	1,139	1,519
SELECTED GROUPS							
(x_a) Adults' clothing, tobacco, beer and miscellaneous§	37	51	81	88	122	136	233
(x_b) Adults' clothing and tobacco	31	40	57	64	84	93	100

* Furniture, carpets, drapery, pottery, brushes, ironmongery, etc.
† Medicine, doctor, dentist, hospital funds, etc.
‡ Excluding National Insurance, Trade Unions and Friendly Societies.
§ Items not separately specified in the family budgets.

TABLE IV.—*Average Expenditure (in pence) of Households containing 2 Adults and 3 Children, in Great Britain, excluding London*

	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	Over 100*s.*
Range of Total Expenditure	40–50*s.*	50–60*s.*	60–70*s.*	70–80*s.*	80–90*s.*	90–100*s.*	Over 100*s.*
Number of Families	15	9	9	7	13	—	5
FOOD							
Meat, bacon and fish	59	97	100	104	115	—	131
Dairy products	76	97	107	110	137	—	176
Fruit and vegetables	11	22	32	30	43	—	62
Other food	109	116	142	141	146	—	178
Total	255	332	381	385	441	—	547
RENT							
(*a*) Average payment on rent or purchase of dwelling, all families	87	77	94	124	124	—	193
(*b*) Average payment on rent by families renting dwellings	87	82	94	124	136	—	186
FUEL AND LIGHT	55	46	66	65	68	—	75
CLOTHING							
(*xc*) Men's and women's apparel	18	20	33	26	35	—	38
Children's apparel	12¾	19½	17½	27¾	30½	—	75¼
Repairs and footwear	20	20	29	27	30	—	38
Total	51	60	80	81	96	—	152
SUNDRIES							
Tobacco and drink	21	27	31	43	47	—	38
Household goods*	9	24	13	15	28	—	21
Medical†	8	19	12	13	31	—	15
Voluntary insurance‡	12	12	9	27	27	—	60
Rest	47	75	97	138	143	—	244
Total	97	157	162	236	276	—	378
TOTAL EXPENDITURE Including rent (*b*)	545	677	783	891	1,017	—	1,338

* Furniture, carpets, drapery, pottery, brushes, ironmongery, etc.
† Medicine, doctor, dentist, hospital funds, etc.
‡ Excluding National Insurance, Trade Unions and Friendly Societies.

Notes on the Diagrams

All the diagrams relate to households in Great Britain, excluding London.

In Diagrams 2 to 19 the scales represent average weekly expenditure, in pence.

Crosses (× × ×)	and lines shown thus:	————	relate to families with	no children.
Circles (o o o)	,, ,,	- - - - - - -	,, ,,	one child.
Dots (● ● ●)	,, ,,	- · - · - · -	,, ,,	two children.
Squares (□□□)	,, ,,	· · · · · ·	,, ,,	three children.

Y denotes total expenditure (in pence).

x_a denotes expenditure on adults' clothing, tobacco, drink and sundries (not separately specified).

x_b denotes expenditure on adults' clothing and tobacco.

x_c denotes expenditure on adults' clothing.

The equations of the regression lines shown in Diagrams 2 to 10 are given in Part I (Table 5).

The equations of the lines shown in Diagrams 11 to 14 and 16 to 19 are given in Part II, Section 3.

Diagram 14 is explained in paragraph 3.3 and Diagram 15 in paragraph 3.5 of Part II.

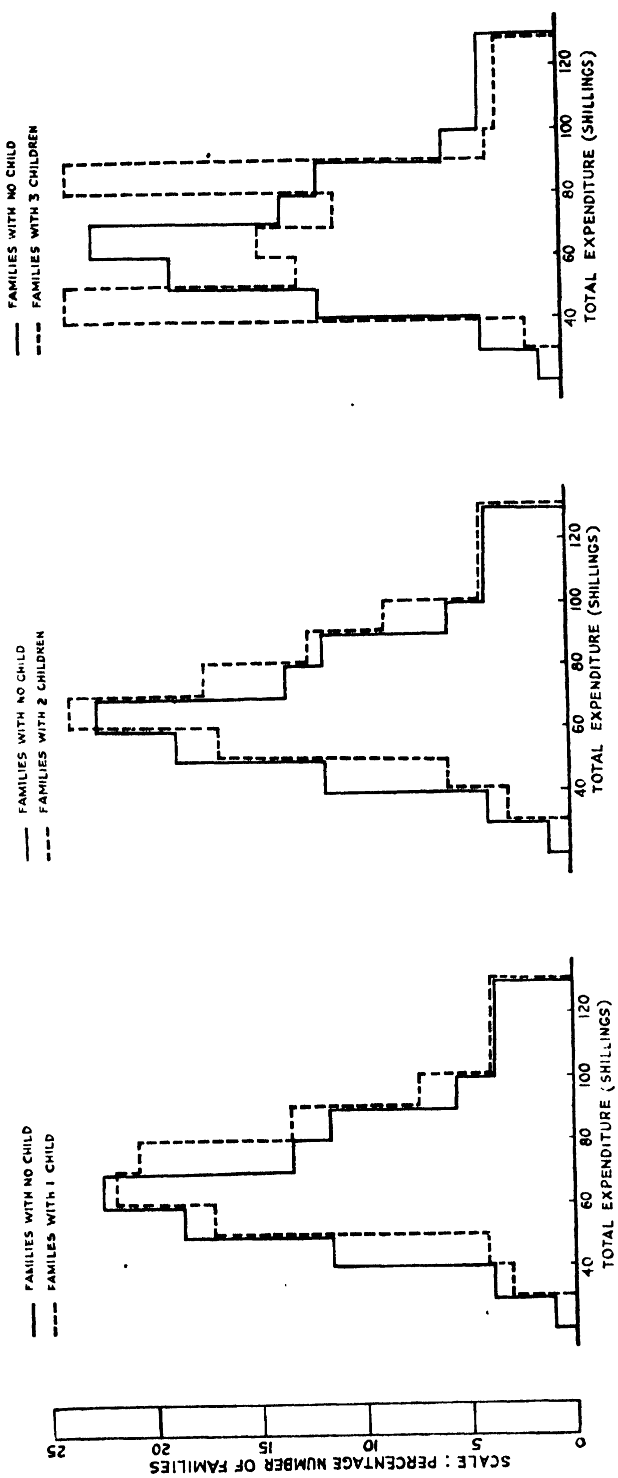


DIAGRAM 1.—*Distribution of families by total expenditure (excluding London)*

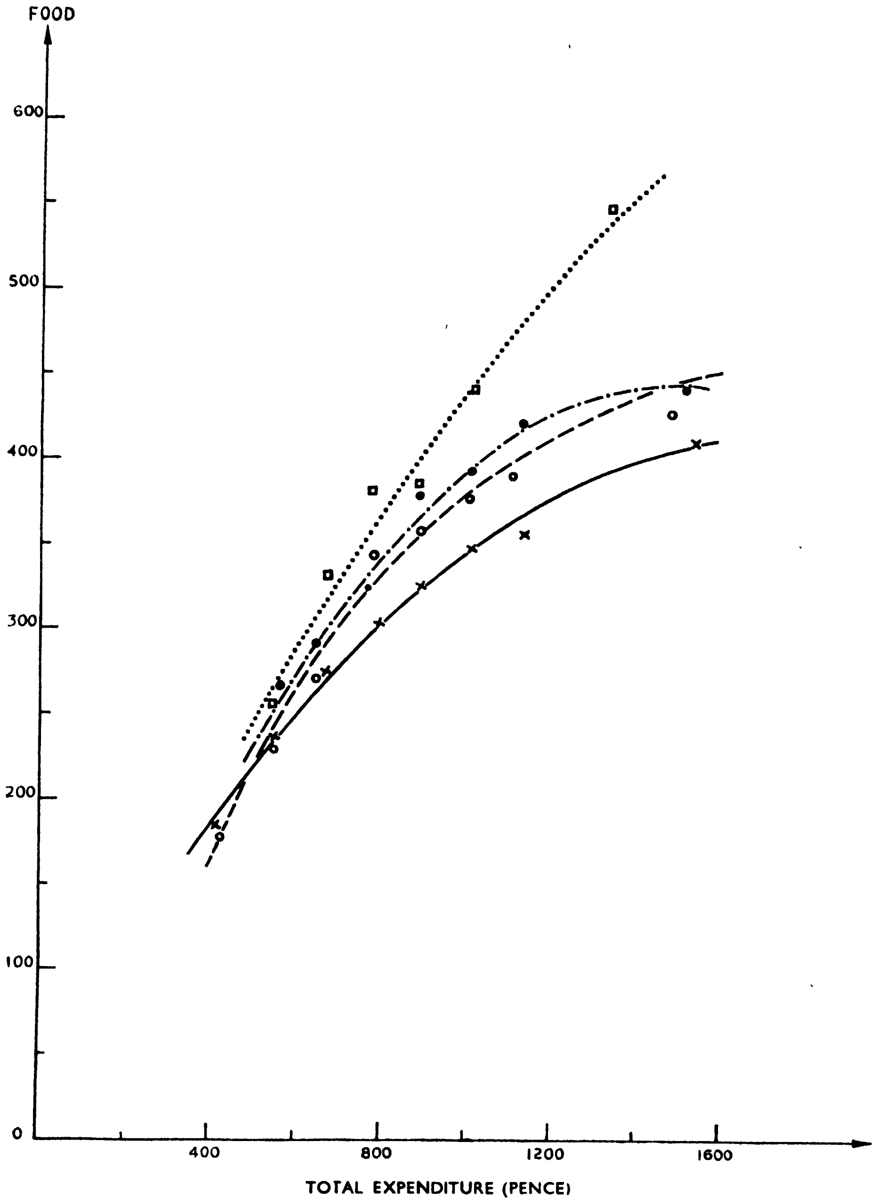

FOOD
600
500
400
300
200
100
0
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 2.

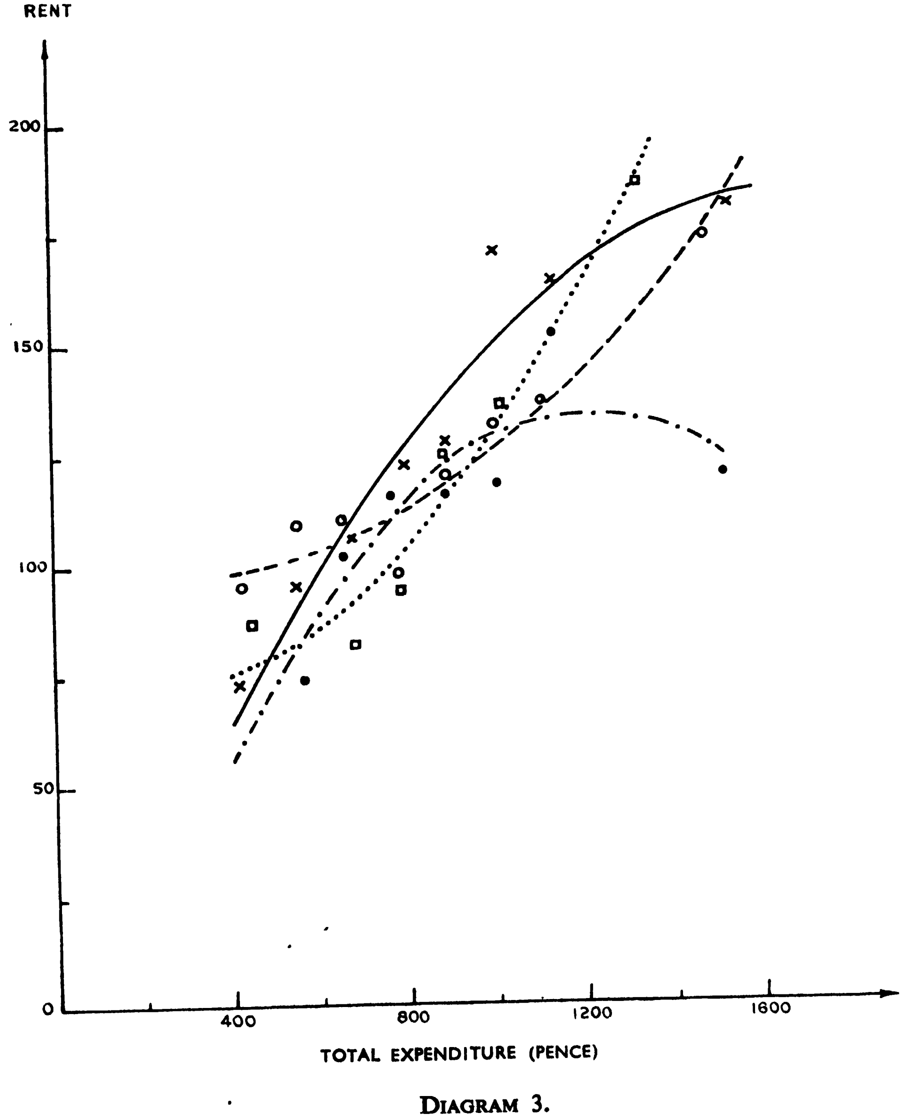

RENT
200
150
100
50
0
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 3.

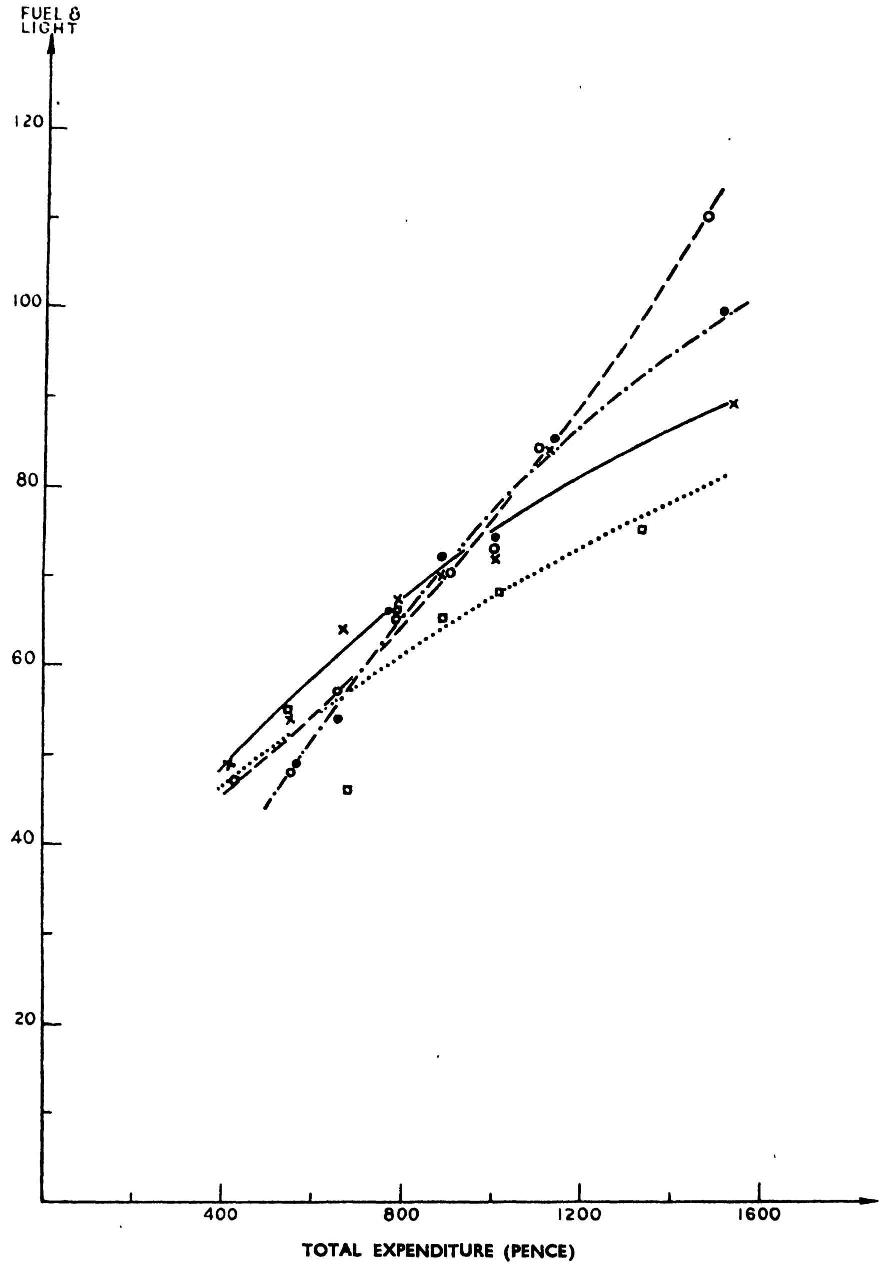

FUEL & LIGHT
120
100
80
60
40
20
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 4

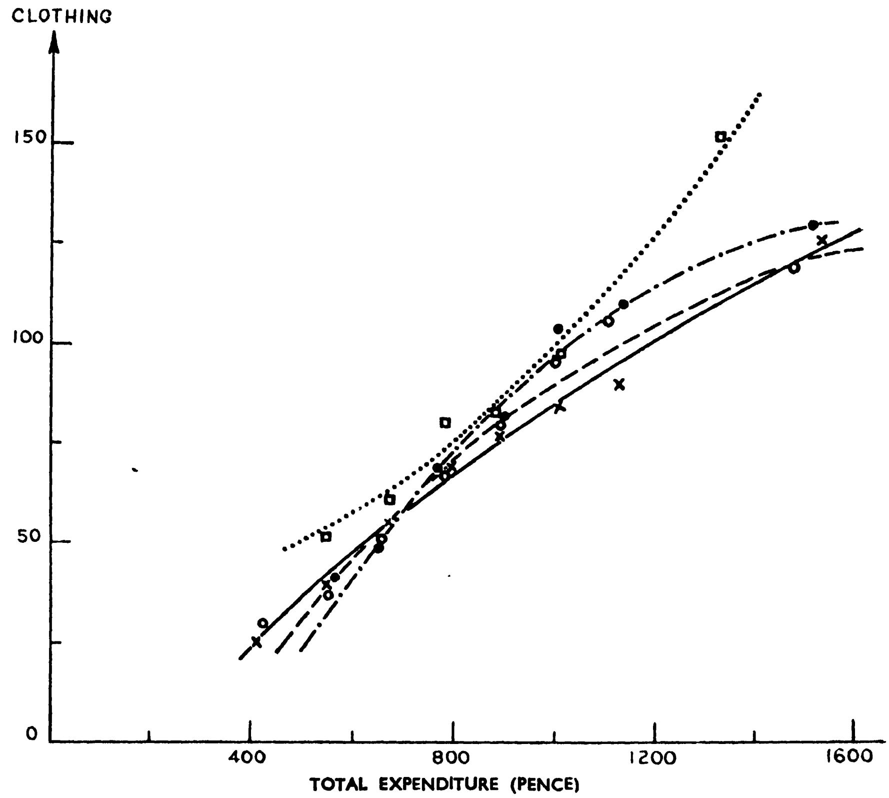

CLOTHING
150
100
50
0
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 5.

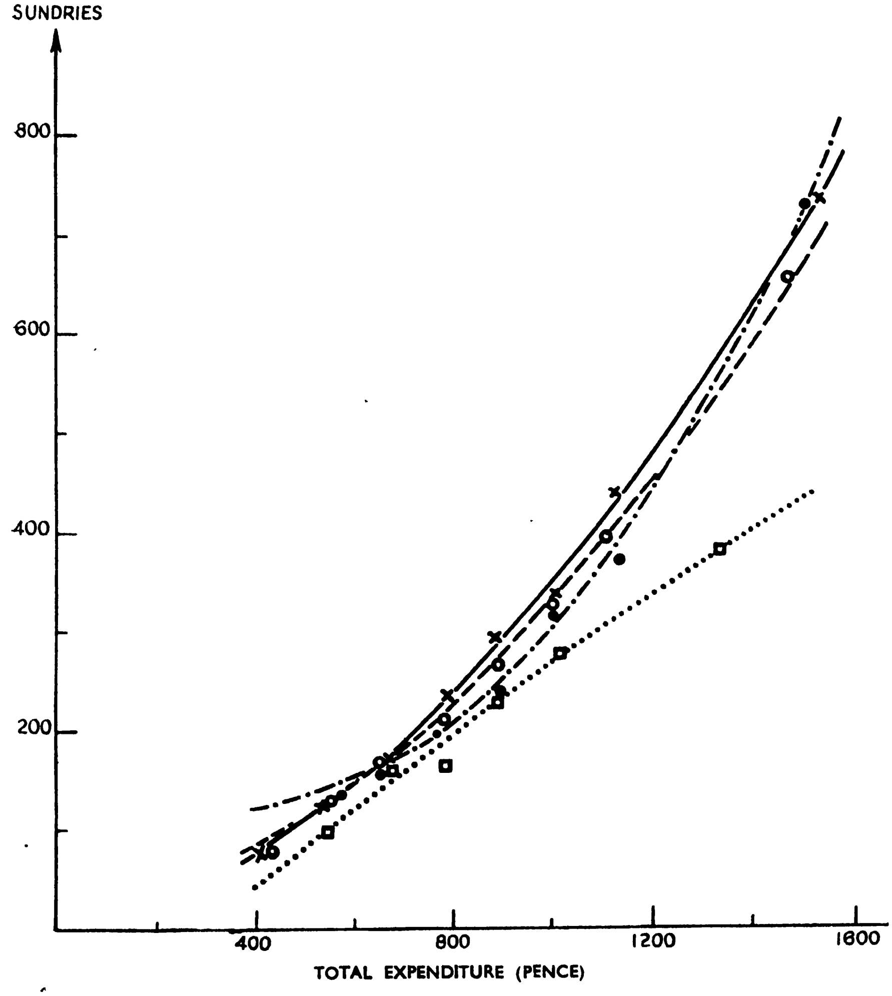
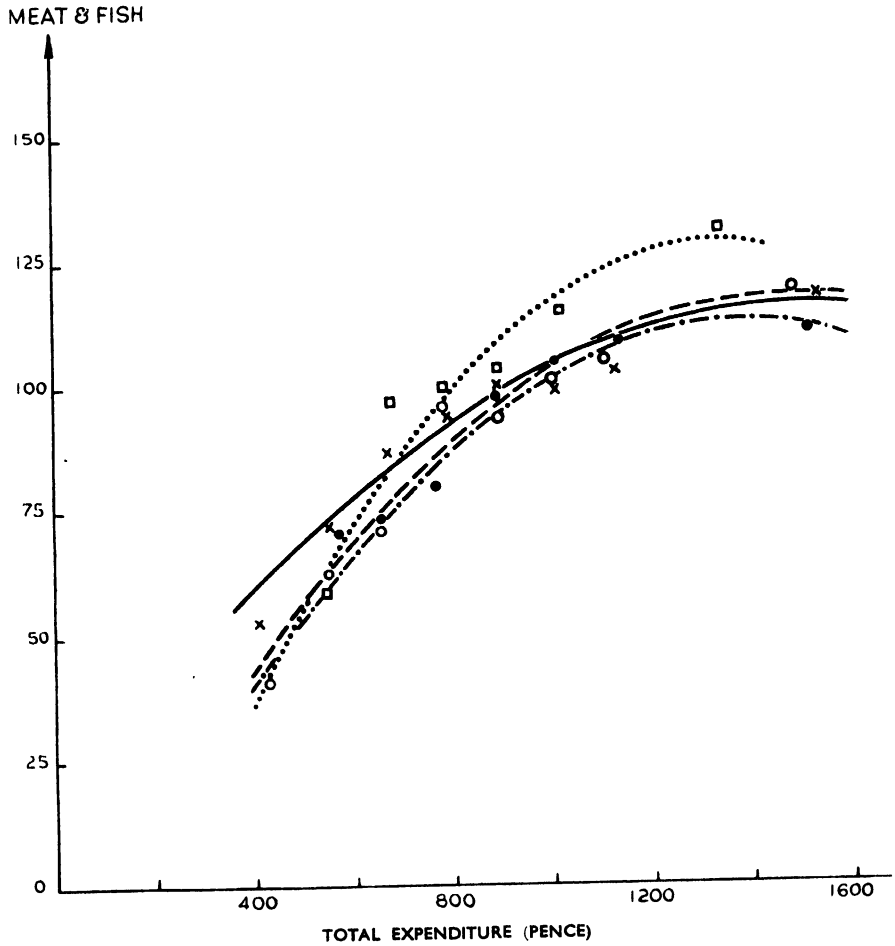

SUNDRIES
800
600
400
200
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 6.



MEAT & FISH
150
125
100
75
50
25
0
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 7.

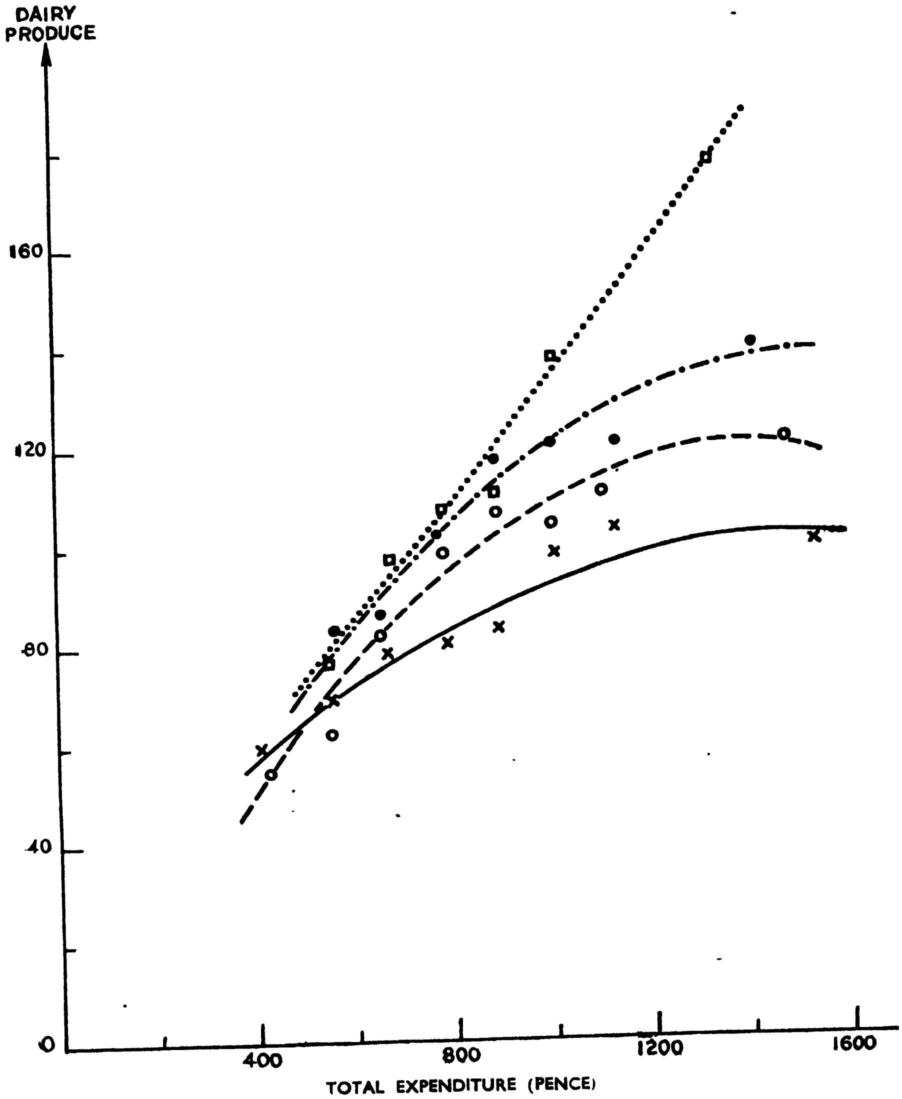

DAIRY PRODUCE
160
120
80
40
0
400
800
1200
1600
TOTAL EXPENDITURE (PENCE)

DIAGRAM 8.

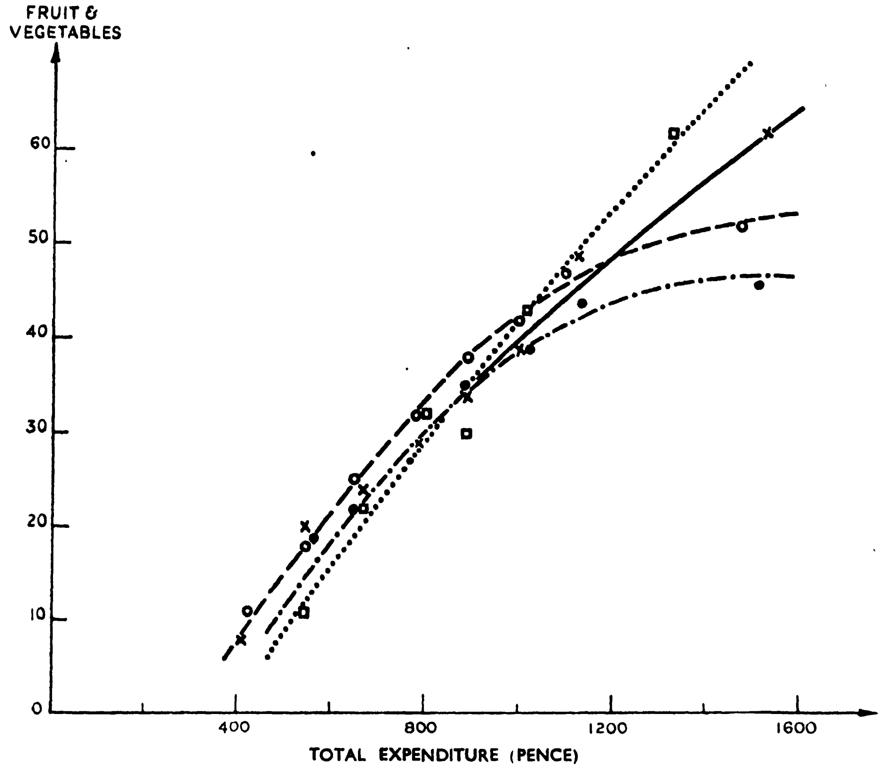


DIAGRAM 9.

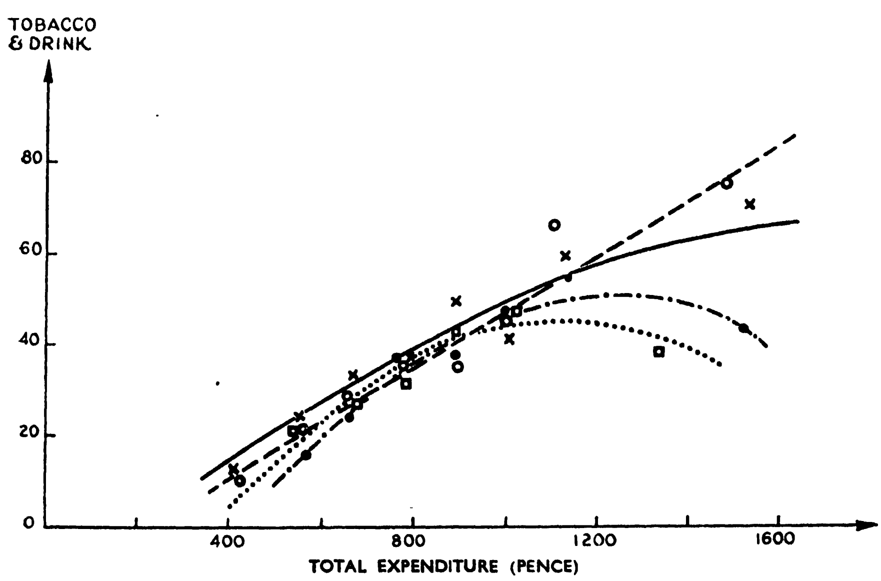


DIAGRAM 10.

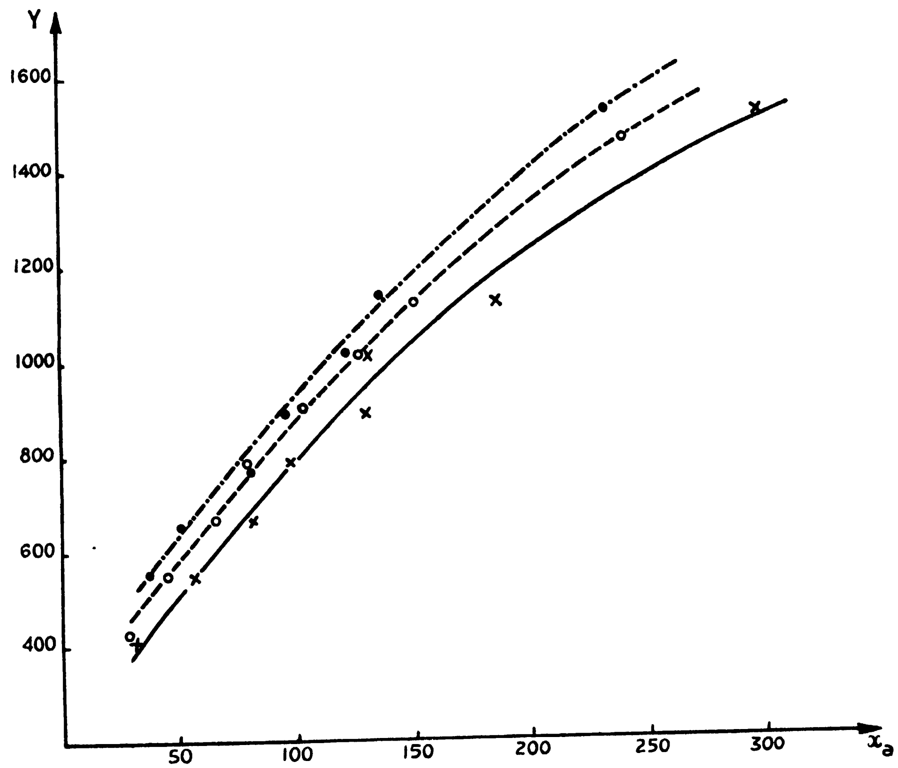


DIAGRAM 11.

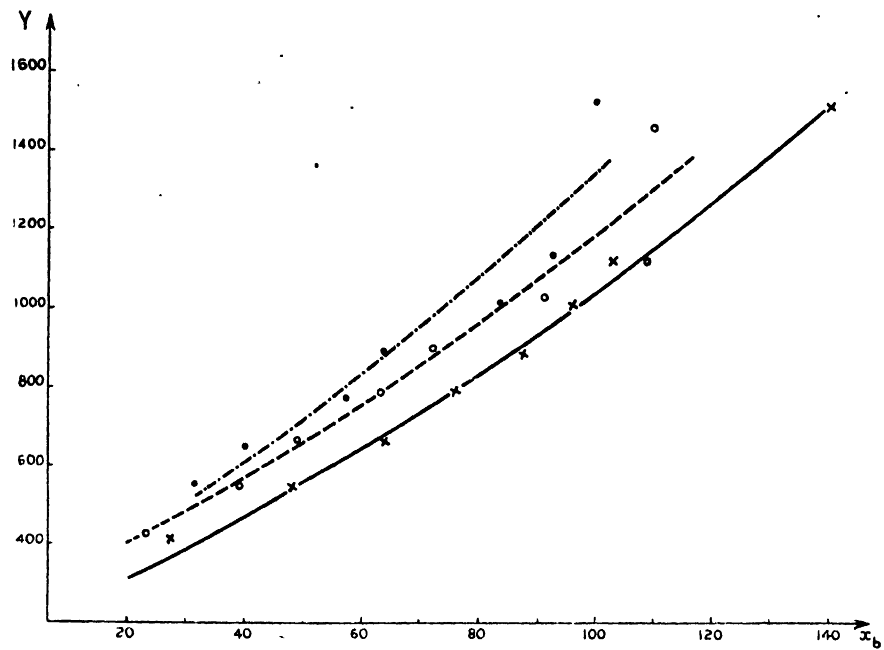


DIAGRAM 12.

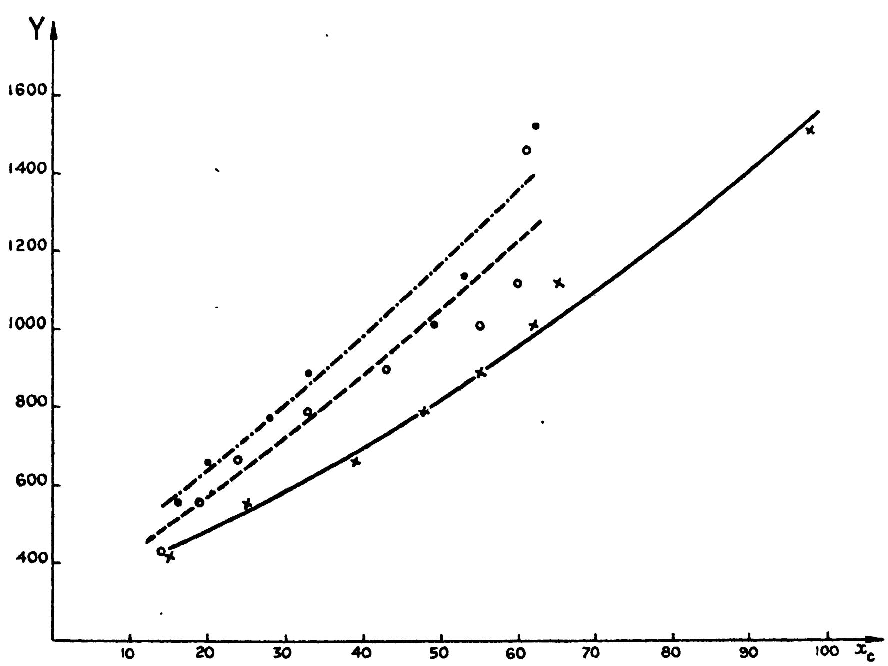


DIAGRAM 13.

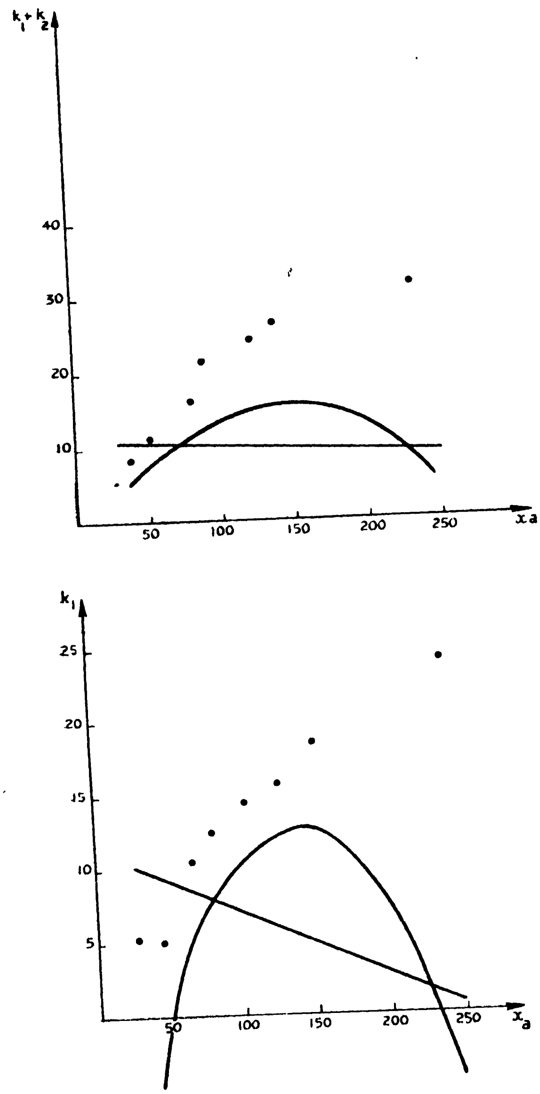
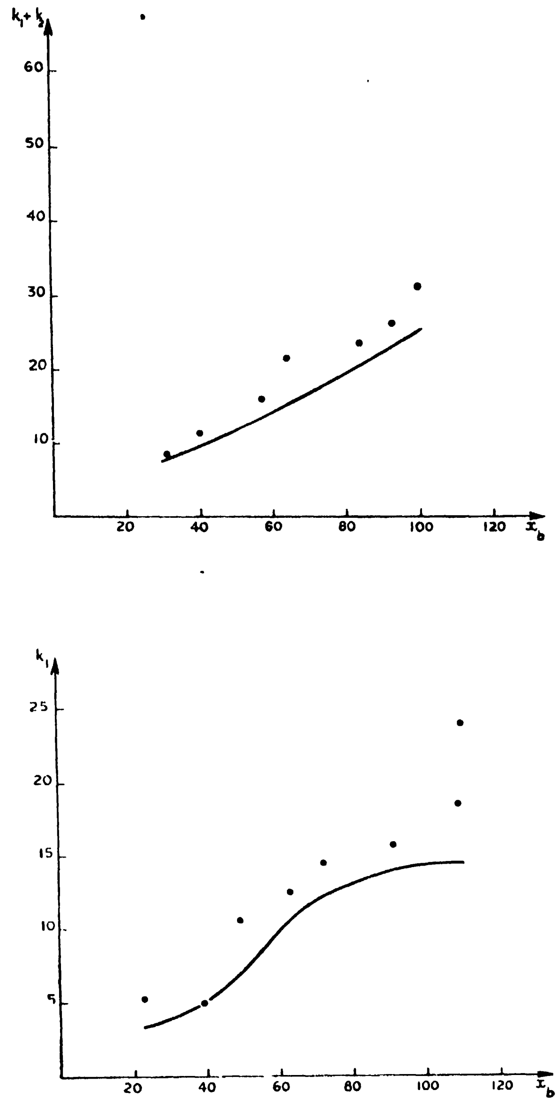

k_1+k_2
40
30
20
10
50
100
150
200
250
x_a
k_1
25
20
15
10
5
50
100
150
200
250
x_a

DIAGRAM 14.



k_1+k_2
60
50
40
30
20
10
20
40
60
80
100
120
x_b
k_1
25
20
15
10
5
20
40
60
80
100
120
x_b

DIAGRAM 15.

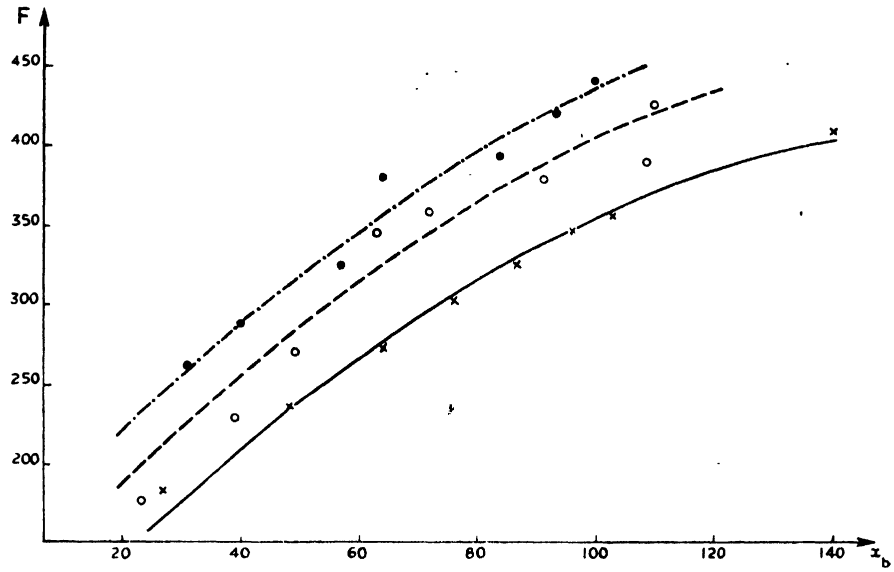


DIAGRAM 16.

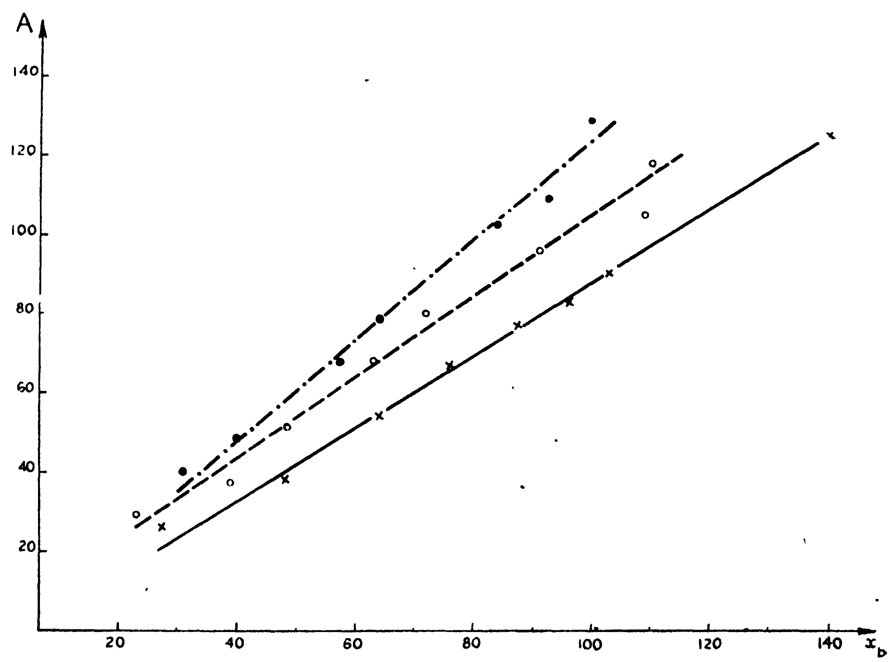


DIAGRAM 17.

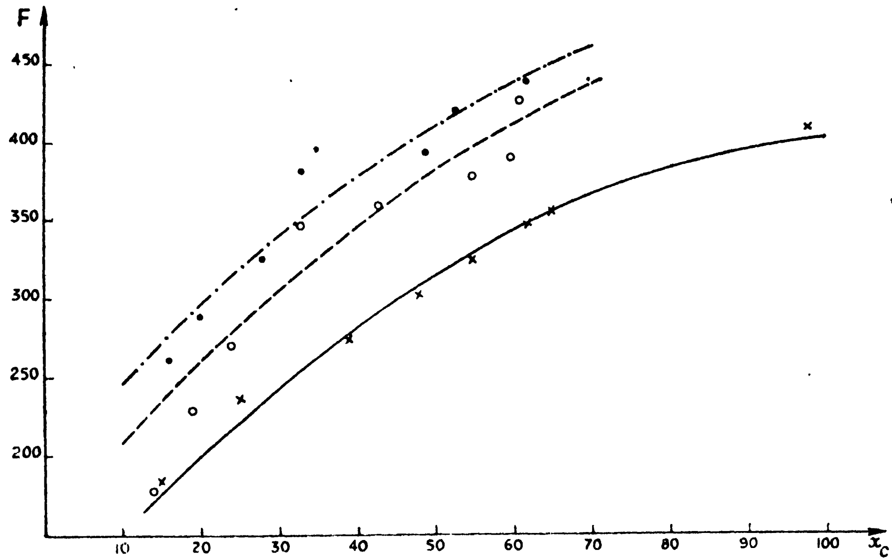


DIAGRAM 18.

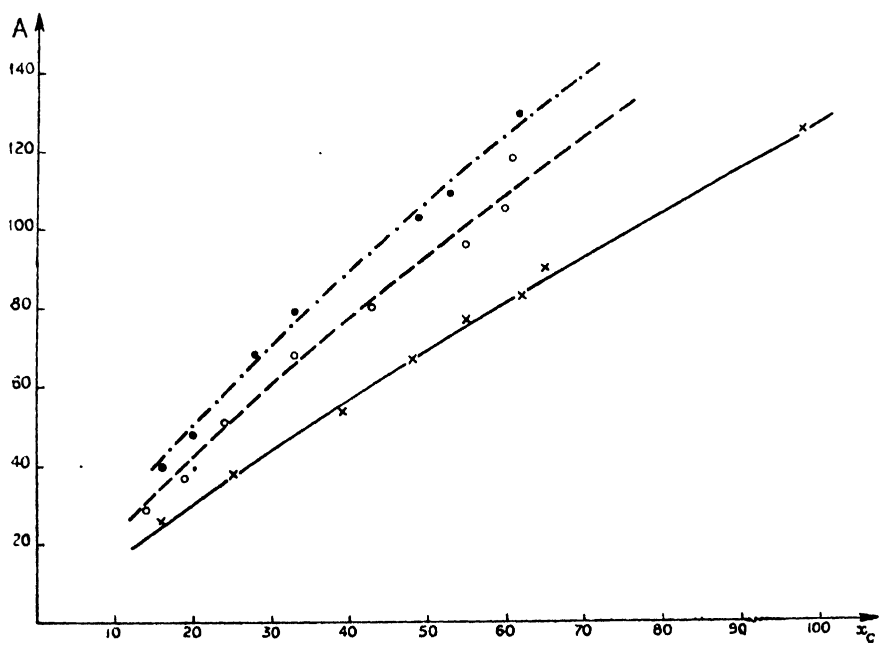


DIAGRAM 19.

Discussion on Mr Nicholson's Paper

Professor R. G. D. Allen: It is with great pleasure that I move a vote of thanks to Mr. Nicholson for a paper which obviously has involved a great deal of labour, and although much of the work may be regarded as experimental, his conclusions give us much to think about. Perhaps I should include in this vote of thanks two official organizations—the Royal Commission on Population for commissioning the second part of this paper, and the Ministry of Labour for making available the detailed budgets of the 1937–38 inquiry. That particular inquiry was unfortunate in that the war came and prevented a full analysis being made, and it is only recently that additional work has been done on the very valuable material that lies in the budgets. I understand the Ministry of Labour have made further analyses, and I hope these will be published.

Mr. Nicholson's sample concerns 704 families in Great Britain, apart from the London group. These families have been selected to contain two adults—one man and one woman—and a varying number of children, but it seems to me, looking at Appendix III—it is not clear, I think, in the text—that an additional factor has been used in selection. All these families were amongst those making returns every week during the year on clothing expenditure, and I wonder whether this might not have imported some bias. Families who find themselves able every week to produce a clothing budget are not necessarily typical of the whole group of families. Perhaps Mr. Nicholson would comment on this.

Another point that arises is that, with only some 700 families in the sample, the cross-classification which Mr. Nicholson has employed involves a considerable number of classes with very few entries. The general difficulty is to get enough entries for a fairly fine analysis without increasing unduly the over-all size of the sample. This is an important matter, particularly if budget inquiries are to be taken at all frequently, as I hope they will be. One way out of this difficulty would be to have a basic sample of large size elaborately stratified on a national basis. Such a national sample could then be used for the taking of small sub-samples, with a variable sampling fraction to ensure that small groups (such as families with three or more children) are adequately represented.

The analysis in Part I takes cognizance of two factors—family income and family size and composition. There are, of course, other factors affecting family expenditure which Mr. Nicholson, like other investigators, has ignored. There are regional differences, including differences in relative prices, as well as differences in habits, tastes, and so on. But Mr. Nicholson gives a more elaborate analysis than has usually been made in the past. A simple method often used is to take equivalence scales, counting, e.g., an adult male as 1·0, an adult female as 0·83, and so on, and so to eliminate the effect of family size. That is a point I will come to later, and one which interests me very much. In Table 3, giving the distribution of families by total expenditure, there is a small point which I am sure Mr. Nicholson can clear up readily. I cannot make the numbers in this Table agree with those given in the detailed tables in Appendix III, and I wonder where the apparent discrepancy arises.

I am interested in the frequency distributions of Diagram 1, obtained from Appendix III. As the shift is made from no-children to one-child and two-children families the income distribution shifts almost unchanged to the right, which means that some fairly simple explanation can be offered. The simplest form would be that, as the family gets older, the number of children increases and the income increases, and therefore the whole distribution shifts to the right. I was interested also to see the amount of shift. Looking at Diagram 1, the amount of shift to the right as the first child is added seems to be rather less than 10*s*. a week. Does that cover the additional cost of the child? This is a question not posed by Mr. Nicholson, who was concerned with getting an estimate of the cost of the first and second child, and not with the further question whether the cost is in fact covered by additional income. The estimated weekly cost of the first child from his Table 14, is 17*s*.; this is subject to a good margin of error, but in any case it would seem to be definitely more than the 10*s*. The shift to the right with the first child suggests that the cost of the first child is apparently not met by the additional income which the family enjoys. Of course, the needs and tastes of the family with one child are not necessarily the same as those of the family with no children. But that is the very broad result, for what it is worth.

The distribution of expenditure among families with three children suggests a bi-modal case—some families have children because they can afford them, and some in spite of the fact that they cannot afford them. But the results for the three-children families are based on only a small number, and little reliance should be placed on them.

In Table 5 Mr. Nicholson sets out the results of his fitting of regression coefficients on expenditure for different family sizes. He finds quite definite evidence, as we see from the diagrams, that the regression is curvilinear for a family of given size. This is something Professor Bowley

and I began to suspect in our earlier work in this direction. In Mr. Nicholson's work this curvilinearity appears for families of a given size, but when families of different sizes are merged it becomes less marked, and the linear pattern becomes more prominent. If these regression equations are to be applicable down to very small incomes, the b_0 coefficient should be zero. With b_0 zero we have a much simpler case, where the coefficient θ, which gives the distinction between "necessities" and "luxuries," becomes equal to b_2. In fact, the distinction between "necessities" and "luxuries" then turns upon convexity, and one can say that a "luxury" is an item where expenditure goes on increasing indefinitely with income, but "necessary" expenditure reaches a peak as income is increased.

Actually, the b_0's are far from zero and not easy to interpret. For example, for families with two children, the negative b_0's might imply that, as income is reduced, nothing is spent on food, rent, fuel and light, and clothing, and even on many sundries such as tobacco and drink. It might appear that expenditure is on items such as medicine—that a family with two children, and with low income, lives almost entirely on aspirins! All this simply stresses the fact that the regressions must be strictly confined to the ranges of income to which they are fitted.

In Part II of the paper the problem that Mr. Nicholson has considered is, in fact, a general problem which has been examined much more in another connection. The problem is to define equivalent standards of living for families in different positions or in different situations. Once equivalent standards of living are found, the corresponding incomes can be deduced and comparisons made. Take, for example, the problem of the definition of an index of the cost of living. If we have families in different price situations and can define equivalent standards of living, the ratio of the incomes which correspond gives us the cost of living index. Mr. Nicholson has tackled the same kind of problem, but in a different context. He has looked at his data and has tried to find expenditures specifically on children and specifically on adults, and to use these to define equivalent standards of living. Of course he finds little of help to him in these particular data. Very few expenditures can be described as specifically on children. Children's clothing is one of them; but even this expenditure may need to be supplemented by replacement expenditure on adult clothing—replacement of items passed on to the children. Recorded expenditure on children's clothes is not directly appropriate to Mr. Nicholson's purposes.

The most interesting part of the paper, from my own point of view, is the conclusion in Part II on the use of equivalent scales for eliminating family size. Mr. Nicholson has said, as I have myself said in another place, that equivalent scales are not appropriate, and that there are significant differences in patterns of expenditure of families of different sizes which are not explainable in any simple way. I was interested to see the orders of magnitude Mr. Nicholson has obtained, and I have extracted and rearranged some of his figures in a rather different form. The expenditure, in shillings per week, of a family of two adults and two children, at the average standard of living as defined by Mr. Nicholson, is given in the following table:

	Total expenditure	*Attributable to two adults*	*Attributable to two children*
Food	$32\frac{3}{4}$	24	$8\frac{3}{4}$
Clothing	$8\frac{1}{4}$	5	$3\frac{1}{4}$
All other items	48	34	14
Total	89	63	26

What I was interested in was the ratio of expenditure on adults to expenditure on children—a ratio which bears upon the equivalence scale. The ratio of spending on children's food, as compared with adults', is about 35 per cent.; for clothing it is much higher—about 65 per cent.; for other items about 40 per cent.; and for total expenditure about 40 per cent. These figures are much lower than the scales for equivalence usually adopted. One I have in mind is a scale Professor Bowley has used: the average of two adults (one male and one female), 0·9; 0·5 for children under six, 0·7 for those from six to ten, 0·83 for those from ten to fourteen, an average of 0·7. Therefore, according to this scale, the ratio of expenditure on children to that on adults is approximately 0·7 : 0·9, or over 75 per cent. It would seem, from Mr. Nicholson's admittedly rough results, that not only is the equivalent scale inappropriate because it does not differentiate between different items, but it is also of the wrong order of magnitude.

In conclusion, Mr. Nicholson's work shows the need for further budget studies specifically designed for particular problems, such as that of determining the cost of children for middle-class as well as working-class families. Mr. Nicholson has helped us along the way towards the proper designing and analysis of such budget studies.

Mr. C. T. SAUNDERS, in seconding the vote of thanks: Mr. Nicholson has developed some new techniques in the study of family budgets which will be both of statistical and practical importance, and these results, experimental as they are, justify the very considerable labour and patience involved in this analysis.

It is obvious that there is increasing interest in family budget inquiries, for a great many practical reasons. The improvement of methods of conducting family budget inquiries is among the matters arousing interest in the United Nations and the I.L.O. statistical organs. Mr. Nicholson's results suggest a number of directions in which family budget inquiries could be improved and standardized to facilitate some of the refined treatments which can be extremely useful.

The first and most obvious is that this kind of work shows how worth while it can be to publish the details of family budget inquiries, and not simply the broad averages. It is always possible for specialists, who are welcomed by the Ministry of Labour and other authorities, to secure detailed data, and work on them; but it is not always easy for all the people who may be able to use material of this kind, particularly from other countries, to obtain the relevant data.

Secondly, this kind of work raises the interesting problem of the use of income data, as well as of expenditure data, in family budgets. Some budget inquiries have purported to separate income and expenditure, but the results which emerged have not been particularly significant. For so refined an analysis it is not really sufficient to rely on expenditure figures as a direct guide to income figures.

The refined treatment adopted by Mr. Nicholson demands a great deal more reliability in the returns of family expenditure. One of the subjects now agitating experts in this field concerns the two alternative ways of getting family budgets, one by a form to be completed in writing, and the other by interview. The tendency in the United States is to believe that the interview method, though at first sight less reliable, is on the whole more useful. The bias inseparable from form filling is substantial enough to affect the results. It includes the bias caused by the actual process of keeping a record of expenditure; the fact of keeping a record may well affect the pattern of expenditure. This form of bias seems completely unavoidable by ordinary methods of sampling.

The paper deals with an analysis based on the experience of ten years ago. How would this work out in a set of family budgets at the present time? Very roughly one can make one approach from the figures in Table 6. The family with two children and total expenditure of 50*s*. represents something like the average working-class family of 1937–38. Again, the family with two children and an income of 100*s*. represents, in money terms, something like the average working-class family at the present time. How would the experience of such a family to-day compare with that of the family earning 100*s*. in 1938? Their food expenditure would have gone up proportionately rather less than total expenditure. Rent, fuel and light, and clothing expenditure would probably have gone up to rather less than the expenditure of the 100*s*. family before the war. The biggest increase in expenditure must fall on the "sundries" item. But within that item expenditure on tobacco and drink, taking the two together, cannot be much more than doubled. Possibly there has been a greater expenditure on medicines. All this suggests the need to distinguish the different effects on the pattern of expenditure of changes in relative income level, of the passage of time, and of the operation of "artificial" factors such as rationing and shortage.

The vote of thanks was put to the meeting and carried unanimously.

Mr. PHILIP LYLE said that he had found Mr. Nicholson's paper extremely interesting. Mr. Nicholson had fitted regression equations with terms in Y and Y^2 separately for the cases of no children, one, two and three children, and the values of the regression coefficients, shown in Table 5, were rather surprising. If, for any particular item, such as food, we compared the changes in any one of the coefficients with the number of children there appeared to be no correlation whatever, the values of the coefficients changing both in magnitude and sign in an apparently random fashion. He felt that in such a case it would be better to fit a multiple regression equation with terms in Y, Y^2, C, C^2 and CY (where $C=$ number of children), the last term to allow for interaction. As this involved the solution of five simultaneous equations for each of the five groups, he had not had time to carry this out, but was able to do so, roughly, by means of approximation. Using the six total expenditure ranges, between 40*s*. and 100*s*. a set of six equi-spaced figures was found which was not significantly different from the actual set in Table 1, 2 or 3 (Table 4 was not included), so this equi-spaced set was used. Further, the weights (number of families in each range) were neglected. These approximations allowed of the data being treated as a 6×3 factorial design, so that the computation of all the regression coefficients in all five sets presented no difficulty.

It appeared that in all five groups (food, rent, etc.) the term in C^2 was not significant, although with more extensive data, particularly for more than two children, there would undoubtedly be

significant curvature. The term in Y^2 was significant only for food and sundries, and the interaction term was significant in every case except rent.

In Table 6, neglecting the last column, which seemed to be unreliable, the mean extra cost in food per child was 10*d*., 1*s*. 9½*d*. and 2*s*. 1*d*. for the three total expenditures taken; the corresponding figures from the multiple regression equation were 1*s*., 1*s*. 10*d*. and 2*s*. 9*d*. respectively.

He joined in thanking Mr. Nicholson for his interesting paper.

Mr. H. S. BOOKER congratulated the author especially upon his attempt to measure the cost of children. He was not completely satisfied with the results, but he had attempted himself to measure the cost of children, so far as food was concerned, and was just as dissatisfied with his own results as with Mr. Nicholson's. He had had some 1,400 budgets, and if, like Mr. Nicholson, he worked only with those of families consisting of man and wife with no children, one or two children, he found the typical married man without children was older then the one with children. Thus, in addition to measuring the influence of children, he was measuring partly the degree to which a young married couple had heartier appetites than the older married couple whose children had left home. If this feature existed in Mr. Nicholson's budgets he did not think it could be discovered, for the Ministry of Labour Survey did not ask for the ages of adults.

He wished to discuss the results of the inquiry in general terms. The cost of children in a working-class household could be expressed in money value, but it was borne usually by the changed standard of living. The aggregate income of the household, and hence its expenditure, tended not to change greatly with the arrival of a child, and that was more true in 1937–38 than now. Realizing the problem, Mr. Nicholson, ingeniously, attempted to measure to what extent income would have to increase to compensate for the increased cost of a child. The solution offered was to find those items of expenditure from which only the adults benefited, and typical items were adults' clothing and, perhaps not so satisfactory in practice, tobacco and beer, so far as they were recorded. For instance, if there was an expenditure of 10*s*. a week on adults' clothing, tobacco and beer, with an income of 70*s*. for families without children and with an income of 85*s*. for families with one child, the difference between the 70*s*. and the 85*s*. gave a measure of the cost of the child. The argument seemed to be entirely reasonable. Let the results, however, be examined to see how reasonable they appeared. It would be foolish not to do so, even though it indicated suspicion of the data, the assumptions, the methods, or perhaps merely the arithmetic. Mr. Nicholson's results showed an average expenditure of about 26*s*. a week on food in families consisting of man and wife only, with an extra 6*s*. a week for the first child and an extra 3*s*. a week for the second child. These additions seemed to the speaker low for the food cost of a child, especially as allowance was made for the parents to maintain their old standard of living. He had worked on this problem and the results were, equally with Mr. Nicholson's, subject to difficulties of interpretation. But he found that two adults spent an average of 21*s*. a week on food, and the effect of the presence of a child aged 6 to 14 (he grouped children according to age) was to increase the expenditure on food by 6*s*. 6*d*. a week—7*s*. for a first, 6*s*. 4*d*. for a second, and only 4*s*. for a third such child. A child aged 1–6 increased expenditure on food by 6*s*. a week. Further, he was quite certain that the standard of living of families with children had fallen, especially for those with children aged 6 to 14. He was certain because expenditure on food associated with a child was 62 per cent., whilst consumption of bread by weight was 86 per cent., of that associated with an adult. The family with the child was using a greater proportion of its food expenditure on a cheap source of calories, and this indicated increased poverty.

This suggested an alternative way of measuring whether a change in family composition was associated with a change in the standard of living. It might legitimately be assumed in this country to-day that all families had enough to eat when "enough" was measured in terms of calories. Hence, if the change in purchases of bread was proportionately equal to the change in expenditure on food, then the standard of living was unchanged. A modification would have to be introduced if the additional person in a family—for example, a baby—required a special kind of food, or if the additional person had discriminating effects on the wastage of bread as compared with the wastage of other foods.

The results of this investigation could be criticized on general grounds. In fact, he criticized the assumption that there were articles used entirely by adults, consumption of which was independent of family composition. Mr. Nicholson fully realized the difficulties and expressed them, so really he was not criticizing the author of this paper. His doubts rather were about applying regression analysis when it depended on the validity of assumptions which were themselves doubtful. In such cases he could not help wondering whether, despite its usefulness for certain purposes, regression analysis was not made to do more work than it could usefully perform. It was a mechanical method, and such methods often did not show where to look for reasons, and

where assumptions or possibly data might be wrong. It might be that the value of the product of regression analysis was less than its cost, and an economist never liked that.

Mr. J. R. CLEMENTSON said that what he had heard on this paper impressed him with the fact that time and labour could be spent on unreliable material. The previous speaker, in his conclusions, had dealt with the point he wished to make. He himself had had to make surveys of family income for commercial purposes, to show what surpluses were available for spending. In Mr. Nicholson's paper 1,000 families were dealt with, and it was assumed that they all had two children, but statistically they had 2·5 children, and many families had as many as six.

No recognition had been given in the paper to family gifts, but in working-class families a coat or a frock was very often given to a child as a present. It was also assumed that none of these families got into debt, though that often happened. They got out of debt again when the children began to pay their bread money. It must be remembered that it was people who were being dealt with, not card indices. It was assumed that they were all good housekeepers or bad housekeepers, good parents or bad parents; but in dealing with families, or any statistical survey of this type, it was well to forget higher mathematics and to get down to simple arithmetic The ordinary man was concerned with pounds, shillings and pence in his family budget. Here was a family in which a child had to have a special milk food costing 10*s*. a week, but the total liquid food bill otherwise for all the family was only 7*s*. 3*d*. a week. He urged that these things should be dealt with in a simple manner, ensuring that the basic facts were correct. For 1937–38 a particular cross-section of the population must have been taken. Did it include people who were out of work ? There appeared to be no recognition of employment or unemployment in this survey.

Mr. WILLSMORE thought that the author had fallen into a trap in using clothing as an index of the scale of living. When a woman had two children to bring up she had not the time to spend as much on her personal appearance as when she had no children, and it was a fact that all married women, quite apart from financial reasons, did not take as much care about their clothes when they had children as they did when they were first married and without a family. Therefore, when the author took two families, one with children and one without, and said that because the expenditure on adult clothing was the same in both the standard of living was the same, he was wrong. It meant that the male adult, in the family with children, was clothing himself better than the male adult in the other family.

Mr. KEMSLEY said that in Table 14 he was puzzled by the percentage of cost of children. He noticed, as Professor Allen had, that the ratio of food cost for the child was 35 per cent. compared with the adult, and of clothing nearly 65 per cent. The figure for "sundries" was about 60 per cent. "Sundries" must include a large amount of goods bought for the household as a whole, such as furniture and other items. He wondered whether sundries could be broken down in rather more detail in order to explain this difference in the proportion of the cost of the child in respect of sundries as compared with food. He thought the amount of food was rather low. Incidentally, Mr. Nicholson calculated that the figure for tobacco for man and wife was 3*s*., and for the four-person family 4*s*., indicating that the extra two children accounted for an expenditure on tobacco and cigarettes of 1*s*.

A further point concerned the ratio of food expenditure for the first and second child. It indicated that, if the standard of living were constant, the average age of the two children in the two-child family was lower than the average age of the single child in the one-child family, since the average price of the various foodstuffs purchased by the families should be more or less constant. It would be interesting to have the average age of the children.

He then turned to a point made by several speakers, namely, the question of representative samples. The original sample, pre-selected by the Ministry of Labour, was over 20,000; 12,000 were collected, and this number was further restricted to 2,000, taking only those who supplied an annual record of clothing expenditure and four-weekly figures for other items. Mr. Nicholson had had to restrict his sample further in order to secure the families required for his analysis. The proportionate reduction was large, and he felt that here some bias might come in.

He felt that with regard to any budget inquiry, the more intensive the survey was made, and the more information that was requested, the more difficult it was to secure a representative sample. What was gained by securing the detailed information from each individual was lost, possibly, by risking a greater bias, involving lack of representativeness in the sample. For that reason many people had been investigating the possibility of getting information by interview.

Like Mr. Booker, he had made experiments in attempting to discover the extra cost of food for children. In doing this, he did not select families of a particular type, because that would have involved throwing away almost half the sample. His solution of the problem was to divide the

persons in a family in a series of types, broadly classified according to sex and age, to describe the family expenditure in terms of the total of a number of parameters, one each for each type, and estimate the parameters by least squares. The advantage was that one's information was not restricted to a particular type of children. It included other families where, for example, there might be a grandmother, or families in which an unmarried brother or sister was living.

Mr. A. J. THAYRE said that, being a stockbroker, statistics of the working class family were rather outside his normal field of study. Various points had occurred to him during the discussion, one of which prompted him to ask about treatment of holiday expenditure. In his own domestic budget he accounted for holidays as a sundry item. Perhaps, more correctly, the food element and the rent element in holiday expenditure should be separated and accounted for under those headings. He gave hypothetical figures to illustrate how large a share of working family expenditure each year might now be attributed to holidays, and suggested that statistics of family expenditure might be seriously biased if this item were not properly treated in future inquiries. He would be interested to learn how the experts treated this particular problem.

Mr. NICHOLSON, in reply, expressed his gratitude to the various speakers—particularly to Professor Allen and Mr. Saunders—for their complimentary remarks, and to others for their stimulating criticisms. His indebtedness to both Professor Bowley and Professor Allen, who were the pioneers in this field, must be obvious to anyone who had read their works. But his obligations were not confined to the influence of their writings. It was Professor Allen who first instructed him in the elements of statistics, and his earliest efforts at research were done under the guidance of Professor Bowley.

Most of his comments on the detailed points raised in the discussion would be reserved for submission in writing, but one or two comments could be made now. First, with regard to the bi-modal appearance of the distribution of families with three children: the total number of such families was very small, and a simple test applied to the figures showed, as was mentioned in the paper, that there was no significance in the bi-modal appearance of this distribution.

Professor Allen had noticed an apparent discrepancy between the total number of families in one table and the total shown elsewhere. The explanation would be found in the third and fourth paragraphs of Appendix III.

Mr. Willsmore had suggested that the use of expenditure on clothing to define the standard of living would cause the estimated expenditure on children to be too low. But his argument implied that families with children would be enjoying a higher standard of living than childless families who spent the same as they did on clothing. If this argument were valid the estimates obtained for the expenditure on children would tend to be, not too low as Mr. Willsmore had suggested, but too high.

He was not sure whether Mr. Booker's figures were strictly comparable with his. Mr. Booker did not say what standard of living his families were enjoying, a question which was obviously crucial to any comparison.

The point of view which had the author's fullest sympathy was that of the speaker who said that he had spent too much time working on this subject.

Mr. NICHOLSON later replied in writing as follows:

Professor Allen had compared the difference between the total expenditure of childless families and that of families having one child with the estimated expenditure of 17*s*. (average) on the child. Table 2 shows that the average family with one child spent only 3*s*. 6*d*. more than the average childless couple. Comparing the upper quartiles, the difference is about 2*s*. 6*d*., and at the lower quartile the difference is 5*s*. 9*d*. It is not only possible but certain that the extra money available to families with one child was less than the amounts spent on the child. The same holds for families with two or three children; indeed, the former had no more money available than families with one child, while the latter actually had less (see Table 2).

I was interested in Professor Allen's remarks about the use of equivalence scales. These scales are based on relative needs, as determined (partly) by nutritional experts, and it is not surprising that they should differ from the figures I obtained, which reflect the actual behaviour of the families. Spending habits are seldom determined by physical requirements alone. This, I think, is one of the reasons why equivalence scales are quite inappropriate for analysing figures of family expenditure.

As to whether there is any bias in the figures because all the families in the sample made weekly returns of expenditure on clothing, it is only possible to speculate. There may, of course, be bias in any figures which are based, as all these working-class budgets were, on voluntary returns.

But it is impossible to know whether the voluntary method introduces any serious distortion until we know what the true figures are.

I also enjoyed hearing the highly sophisticated views of Mr. Lyle. It would be very nice if we could find a form of equation which would explain the differences between the expenditure figures of families of different size. But I am somewhat doubtful whether the form of equation which he suggested would fit the observations very well. Mr. Lyle found that children appeared to have no curvature, and I do not like the idea of putting them into straight jackets.

Mr. Kemsley asked for a subdivision of the estimated expenditure, attributable to the children, on sundries. The reasons against doing this are carefully explained in the paper. Estimates of the amounts spent, on behalf of the children, on particular items are bound to be unreliable; and I emphasized that if the total expenditure on the item in question is small the estimate is likely to have a very wide margin of error. The queer result for tobacco is not at all surprising. For it is well known that the amounts spent on drink and tobacco, shown in these budgets, are quite untrustworthy.

Lastly, here are the figures, which the Ministry of Labour have kindly supplied, showing the average ages of the children. In families with one child, the average age of the child was 5·3 years. In families with two children, the average age of the younger child was 3·2, and of the elder child 6·8. The differences in age, even allowing for possible economies of scale, can hardly explain why the extra child, in families with two children, appears to consume much less than the single child.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

William Brass.
James Cedric Davies.
William R. S. Doll.
John Leonard Duff.
T. W. Fazakerley.
Joan Winifred Floyd.
Thomas Alfred Humphreys.
Ronald Arthur Jackson.
William Leofric Kesteven.
James Joseph McErlain.
James Ferguson Mitchell.
John Murdoch.
Paul Schiller.
N. P. Srivastava.
Raghavendra R. Umarji.
Denis Weaver.
Edward Victor Whillock.

THE SOURCES AND NATURE OF STATISTICAL INFORMATION IN SPECIAL FIELDS OF STATISTICS*

INTERNATIONAL RUBBER STATISTICS

By W. G. G. KELLETT

Introduction

1. IT is proposed in this paper to deal with the statistics of the commodity and not with the statistics of the various products manufactured from rubber. Moreover, since the commodity nowadays embraces natural rubber, the so-called synthetic or chemical rubbers, reclaimed rubber and scrap rubber, it has been decided to deal principally with the first two types, touch on the third type and ignore the statistics of scrap rubber.†

2. Natural rubber is obtained from the latex of certain trees and plants. Latex is a milky liquid in which tiny particles of solid matter are suspended. The latices of rubber-bearing plants contain varying proportions of pure rubber hydrocarbon, or, as it is sometimes called, *caoutchouc*.

3. The most important source of natural rubber is the latex of the *Hevea brasiliensis*, and this is the tree which has been developed commercially in the rubber plantations.

4. Natural rubber was originally obtained from the wild rubber trees in the Amazon Valley, and also from various wild trees and shrubs in other parts of America, Africa and Asia. Up to the end of the last century natural rubber was obtained solely from wild sources.

5. Towards the end of the 19th century British interests introduced the rubber plantation industry to the countries of South-East Asia, using planting material from the Amazon. In a short time Dutch and French interests followed suit, to be followed by Asiatic growers, mainly the Chinese and the local inhabitants of Malaysia.

6. To-day the great bulk of natural rubber comes from the plantations of the Far East.

7. Commercially, natural rubber is marketed in various forms. Within the last quarter of a century there has been a growing demand for natural rubber in the form of liquid latex. The latex from the tree is first of all prevented from solidifying by the addition of a chemical anti-coagulant and is then concentrated. Concentration is usually done by centrifuging or creaming, and the concentrated latex is marketed in drums or tanks.

8. Solid rubber is usually marketed in the form of sheets. The method of preparation is simple. The latex collected from the trees is coagulated by the addition of an acid. The coagulum is rolled, dried and smoked before marketing. The smoking kills bacteria and so prevents deterioration. The sheets are normally ribbed, and rubber in this form is sold as *ribbed smoked sheet*. There are various grades of these sheets, depending largely on the amount of impurities present.

9. Solid rubber is also marketed in the form of crêpe. Preparation is much the same as in sheet rubber. Rollers revolving at different speeds produce the crêping effect. Thin crêpe for medical purposes and thick crêpe or sole crêpe are not smoked, as colour is important, but are preserved by the addition of a chemical. Very inferior grades of rubber containing a high proportion of extraneous matter, such as bark scraps, are also sold in the form of crêpe.

10. Synthetic rubber is really a misnomer, in that nobody has succeeded in synthesizing rubber as it occurs in nature. Despite this, there are to-day many materials referred to as synthetic, or, perhaps more correctly, chemical rubbers. Many of these materials are more closely allied to the plastics than to rubber. To narrow the field, it has been agreed that, except in special circumstances, only copolymers of butadiene and styrene (Buna-S type),‡ copolymers of butadiene and

* It is the intention that most of the articles in this series shall deal with statistics relating to the U.K., but in certain fields, particularly those concerned with a raw material such as rubber, a survey of international statistics will be given.

† Scrap rubber is mainly used as the raw material for obtaining reclaimed rubber. A few countries, such as Spain, import considerable quantities of scrap rubber for soling shoes.

‡ The so-called Russian rubberlike materials SKA and SKB are straight polymers of butadiene, but are usually regarded as being "synthetic rubber."

acrylonitrile (Buna-N type or perbunans) together with neoprene and butyl type rubbers shall be regarded as being "synthetic rubber."

11. Historically it is probable that neoprene, which is a polymer of chloroprene, was the first of these synthetic rubbers to be produced commercially. This material, which is a product of the du Pont Company of America, was first marketed in 1932. About the same time the first Russian synthetic rubber was produced. A few years later the Germans developed their Buna* rubber, a stiaight polymer of butadiene. Within a short time the Germans had improved their original product by the combination of other materials with butadiene. Thus by 1937 the original Buna had been superseded by Buna-S (a copolymer of butadiene and styrene) for general purposes, and by Buna-N or Perbunan (a copolymer of butadiene and acrylonitrile) for special purposes. Following these discoveries by the Germans, rapid improvements in the two materials were made by American chemists, and by 1940 improved types of Buna-N were being produced in the U.S.A. In 1942, when the Japanese had cut off the supply of natural rubber and the American Government launched its vast synthetic rubber programme, it was decided that the principal type of synthetic rubber to be produced should be the American refinement of the German Buna-S, which was in American terminology called GR-S (*G*overnment *R*ubber with *S*tyrene). The American refinements of Buna-N, a number of which were, and still are, being marketed under trade names, were called, in general, N-type rubbers. It was also decided in 1942 to commence large-scale production of another American synthetic rubber which had been developed by the Standard Oil Company of New Jersey, the so-called Butyl rubber, or, as it became in official jargon, GR-I (*G*overnment *R*ubber with *I*sobutylene). This material, which is specifically used in the manufacture of inner tubes, where its high degree of impermeability to air gives it an advantage, is a copolymer of isobutylene with a small amount of butadiene or isoprene.

12. At the moment, apart from production in Russian territories for which no statistics are available, the manufacture of synthetic rubber, excluding trifling amounts of neoprene type rubbers produced in Sweden and Czechoslovakia, is confined to the U.S.A. and Canada. In Canada, where production commenced in 1943, the types produced are GR-S, butyl and certain N-type rubbers.

13. Synthetic rubber, like natural, is mainly marketed in solid form, but there is also a demand for GR-S, neoprene and N-type latices.

14. Reclaimed rubber, or, as it is more commonly known in the industry, reclaim, is, as the name suggests, a material which has been recovered from old rubber goods, such as tyres and tubes. The "caoutchouc" content of reclaimed rubber is about 55 per cent. of its weight, the remaining weight consisting of materials that have to be used in the manufacture of new rubber products. Reclaim is a compounding ingredient, and its use as such in products not requiring high resistance to abrasion or oxidation is fundamental and extensive. It has also been used as a substitute for new rubber, increasingly when new rubber is scarce or its price unusually high; it is thus an extender of new rubber, and an economic competitor under certain conditions.

Rubber Production

NATURAL RUBBER

Acreage

15. As it is obviously impossible to measure the area under rubber in territories where it occurs as a wild tree or shrub, such as the Amazon Valley, an examination of acreage will be confined to plantation rubber.

Plantation rubber, as has already been mentioned, is predominantly produced from the *Hevea brasiliensis*. The produce of this tree is termed *para* rubber. A few other trees, such as the *Manihot glaziovii*, which yields *ceara* rubber, have been cultivated in plantations, but their number is negligible.

16. Plantations are themselves divided into two broad categories.

The larger plantations, which are usually owned by Europeans or managed on European lines, are referred to as "estates," and their produce is termed "estate rubber." In most territories holdings of 100 acres and over are regarded as being estates.

* The name Buna was coined from the first two letters of Bu-tadiene and Na-trium (Sodium), the latter being used in the original polymerization process.

The smaller plantations, which are usually owned by the local inhabitants, or in parts of the Far East by Chinese or Indian immigrants, are referred to as "smallholdings" and their produce is termed "smallholders' rubber," or more loosely, but much more commonly, "native rubber." In most territories holdings of under 100 acres are regarded as smallholdings.

17. So far as acreage statistics for estates are concerned these have been always fairly easily available and have also been fairly accurate. The unsettled conditions in some territories, of course, prevent an up-to-date assessment at present.

In regard to smallholdings, however, it has been impossible up to the present to obtain anything better than estimates for the area planted with rubber. In some territories these estimates are probably fairly accurate, but in some of the larger areas, particularly in Indonesia, the margin of error is enormous.*

18. Bearing in mind what has been said in the previous paragraph as to the unreliability of estimates of the area under smallholders' rubber and the absence of up-to-date figures in respect of estate rubber in some territories, the following table gives a fair summary of the acreage under plantation rubber:

TABLE 1

Planted Acreage

Territory	*End of*	*Estates* ('000 *acres*)	*Smallholdings* ('000 *acres*)	*Total* ('000 *acres*)
Malaya	1947	1,946	1,398	3,344
Indonesia	1940	1,567	..	..
Ceylon	1948	371*	290*	661
Thailand	1940	..	419	419
Indo-China	1942	312	20	332
Sarawak	1940	18	222	240
Belgian Congo	1946	131	68	199
India	1944	92*	60*	152
Other British Borneo	1940	81	69	150
Nigeria	1946	19	101	120
Burma	1940	68	43	111
Liberia	1944	77	..	77
Other Africa	1945	75*	..	75
Latin America	1944	..	..	30
Oceania	1943	25*	..	25
Other Asia	1940	10*	..	10
	Total:	4,800*	2,700*†	7,500*†

† Excluding Indonesian smallholdings.
* Partly estimated.

Including Indonesian smallholdings and making allowances for areas cut out during the Japanese occupation, and for areas planted recently, total world acreage under plantation rubber might be taken at around 10 million acres, more than half of which is cultivated by smallholders.

19. The great bulk of the area under rubber is planted with *Hevea brasiliensis*. While practically the whole of the smallholdings' area is planted with ordinary or seedling *Hevea*, a significant part of the estates' area is planted with high-yielding trees, particularly in Indonesia, Malaya, Indo-China and Liberia. These high-yielding varieties are usually obtained by grafting, and are commonly referred to as *budgrafted* rubber or *budgrafts*. It is also possible to obtain high yields

* During 1934–1936 the Government of Indonesia endeavoured to assess the acreage under rubber by means of a tree count. The trees on each native holding were counted, and an estimate was made of the planting density in various regions based on sample plots. (It is not known whether there was any attempt made to employ scientific sampling methods.) The planted acreage was calculated by dividing the number of trees counted in each district by the estimated density of planting and aggregating the results. A total of 582,365,735 (*sic*) trees was counted and a planted area of 1,683,202 acres was calculated. It was, of course, realized that the method employed was unsatisfactory, and in 1938 a proper scientific survey was begun. By the beginning of 1942 the field work had been completed but the results had only been partially tabulated. From the known results it was found that some 46 per cent. of the areas covered in the earlier tree count had been measured, and that the measured acreage exceeded the previous estimate by 90 per cent. On this partial result the total acreage has been estimated by the Indonesian Government at 3,200,000 acres.

by using specially selected seed, commonly called *clonal* seed. The following table gives the division of estate rubber as between ordinary or *seedling* rubber and high-yielding rubber in a few of the principal territories:

TABLE 2

Acreage of Estate Rubber Divided as to Planting Materials

Territory	*End of*	*Ordinary* ('000 *acres*)	*High Yielding* ('000 *acres*)	*Total* ('000 *acres*)
Malaya	1947	1,545	401	1,946
Indonesia	1940	1,008	559	1,567
Ceylon	1940	322	37	359
Indo-China	1940	165	146	311
Liberia	1944	16	61	77

Yields

20. Unfortunately, for various reasons which will not be enumerated it is extremely difficult to assess accurately average annual yields obtained from plantation rubber. There is a marked difference between the average yield obtained by one class of producers and that obtained by another class within a territory, and there is also variation between territories. With high yielding material yields of two to three times those obtained from seedling rubber under similar conditions have been obtained, and this ratio will undoubtedly be increased.

21. While no accurate yield figures are available, it is known that because of their practice of planting more trees per acre, smallholders' yields per acre from seedling rubber are greater than those obtained by estates from the same material. Smallholders' yields per tree are, however, smaller than estates' yields per tree.

22. Again, while yields from similar planting material are roughly the same in Malaya and Indonesia and the contiguous territories, there is a very definite diminution in yield from similar material planted in India and Burma, and in some of the African territories.

23. The rubber tree is usually tapped about six years after planting and is usually considered to have an economic life of up to 30 years. Full maturity is probably not achieved until the tree is about 14 years old. The following table, based on information obtained from estate sources in Indonesia during the 1930's, gives a rough idea of the relative yields of budgrafted and seedling rubber before 1941. Yields obtained from budgrafted rubber since the war have been substantially greater than those given in the table and even greater yields are anticipated in the future:

TABLE 3

Scale of Yields per Acre (Approximate)

Age, Years	*Seedling (lb. per annum)*	*Budgrafted (lb. per annum)*
6–7	150	275
7–8	225	425
8–9	300	550
9–10	375	675
10–11	425	775
11–12	475	900
12–13	500	975
13–14	525	1,025

24. In India and Burma the average yields obtained from mature rubber are probably only about 50 per cent. of those shown in the above table. In Malaysia the average yield per acre obtained from smallholders' mature seedling rubber is probably somewhat above 500 lb. per annum, while the average yield per acre from mature seedling rubber on estates is probably somewhat less than 500 lb. per annum.

Production

25. Although rubber is produced continuously throughout the year, output is subject to important seasonal variations. These variations are principally due to climatic conditions, but religious festivals and the deflection of labour, particularly on smallholdings, to other seasonal activities such as padi planting and harvesting also have their effect. The most important variation in production takes place during the "wintering" period when the trees are shedding their foliage. The decline in output during the period of six weeks or so of "wintering" is in most territories very important. In most of the Far Eastern territories "wintering" takes place in the early part of the year, February being the month most affected, but in some territories such as Java and Burma the great fall in output occurs around July/August.

26. In a few territories such as Malaya, Indonesia, Indo-China and Ceylon, reliable monthly figures for production on estates are available, but in no territory are there any reliable data on smallholders' output. In fact, with the exception of estate production data in the few territories named, the only method of assessing production is by taking net export figures and adjusting for local consumption and stocks. As there is normally very little domestic consumption in most exporting countries, and as variations in stocks are mostly negligible, for most countries net exports do indicate production fairly accurately. Of course, as it may take several weeks to prepare rubber for export, rubber actually produced in, say, March, may not be exported until April or May.

27. In the Federation of Malaya, where a substantial entrepôt trade somewhat complicates matters, smallholders' production up to the end of 1948 was calculated from the following formula:

> Smallholders' production during month = Total stocks at end of month + Gross exports during month + Local consumption − Stocks at beginning of month − Imports − Estate production.

As declared stocks did not include stocks held by smallholders themselves, which, although individually small, might be substantial in aggregate, this method was far from satisfactory, the chief objection being the large fluctuations which occurred between months. Since the beginning of 1949 an attempt has been made to estimate smallholders' production on monthly purchases by dealers, but whether this will achieve any better results is still doubtful.

28. In the following table indices of seasonal variation for various producing territories are given. Some of these indices have already been published by the U.S. Department of Commerce; those for Malayan (European) estates and Thailand are original. The indices for Malayan estates are based on the period 1928–1934, those for Thailand for the period 1934–1941, the Indonesian indices on the period 1928–1931.

TABLE 4

*Indices of Seasonal Variation in Rubber Production**

	Malaya, European Estate	*Indonesia*		*Thailand, exports*	*Indonesia, native exports*
		Java Estate	*Other Estate*		
January	8·7	9·0	9·1	9·3	8·2
February	6·7	8·3	7·7	9·7	6·8
March	7·2	8·9	6·8	8·8	8·0
April	7·3	8·8	7·2	4·9	7·1
May	7·8	9·2	8·2	6·3	8·4
June	8·1	8·7	8·5	8·2	8·6
July	9·0	8·3	9·0	9·9	10·2
August	9·4	6·3	8·7	9·3	9·3
September	8·4	7·3	8·4	8·3	8·1
October	9·1	7·9	8 8	8·5	8·8
November	8·6	8·3	8·4	7·3	7·7
December	9·7	9·0	9·2	9·5	8·8
	100·0	100·0	100·0	100·0	100 0

* Indices are expressed as percentage of annual total.

29. In addition to the output of plantation rubber there is still a small quantity of wild rubber being produced. Because of the high production costs, it is only at times of very high rubber prices that any appreciable amount of wild rubber is produced. During the recent war, when the Far Eastern plantations were over-run, wild rubber production was stimulated in Latin America and Africa, but since the re-occupation of the rubber areas of Asia the output of wild rubber has dropped steeply. With the exception of some South American countries where domestic production of wild rubber is protected there is virtually no wild rubber being produced.

30. The following table illustrates the growth in natural rubber production throughout the present century, and also shows the change-over from the wild to the plantation product:

TABLE 5

Natural Rubber Production. ('000 *tons*)

	Plantation Rubber										
	*British territories**			*Other territories*			*Total*			*Wild rubber, Total*	*Grand Total*
Year	*Estate*	*Small-holding*	*Total*	*Estate*	*Small-holding*	*Total*	*Estate*	*Small-holding*	*Total*		
1900	0·5	..	0·5	..	..	..	0·5	..	0·5	44·5	45·0
1910	8·0	0·5	8·5	4·0	..	4·0	12·0	0·5	12·5	82·5	95·0
1920	167·5	60·0	227·5	75·0	5·0	80·0	242·5	65·0	307·5	35·0	342·5
1925	190·0	90·0	280·0	120·0	90·0	210·0	310·0	180·0	490·0	37·5	527·5
1929	317·5	242·5	560·0	167·5	117·5	285·0	485·0	360·0	845·0	25·0	870·0
1932	265·0	202·5	467·5	167·5	67·5	235·0	432·5	270·0	702·5	7·5	710·0
1941†	472·5	305·0	777·5	395·0	400·0	795·0	867·5	705·0	1,572·5	27·5	1,600·0
1944	117·5	32·5	150·0	110·0	27·5	137·5	227·5	60·0	287·5	72·5	360·0
1945	95·0	40·0	135·0	47·5	5·0	52·5	142·5	45·0	187·5	62·5	250·0
1946	267·5	272·5	540·0	45·0	205·0	250·0	312·5	477·5	790·0	47·5	837·5
1947	450·0	365·0	815·0	85·0	325·0	410·0	535·0	690·0	1,225·0	35·0	1,260·0
1948	500·0	377·5	877·5	182·5	432·5	615·0	682·5	810·0	1,492·5	27·5	1,520·0
1949	497·5	347·5	845·0	247·5	362·5	610·0	745·0	710·0	1,455·0	27·5	1,482·5

* Including India.

† Owing to the outbreak of hostilities in the Far East, the production figures for 1941 are incomplete. The figures given are estimates based on outputs during the first ten months in the principal territories.

31. It should be noted that most of the figures in Table 5 are partly estimated and rounded off, as, among other reasons, it is impossible to get a precise division between estates' and smallholders' rubber. Prior to 1941 all the figures represent net exports, but from then onwards production figures, where available, have been included. By and large, it is believed that the table gives a fair picture of the actual position. All figures contain the dry rubber content of latex produced or exported.

32. In regard to latex production, the following table of net exports illustrates the growth of the output of rubber in this form:

TABLE 6

Natural Rubber Latex Exports from Principal Areas

(Figures in long tons of dry rubber content)

Year	*Malaya*	*Indonesia*	*Ceylon*	*Indo-China*	*North Borneo*	*Liberia*	*Others*	*Total*
1922	63	n.a.	n.a.	..	..	..	..	n.a.
1927	2,634	83	..	..	..	..	..	2,717
1932	5,193	3,488	2	..	..	..	..	8,683
1937	19,408	13,651	4	7	307	1,139	..	34,516
1940	22,269	17,559	10	7	644	4,259	..	44,748
1947	26,595	53	446	170	..	9,168	96	36,528
1948	39,956	18	625	241	..	6,006	224	47,070
1949	45,731	1,150	879	293	..	7,500	500	56,053

Exports and Stocks in Producing Areas

33. In considering natural rubber production it will, perhaps, not be out of place to consider stocks of rubber held in producing areas and exports from producing areas, which, as has already been mentioned, are in many cases the only available measure of actual production.

34. In regard to stocks in producing areas, these usually bear a fairly steady ratio to the rate of production. In practice there is normally a stock in producing areas equivalent to roughly 2 months' current production. So far as these stocks are concerned, while most of the important territories can supply information regarding stocks on estates, in hands of dealers and at ports, there is usually no information available for stocks held by smallholders. Occasionally, particularly in Malaya, where the situation is somewhat complicated owing to the importance of Singapore as a world rubber market, dealers may hold high stocks as a speculation against rising prices, but in general a stock equal to 2 months' production is the rule in producing areas.

35. So far as exports are concerned, the special position of Malaya calls for a word of explanation regarding exports from that territory. In the first place Singapore, and to a lesser extent, Penang, serve as ports for many of the areas adjoining Malaya as well as Malaya itself, and secondly, a great amount of "native" rubber, particularly from Indonesia, is imported into Malaya for the purpose of being re-processed, and is then re-exported. It is obvious, therefore, that shipments of rubber from Malaya are no measure of actual Malayan production. In spite of the fact that it is obviously nett exports which should be taken as Malayan produce, the error is frequently made of considering all rubber from Malaya as being of Malayan origin.

36. The point mentioned above is of the greatest importance when considering the dollar earnings of British rubber. Under an Anglo-Netherlands agreement the Indonesian Government is awarded dollars on the basis of the following formula:

$$\text{Dollars paid to Indonesia} = \begin{array}{c}\text{Value of Indonesian}\\ \text{rubber imports}\end{array} \times \frac{\text{Value of gross Malayan rubber exports to U.S.}}{\text{Value of total gross Malayan rubber exports.}}$$

37. Another point in connection with rubber imported into Malaya and re-exported therefrom which complicates the statistical position is the fact that much of the rubber thus imported is of low quality and contains a high percentage of moisture. At the moment there is no accurate assessment of the moisture content of this rubber; in some parts it is arbitrarily assessed at 70 per cent. dry rubber content, and at others the assessment is as high as 85 per cent. dry rubber content. In view of the large quantities imported, it is quite possible to have a discrepancy of the order of 10,000 tons in annual import figures.

SYNTHETIC RUBBER

Productive Capacity

38. So far as is known, production of the four main types of synthetic rubber is confined at the moment to the United States of America, Canada, Russia and Russian-occupied Europe, Czechoslovakia and Sweden. The following table gives the estimated productive capacity of the plants in these territories:

TABLE 7

Synthetic Rubber Productive Capacity (long tons)

	Butadiene styrene types	*Neoprene types*	*Butadiene acrylonitrile types*	*Butyl types*	*Grand total*
U.S.A.	690,000	69,000	18,000	68,000	845,000
Canada	40,000	—	*	10,000	50,000
Russia	125,000	25,000	..	..	150,000
Czechoslovakia	..	1,500	..	..	1,500
Sweden	..	800	..	..	800

* While there are no specific butadiene-acrylonitrile type plants in Canada, these types of rubber may be produced in plants of the butadiene-styrene type with only minor modifications. This is in effect being done.

39. In regard to the figures given in the above table the following comments may be made:

U.S.A.

The government owned synthetic rubber plants at January 1st, 1949, as follows:

	Number of plants	Capacity (long tons per year) Rated	Demonstrated
Copolymer plants (GR-S):			
Active	9	375,000*	453,750
Standby	4	225,000	281,250
Surplus	1	30,000	37,500
Sold	1	60,000	75,000
Total	15	690,000	847,500
Butyl plants:			
Active	2	55,333	66,000
Standby	1	12,667	14,000
Total	3	68,000	80,000
Neoprene plants:			
Sold	1	60,000	60,000

* The active copolymer plants (GR-S) are expected to have a demonstrated capacity of 405,000 tons in operations producing both cold and normal GR-S when the present conversion of part of the facilities to cold rubber production has been completed.

In addition to these government plants, there are also a privately built neoprene plant and privately built butadiene-acrylonitrile type plants.

Canada

The Canadian government plant is operated by the Polymer Corporation at Sarnia in Ontario. The plant makes butyl rubber and the two types of butadiene copolymers. The estimates given represent rated or design capacity, but actual capacity as demonstrated is undoubtedly higher.

Russia

The figures given for Russia are largely based on the pre-war position. Russian synthetic rubbers consist of two main types, SKA and SKB, which are both straight butadiene polymers, and Sovprene, a chloroprene polymer. SKA is made from petroleum derived butadiene, while SKB is made from butadiene derived from agricultural alcohol.

In June, 1941, there were eight SKB plants in operation in Russia. Two of these had an annual capacity of 20,000 tons, two others of 15/20,000 tons, three had an annual capacity of 15,000 tons and one other had a capacity of around 5,000 tons per annum. Total capacity was thus around 125,000 tons per annum.

Two SKA plants, one of which had a design capacity of around 12,000 tons, had been planned in 1939, but it is not known whether either of these were in production by June, 1941.

As regards Sovprene, there was definitely one plant of 25,000 tons rated annual capacity in operation in June, 1941. Two others, each of around 15,000 tons annual capacity, were about to start operations.

Many of the Russian plants were destroyed during the war, and there is also the possibility of the transfer of German plants to Russia after the war. The capacity figures given in Table 7 should, therefore, be treated with the greatest reserve.

Production

40. The first synthetic rubber produced on a commercial scale was the methyl rubber made by the Germans during the first World War. This rubber, which was discovered by a Russian named Kondakoff, was of a very poor quality; in spite of this, some 2,500 tons were produced during 1915/1918.

41. As already mentioned, the first of the present types of synthetic rubber, commercially produced, was neoprene, which was marketed by the du Pont Company in 1932. This was soon followed by the Russian synthetic SKB. Table 8 gives production statistics for synthetic rubber since 1933:

TABLE 8

Production of Synthetic Rubbers (long tons)

Year	*Canada*	*Germany*	*U.S.A.*	*U.S.S.R.*	*Total*
1933	..	..	..	2,200	..
1934	..	..	..	11,100	..
1935	..	..	..	25,600	..
1936	..	..	..	44,000	..
1937	..	2,000	..	56,000	..
1938	..	5,000	1,000	64,000	70,000
1939	..	21,998	1,750	78,500	102,250
1940	..	39,826	2,560	100,000	142,500
1941	..	69,361	8,114	120,000	200,000
1942	..	98,135	22,476	..	..
1943	2,522	115,754	231,767	..	..
1944	34,829	101,624	764,072	..	..
1945	45,717	..	820,352	..	..
1946	50,981	15,557*	740,026	..	..
1947	42,393	8,229*	508,702	..	..
1948	40,455	3,388*	488,343	..	..
1949	46,642	..	393,690	..	..

* British Zone only since 1945. Production was suspended in July, 1948.

42. The United States Government had become interested in the erection of synthetic rubber plants during 1941, and at the time of Pearl Harbour plans had been drawn up for the building of some 40,000 tons annual capacity in addition to the plants already operated by private industry. After the loss of the Far Eastern rubber areas, plans were speeded up to increase synthetic rubber production capacity to a total of around one million tons per annum. Elsewhere such a phenomenal programme would undoubtedly have come to grief, but it is a lasting memorial to American enterprise that within three years their planned objective had been achieved.

43. While most synthetic rubber is produced in solid form, a number of synthetic latices are manufactured in the U.S.A. and Canada. Production figures of these latices are included in the total production figures given in Table 8 on the basis of the weight of solids in the latices. The principal latices produced are those of neoprene and the butadiene-styrene type of synthetic rubber, but latices of the butadiene-acrylonitrile or N-type rubbers are also made. There are no figures available for synthetic latex production outside the U.S.A. The U.S. figures are given in Table 9 in terms of the weight of solids in the latices:

TABLE 9

U.S. Production of Synthetic Latices (long tons)

Type	1941	1942	1943	1944	1945	1946	1947	1948
Neoprene	59	210	1,512	4,683	7,077	13,603	6,089	5,022
GR-S	..	..	194	6,580	15,176	24,810	22,474	21,018

RECLAIMED RUBBER

Productive Capacity

44. Practically every country which has a substantial rubber products manufacturing industry has also a rubber reclaiming industry. Reclaimed rubber is not primarily a substitute for raw

rubber, but is complementary to it. The following data in Table 10 regarding estimated reclaiming capacity in various countries in 1945 should be treated in most cases with considerable reserve:

TABLE 10

Rubber Reclaiming Capacity in Principal Countries (long tons)

U.S.A.	325,000*	Australia	9,000	Sweden	2,500
Germany	50,000	Argentina	7,500	South Africa	2,500
U.K.	30,000	Italy	7,500	Spain	2,000
U.S.S.R.	30,000	Canada	6,000	Brazil	1,500
Japan	20,000	China	5,000	Mexico	1,000
France	15,000	India	4,000	New Zealand	1,000

* Current estimate of maximum annual reclaiming capacity is about 350,000 tons.

Production

45. Figures of actual production of reclaimed rubber are available for only a few countries. Table 11 gives a few recent figures for reclaimed rubber production in various countries:

TABLE 11

Reclaimed Rubber Production in Various Countries (long tons)

Year	*U.S.A.*	*U.K.*	*France*	*Australia*	*Canada*	*India*	*New Zealand*
1941	274,202	..	9,987	..	..	..	..
1942	285,114	18,275	6,625	..	..	..	..
1943	303,991	36,587*	7,889	6,771	4,708	2,326	576
1944	260,607	28,690	7,194	7,206	3,153	3,728	696
1945	243,309	22,317	9,210	5,819	3,307	5,241	712
1946	295,612	27,798	15,360	6,410	3,967	1,645	..
1947	291,395	22,431	..	5,973	4,125	..	..
1948	266,861	24,614	..	5,305	4,323	..	..
1949	223,990	20,946	..	..	3,400	..	..

* Includes crumb. Actual reclaim production was about 27,000 tons.

46. In addition to the above information the following figures of Russian pre-war reclaimed rubber production are available: 1933, 17,000 tons; 1934, 18,500 tons; 1935, 21,500 tons; 1936, 26,600 tons; 1937, 28,000 tons; 1938, 23,000 tons; 1939, 21,000 tons.

Rubber Consumption

NEW RUBBER

General

47. Whereas in the section on "Rubber Production" it was more convenient to deal with the various types of rubber separately, in this section the consumption of natural and synthetic rubber will be considered together under the heading "New Rubber."

48. By the term "consumption" is generally meant the amount of raw rubber which has been processed, i.e. turned into manufactured goods, during the period under review. In some cases this is measured by the quantity of rubber "mixed" by manufacturers, while in others it may be the amount withdrawn from factory stocks.

49. At present comparatively few countries collect and publish accurate statistics relating to their consumption of rubber. Data have been available for many years in regard to U.S. rubber consumption, and there have been satisfactory statistics of U.K. rubber consumption since 1935. In addition to these two countries, consumption data are regularly available for France, Germany (at present the Bi-zone only) and the Netherlands in Europe and for Australia, Canada and India. Several other countries supply consumption data periodically, but in most cases one is forced to make use of import statistics as a means of estimating rubber consumption.

50. Normally, since the smaller consuming countries rarely vary their stocks, net imports give a reasonably accurate measure of consumption. After the end of World War II, however,

when most countries were completely denuded of rubber, the first essential was to build up working stocks. For this reason net imports were not a true reflection of consumption, and in the official statistics of the Rubber Study Group it was decided to estimate consumption from net imports on the assumption that stocks would be maintained at around 2 to 3 months' consumption, at the same time making allowance for abnormal stockpiling by some countries such as Argentina and Italy. As a result of this artifice, it has been possible to indicate a smooth increase in consumption and working stocks. At the moment, as working stocks have been accumulated in most countries, estimated consumption in countries not reporting consumption is roughly the same as net imports.

51. The most important part of rubber consumption is for transportation purposes, i.e. the manufacture of automobile and truck and bus tyres and inner tubes, bicycle and motorcycle tyres, etc. The percentages of rubber used in transportation in the leading consuming countries in recent years are given in Table 12:

TABLE 12

Rubber Used in Transportation Products as Percentage of Total Rubber Consumption

Year	*U.S.A.*	*U.K.*	*Canada*	*France*
1946	75	64	73	64
1947	74	58	75	63
1948	70	60	73	58

These countries are also the best equipped in motor transport, so that the percentage of rubber used in transportation in the rest of the world is likely to be below 60. Weighting in accordance with total rubber consumption in the above countries, it is probable that the percentage of rubber consumption used in transportation in the world as a whole is around 65.

52. The growth in rubber consumption during the present century may be shown as in Table 13. In this table no attempt has been made to distinguish between synthetic and natural rubber or between the plantation and wild varieties of the latter. In the earlier years, figures are either net imports, or at best net imports corrected for annual changes in reported stocks.

TABLE 13

Natural and Synthetic Rubber Consumption (1,000 *tons*)

	U.S.A.	*U.K.*	*Continent of Europe**	*Rest of world*	*Total*
1900	20·5	10·0	20·0	2·0	52·5
1910	42·5	20·0	35·0	2·5	100·0
1920	206·0	24·0	47·5	20·0	297·5
1925	388·5	30·0	98·0	36·0	552·5
1929	467·5	72·5	175·0	90·0	805·0
1932	337·0	78·0	177·5	100·0	602·5
1937	543·5	114·6	278·2	156·2	1,092·5
1941	781·3	156·5	163·2	211·5	1,312·5
1946	1,039·3	126·8	145·7	155·7	1,467·5
1947	1,122·3	156·4	242·0	214·3	1,735·0
1948	1,069·4	196·3	390·0	244·3	1,900·0
1949	990·6	186·6	441.0	256·8	1,875·0

* Excluding Russian produced synthetic rubber.

The above table not only illustrates the rapid growth in rubber consumption which has taken place during the first half of this century, but also indicates the importance of the U.S.A. as a rubber-consuming country.

53. As shown in Table 12, roughly 70 per cent. of all the rubber consumed in the U.S.A. is used in transportation products. In practice by far the greater portion of the rubber consumed goes into the manufacture of automobile casings. For this reason, although it has been stated earlier that this paper is concerned with the statistics of rubber as a raw material and not with the statistics of manufactured products, it will be useful to comment shortly on U.S. tyre statistics.

54. Thanks to the excellent statistical service of the Rubber Manufacturers' Association of America, there is a wealth of statistical data for the U.S. tyre manufacturing industry. Monthly

figures of tyre production, sales and inventories held by manufacturers are available over a long number of years. Sales figures are subdivided as to original equipment, i.e. sales to automobile manufacturers, replacements, i.e. sales to dealers, garages, etc., and exports.

55. This subdivision of tyre sales is important, as sales for replacements are, in particular, strongly subject to seasonal variation. The significance of this, often overlooked factor is its seasonal influence on rubber consumption. The following indices of seasonal variation in tyre casings replacement sales in the U.S.A. are based on data covering the period 1929–1941:

TABLE 14

Indices of Seasonal Variation in U.S. Tyre Replacements

	Passenger car casings		*Truck and bus casings*	
	Average month (%)	*Annual total* (%)	*Average month* (%)	*Annual total* (%)
January	81·8	6·8	84·9	7·1
February	68·2	5·7	71·7	6·0
March	83·1	6·9	86·6	7·2
April	101·7	8·5	91·0	7·6
May	120·3	10·0	101·9	8·5
June	135·0	11·3	121·7	10·1
July	135·3	11·3	128·3	10·7
August	132·5	11·0	130·0	10·8
September	111·9	9·3	114·9	9·6
October	87·8	7·3	106·3	8·9
November	74·7	6·2	85·3	7·1
December	67·7	5·7	77·4	6·4
	1,200·0	100·0	1,200·0	100·0

56. In so far as the relative consumption of natural and synthetic rubber is concerned, it is regrettable that in regard to the largest consuming country it is impossible to draw any conclusions as to the demand for either variety. This is partly due to the fact that under present U.S. legislation there is a minimum mandatory usage of synthetic rubber laid down by Government. On the other hand, it is impossible to draw any firm conclusions from countries outside the U.S.A. with the possible exception of Canada, because (*a*) synthetic rubber can only be obtained from dollar countries, and (*b*) very few countries have had sufficiently long experience of using synthetic.

57. So far as the U.S.A. is concerned, the present mandatory use is confined to transportation equipment, while the non-transportation field, where in many cases synthetic would be preferred on its merits, is completely free.

58. Despite the limitations on its value set out in the two previous paragraphs, the following table indicates the relative use of natural and synthetic rubber during the last few years. In every case the consumption of Russian synthetic rubber has been excluded.

TABLE 15

Synthetic Rubber Consumption as a Percentage of Total New Rubber Consumption

12 months ending	*United States*			*Canada*			*Rest of world, Total*	*Grand Total*
	Transport	*Non-Transport*	*Total*	*Transport*	*Non-Transport*	*Total*		
December, 1946	71·1	79·1	73·2	76·6	73·2	75·7	31·2	62·2
,, 1947	46·7	58·8	49·8	47·2	48·2	47·5	6·6	36·0
March, 1948	43·5	52·2	45·9	42·0	41·2	41·8	4·6	31·7
June, 1948	40·9	47·4	42·7	36·9	36·5	36·8	3·8	28·6
September, 1948	40·4	44·8	41·7	34·3	35·3	34·6	3·0	27·0
December, 1948	40·5	43·3	41·3	32·6	34·4	33·1	2·2	25·3
March, 1949	41·2	42·8	41·7	31·9	34·1	32·5	2·1	24·7

59. In regard to consumption of the various types of synthetic rubbers, detailed statistics are available for only a few countries. By far the greatest consumption, however, is that of the butadiene-styrene copolymers, i.e. GR-S. These rubbers are used in tyre manufacture, particu-

larly in passenger car sizes. Butyl rubber is used for inner tube manufacture, where its excellent impermeability gives it a decided advantage over natural rubber. This advantage, however, has up to now been outweighed to a certain extent by other inherent disadvantages, such as its poor resistance to low temperatures. Neoprene is probably the most akin to natural rubber of all the synthetics, and may be used for practically all purposes for which natural rubber or GR-S is employed, but its relatively high price precludes it from many fields. It is mainly used in cable making and mechanical goods, where its high oil-resistant qualities make it preferable to GR-S or natural. The butadiene-acrylonitrile or N-type rubbers are special purpose rubbers mainly used where oil-resistance is of paramount importance. The relative uses of the various types of synthetic rubber during 1948 are given in the following table:

TABLE 16

Synthetic Rubber Consumption in 1948 by Types

	Butadiene styrene rubber	*Butyl rubber*	*Neoprene rubber*	*N-type rubber*	*Total synthetic rubber*
U.S.A.:					
Transportation	243,400	57,531	589	55	301,575
Non-transportation	101,913	1,339	31,529	5,716	140,497
Total	344,313	58,870	32,118	5,771	442,072
Canada:					
Transportation	12,082	2,588	8	..	14,678
Non-transportation	4,818	45	905	108	5,876
Total	16,900	2,633	913	108	20,554
U.K.	635	125	1,472	323	2,555
France	5,250*	2,000*	54*	98	7,402
Australia	130	17	25	18	190
Rest of World (estimated)	6,500*	250*	250*	250*	7,250
Grand Total	374,000	64,000	35,000	6,500	480,000

* Estimated.

60. The figures of consumption quoted above both for natural and synthetic rubber contain figures of natural and synthetic latices consumed in terms of dry solids weight. So far as latex consumption is concerned, it is generally expected that a large potential market exists for articles made from rubber in its liquid state, particularly for foamed sponge goods. The marked superiority of "foam rubber," as it is generally called, in upholstery, mattresses, etc., does suggest that there will eventually be a large rubber consumption in this field.

61. Latex consumption figures are again only available for a few countries, but the growth of rubber consumption in the form of latex may be judged from the following table, the figures in which have been estimated to a large extent from export and import statistics. All figures represent dry solids weight.

TABLE 17

Estimated Consumption of Natural and Synthetic Latices (in tons d.r.c.)

Year	*Natural latex*	*GR-S latex*	*Neoprene latex*	*Total Synthetic latices*	*Total latices*
1922	250	..	..	..	250
1925	2,000	..	..	..	2,000
1930	5,500	..	..	..	5,500
1935	20,000	..	..	..	20,000
1940	40,000	..	..	..	40,000
1945	4,750	14,250	6,000	20,250	25,000
1946	8,750	23,750	12,500	36,250	45,000
1947	25,000	23,000	7,000	30,000	55,000
1948	47,500	22,500	5,000	27,500	75,000
1949	60,000	20,000	..	..	..

Imports and Stocks in Consuming Areas

62. In many countries statistics of rubber imports are recorded in a very loose manner. A few countries differentiate between natural rubber, synthetic rubber and reclaimed rubber, and a few, a very few, even report latices separately, but in most cases natural and synthetic rubber are lumped together, and not infrequently reclaimed rubber is also included. In quite a few countries waste or scrap rubber and related materials, such as guttapercha and balata, are also reported as *raw* rubber. It is desirable to distinguish the various types of rubber imported in countries not giving sufficient detail, and this is usually possible by reference to the value of the goods entered, and by cross checking from data supplied by exporting countries.

63. A good many countries, the most important being the U.S.S.R., do not publish any adequate import statistics of rubber, and in these countries imports have to be estimated from reported export, re-export and transhipment statistics, allowing a time-lag for the length of voyage.

64. In regard to stocks of natural rubber in consuming territories, these are at the moment divided into those held by government for purposes of national security and those held by industry, i.e. dealers and manufacturers. The latter category may be further subdivided into (*a*) stocks held in territories where, besides manufacturing demand, there is considerable trade in rubber, e.g. the U.K. and the U.S.A., and (*b*) stocks held in territories for domestic manufacturers' requirements only.

65. Industry stocks in the U.K. are held by manufacturers and dealers. Dealers' stocks or U.K. warehouse stocks, as they are usually called, had a special significance before the war. London was the most important rubber market in the world, and dealers held considerable stocks as a hedge against normal trade demands in the U.K. and elsewhere, and purchases by speculators. To-day, with speculation discouraged and the growing tendency of European consumers to import direct from producing areas, warehouse stocks in the U.K. are only a fraction of their pre-war level. In the following table average U.K. warehouse stocks are given compared with total world consumption for the period between the two world wars and during 1947 and 1948.

TABLE 18

U.K. Warehouse Stocks of Natural Rubber in Terms of World Consumption

Year	*Average U.K. warehouse stocks* ('000 *tons*)	*Average Monthly world consumption* (000 *tons*)	*Stock in terms of months' consumption*
1919	26	26	1·0
1920	32	25	1·3
1921	76½	23	3·3
1922	78	33½	2·3
1923	65	37	1·8
1924	52½	38½	1·4
1925	11	46	0·2
1926	28½	45	0·6
1927	67	49½	1·4
1928	43	57	0·8
1929	44½	67	0·7
1930	104½	59	1·8
1931	133	56½	2·4
1932	111*	57½	1·9
1933	94	68	1·4
1934	106½	76½	1·4
1935	166	78	2·1
1936	121	86½	1·4
1937	52	91	0·6
1938	86	78	1·1
1939	58	91	0·6
1947	13	92	0·1
1948	18	118	0·2
1949	16½	119	0·1

* While public warehouse stocks decreased in 1932, it is known that large stocks were being held in private warehouses and by manufacturers.

66. In regard to U.K. manufacturers' stocks, figures of these have been available regularly since the end of 1934. Table 19 shows the actual and relative size of these stocks at the end of each year since data were first available:

TABLE 19

U.K. Manufacturers' Stocks of Natural Rubber ('000 tons)

	Actual Stock	*U.K. Consumption during previous 12 months*	*Stock in terms of months' consumption*
1934	43	90	5·5
1935	45	95	5·7
1936	24	99	2·9
1937	22	115	2·3
1938	19	107	2·1
1939	24	123	2·3
1940	35	147	2·9
1941	33	157	2·5
1942	18	97	2·2
1943	15	74	2·4
1944	10	45	2·7
1945	8	27	3·6
1946	17	97	2·1
1947	27	154	2·1
1948	31	194	1·9
1949	24	184	1·6

From the above table it might be inferred that a U.K. manufacturers' stock of around 2 months' current consumption might be regarded as a "normal" figure.

67. Stocks held in the U.S.A. may, like those in the U.K., be divided into two categories, stocks held by manufacturers and stocks held by dealers and importers. But, whereas U.K. warehouse stocks were, in pre-war days at any rate, held against purchases by manufacturers all over the world, the stocks held by dealers and importers in the U.S.A. are held against purchases by American manufacturers. All the trade stocks in the U.S.A. may therefore be regarded as manufacturers' stocks. Table 20 gives details of the U.S. stock position since 1925 except during the years 1942 to 1945 inclusive when all stocks were taken over by government:

TABLE 20

U.S. Trade Stocks of Natural Rubber

	Manufacturers'		*Dealers', etc.*		*Total Stocks*	
End of	*Actual Stocks* ('000 *tons*)	*% of total*	*Actual stocks* ('000 *tons*)	*% of total*	*Actual* ('000 *tons*)	*In terms of previous 12 months' consumption*
1925	41½	81	9½	19	51	1·6
1926	62	85	10½	15	72½	2·4
1927	80	81	19	19	99	3·2
1928	50½	77	15	23	65½	1·8
1929	68½	58	50	42	118½	3·0
1930	137½	68	63½	32	201	6·4
1931	214	66	108	34	322	10·9
1932	309	82	70	18	379	13·5
1933	251	69	114	31	365	10·6
1934	243½	68	111½	32	355	9·2
1935	180	59	123	41	303	7·4
1936	156½	70	66½	30	223	4·7
1937	167	64	95	36	262	5·8
1938	112	54	97	46	209	5·7
1939	90½	73	34	27	124½*	2·5
1940	144	82	32	18	176	3·3
1941	152	92	13	8	165	2·6
1946	69	100	..	..	69	3·0
1947	..	..	..	..	129	2·8
1948	..	..	..	..	142	2·7
1949	..	..	..	..	105	2·2

* Omitting 1,373 tons of "barter" stock.

It will be noted that there have been wide fluctuations in the stocks held by the American industry, but that throughout the '30's a high level was maintained. According to an official U.S. publication* issued at the beginning of 1946, the natural rubber inventory required by the industry "in accordance with pre-war practice" would be a stock equivalent to 5 months' consumption.

68. Stocks in other consuming countries are mainly held by manufacturers, and it is generally assumed that the practice followed by manufacturers in other countries will not differ greatly from that followed by American and British manufacturers. Some countries are now furnishing stock figures, but where such figures are not available, it is generally assumed that stocks are maintained at around 2 to 3 months' current consumption (see para. 50 above).

69. Although they are not strictly speaking "stocks in consuming territories," reference might be made here to "stocks afloat." The measurement of the quantity of rubber afloat at any time is one of the major gaps in present rubber statistics. No exact data on the amount in transit between producing and consuming areas are available. Before the war stocks afloat at the end of a month were calculated by taking the shipments during the month and adding to them one-third of the shipments during the previous month. Since the war, experience has indicated that shipments during the whole of the previous month should be included.

70. No reference has been made to stocks of synthetic rubber, as so far there has been no trading in synthetic rubber on the same scale as in natural. Synthetic stocks may therefore be regarded as being held merely for current consumption requirements.

Reclaimed Rubber

71. Reclaimed rubber has its own particular fields of use, and long experience has shown that because of, among other things, the characteristics it imparts to certain finished products its continued use can be depended upon even when the price of new rubber is low.

Advantages cited for the use of reclaim are:

(1) Cheapness in comparison to raw rubber.
(2) Less power required for mastication.
(3) It is uniform and can be made to specification.

The extent, however, to which reclaimed rubber can be used is limited, as otherwise it would have an adverse effect on the durability and other properties of the finished article, e.g. articles containing a high percentage of reclaim have poor extensibility and poor resistance to abrasion.

72. Reclaimed rubber, which was, of course, made originally from natural rubber scrap, can now be made also from synthetic rubber or from a mixture of both types.

73. Except in the case of the U.S., consumption data regarding reclaim are scarce. For the U.K. consumption statistics are available since 1935, but for other countries only occasional figures are available.

74. The following statistics cover the use of reclaimed rubber in the U.S.A. since 1921, and show how consumption has varied with the price of raw rubber:

* *First Report of the Inter-Agency Policy Committee on Rubber.*

TABLE 21

U.S. Consumption of Reclaimed Rubber ('000 *tons*)

Year	*Consumption of reclaimed rubber*	*Consumption of new rubber*	*% Reclaim to new rubber*	*Price of reclaim (U.S. cents per lb.)*	*Price of natural (U.S. cents per lb.)*
1921	41·4	177·8	23·3	12·7	16·4
1922	54·5	301·5	18·1	10·2	17·3
1923	69·5	319·4	21·8	9·6	29·6
1924	76·1	328·8	23·1	8·9	26·1
1925	137·1	388·5	35·5	10·4	72·5
1926	164·5	366·2	44·9	10·5	49·4
1927	189·5	373·0	50·8	8·8	37·8
1928	223·0	437·0	51·0	8·2	22·3
1929	212·7	467·4	45·5	7·9	20·5
1930	153·5	376·0	40·8	6·6	10·2
1931	123·0	355·2	34·6	5·1	6·1
1932	77·5	336·7	23·0	4·2	3·4
1933	85·0	412·4	20·6	4·7	5·9
1934	100·6	462·5	21·8	5·3	11·0
1935	117·5	491·5	23·9	5·4	13·9
1936	141·5	575·0	24·6	5·4	12·9
1937	162·0	543·6	29·8	6·2	12·3
1938	120·8	437·0	27·6	6·1	14·6
1939	170·0	593·7	28·6	6·0	17·5
1940	190·2	651·1	29·2	6·0	19·9
1941	251·2	781·3	32·2	6·2	22·1
1942	254·8	394·4	64·6	6·5	22·5
1943	291·1	488·6	59·6	6·5	22·5
1944	251·1	710·8	35·3	6·7	22·5
1945	241·0	799·0	30·2	7·0	22·5
1946	275·5	1,039·3	26·5	7·2	22·5
1947	288·4	1,122·3	25·7	8·0	21·0
1948	261·1	1,069·4	24·4	8·1	22·0
1949	222·7	990·6	22·5	8·4	17·6

75. Reclaimed rubber consumption in the U.K. since 1935 is given in Table 22:

TABLE 22

U.K. Consumption of Reclaimed Rubber ('000 *tons*)

Year	*Consumption of reclaimed rubber*	*Consumption of new rubber*	*% Reclaim to new rubber*	*Price of reclaim (d. per lb.)*	*Price of natural per lb. (s. d.)*
1935	7·7	95·0	8·1	..	0 6
1936	7·9	99·2	8·0	..	0 $7\frac{3}{4}$
1937	10·3	114·6	9·0	..	0 $9\frac{1}{2}$
1938	8·9	106·9	8·3	$3\frac{17}{32}$	0 $7\frac{7}{32}$
1939	10·4	122·7	8·5	$3\frac{23}{32}$	0 9
1940	9·8	147·1	6·7	$4\frac{5}{8}$	1 $0\frac{1}{16}$
1941	8·2	156·6	5·2	$5\frac{13}{32}$	1 $1\frac{19}{32}$
1942	33·8	97·1	34·8	$5\frac{1}{2}$	1 $1\frac{3}{4}$
1943	42·0*	77·7	54·0	$5\frac{1}{2}$	1 6
1944	29·8	87·0	34·2	$5\frac{1}{2}$	1 6
1945	26·7	91·0	29·3	$5\frac{1}{2}$	1 6
1946	25·3	126·8	19·9	$5\frac{1}{2}$	1 $4\frac{7}{8}$
1947	22·5	156·4	14·1	$5\frac{1}{2}$	1 $0\frac{11}{32}$
1948	23·5	196·3	12·0	$5\frac{23}{32}$	1 $0\frac{29}{32}$
1949	20·2	186·6	10·8	$5\frac{1}{2}$	0 $11\frac{25}{32}$

* Includes crumb.

A Few Statistics on the Length of English Words

By Sir William P. Elderton, K.B.E., F.I.A., etc.

In March, 1946, Professor A. C. Aitken, F.R.S., wrote a letter to me in which he mentioned that his friend, Lt.-Col. J. B. Molony, had counted the numbers of monosyllables, dissyllables, etc., in Fitzgerald's *Rubáiyát of Omar Khayyám*, and that the distribution was close to a geometrical progression with a common ratio of 2/9. This seemed to me of interest, and as, a few months later, I wanted some examples of geometrical progressions for a lecture I was preparing for the Insurance Institute of Norwich, I made a first attempt at counting syllables with Gray's "Elegy," a poem of just under 1000 words, and followed it up with a few other counts.* It seemed possible that if extended considerably, we might even find a way of measuring literary vocabulary alternative to that given by G. Udny Yule in his *Statistical Study of Literary Vocabulary* (Cambridge University Press, 1944). I do not pretend that I have made sufficient counts to test this last point, and the object of this paper is merely to give some examples of counts made from a few authors, and then to show how some skew correlation distributions of the kinds mentioned in the paper on cricket scores can be obtained from English literature.

As I have mentioned, my first count was with Gray's "Elegy." The advantage of using a poem is that the metre shows, except in the case of some modern writers, how many syllables the author intended to be attributed to each word; the disadvantage is that in verse a syllable may be introduced that would not be shown in ordinary speech or ignored in some cases when it would generally be shown. Thus, in the "Elegy," I overlooked at first that Gray intends "glimmering" to have two syllables only, and I subsequently found that in his manuscript† he had written "glimm-'ring," though unfortunately it does not so appear in the edition of his work I was using. If I had considered the scansion carefully I should have seen that I had too many syllables in the line, and if we are counting syllables in verse we must abide by what the author meant even if we say, in defence, that he invented a new word "glimring" because it conveyed what he felt better than the existing word "glimmering." Similarly, if a poet used "hushéd" I should count it as two syllables, though in the ordinary way it would be spoken as and counted as a word of one syllable. In verse the metre is decisive as to the syllables, but in prose it may be difficult to decide, and I have followed the *Concise Oxford English Dictionary* whenever it is definite. A few examples may help:

2 syllables: brilliant, sovereign, anxious, ancient, every.
3 syllables: criticism, ingenious.
4 syllables: contemplation.

The *O.E.D.* gives the pronunciation of the last five letters of 'anxious' as -ksh*us*; and schism as sĭ-zm. The word ending "ism" is counted as one syllable, though it is hard to say without implying a slight vowel sound between the "s" and the "m." Gray, in the "Elegy," needs 3 syllables only in "inglorious" and "ingenious" and these are generally so pronounced, but some people seem to make four syllables. There is a difficulty in the "Elegy," and for that matter in other verse, in dealing with the words "many a," which occur in such lines as

"Full many a gem of purest ray serene."

The scansion can be made by regarding "many a" as one word and treating the "y" as a consonant. I have done this; it would come to the same thing it we assumed that the word "a" has been slurred away.

* The lecture was given on March 17th, 1947, and entitled "Cricket Scores, Fire and Accident Claims and Gray's 'Elegy'." I had mentioned at the Royal Statistical Society in November, 1944, in my reply at the end of the discussion on the papers on cricket scores by me and by G. H. Wood, that fire claims, in common with many other things, gave a J-shaped distribution (see *J.R.S.S.*, vol. cviii, pp. 1–39, and the corrections to misprints in the discussion, p. 261).

† I have not seen the MS., but the word occurred among a few verses reproduced photographically in the *Illustrated London News*.

With all these adjustments we reach col. (2) of Table 1, and in col. (3) I have shown the distribution of syllables if all the words are treated as in prose. It may be mentioned that the 713 words of one syllable in Col. 2 are not the same 713 words as those in Col. 3.

TABLE 1

Number of syllables	*Number of words in Gray's "Elegy" having the syllables shown in Col. (1)*			
	Words adjusted to metre		*Words treated as if prose*	
	Number	*Per mille*	*Number*	*Per mille*
(1)	(2)	(2a)	(3)	(3a)
1	713	732	713	724
2	226	232	224	228
3	26	27	38	39
4	8	8	8	8
5	1	1	1	1
Total	974		984	
Average number	1·31		1·33	

Before proceeding to the next step we may mention two things:

(I) If we mean to compare the lengths of words used by different authors, or even by the same author at different times, it will be well to decide in advance how hyphened words should be counted. For instance, it would seem wrong to count "yew-tree," "yewtree," "yew tree" differently merely because of an author's or printer's taste. My personal inclination would be to separate the linked words except in cases in which an author puts a hyphen where he might have used a single word, e.g. today, tomorrow, noontide, pathway. For Table 1 a hyphen was considered to make the word one, so that "incense-breathing" was one word of 4 syllables.

(II) Yule, in the book already mentioned, has drawn attention to the repetition of a given noun in a particular work: this may be due to the subject matter or to the author's style. If a word of 4 syllables is repeated many times it follows that we shall find an excess of 4-syllable words unless there is an offset by an appropriate number of repetitions of words of other lengths.

The next step was to work on some of Gray's other poems, and Table 2 gives the result, with a subdivision showing the effect of Gray's hyphens.

The length of words varied little between the different poems, but the figures may be compared with those that were obtained from words used by Gray in his letters. I took a few letters written to six correspondents and, judging from this set, Gray used more one-syllable words and more long words in his letters than in his verse but fewer words of two syllables. Some long words, natural in prose, may be unsuitable to Gray's metres, where the feet are always of two syllables.

The next counts made were from Carlyle and Macaulay, both of whom use more long words.

In each book two separated samples of about 500 words were counted, each 500 being continuous.

With Macaulay I counted nearly 2,000 words from the Essay on Clive chosen at random from four different pages in a similar way, and also made a couple of small counts from the Essays on Hastings and Milton. The results are given in Table 5, from which it will be seen, if these small samples suffice, that about 15 per cent. of the words used by Macaulay were of 3 or more syllables, and that only two-thirds of the words were of 1 syllable.

The proportion of 15 per cent. was checked with a much larger sample by estimating the number of words on a page and counting the words of 3 or more syllables.

TABLE 2

Number of syllables (1)	Elegy (2)	Sonnet on West (3)	Eton (4)	Adversity (5)	Spring (6)	Favourite Cat (7)	Total (8)	Per mille (8a)
	Number of words in Gray's Poems having the syllables shown in col. (1)							
	(A, as versified, hyphened words counted as one)							
1	713	75	392	205	206	178	1,769	723
2	226	31	130	81	61	56	585	239
3	26	1	24	7	6	6	70	29
4	8	—	4	5	6	—	23	9
5	1	—	—	—	—	—	1	·4
Total	974	107	550	298	279	240	2,448	
Average number	1·31	1·31	1·35	1·33	1·33	1·28	1·33	
	(B, as if prose, hyphened words counted as one)							
1	713	76	396	206	206	179	1,776	719
2	224	29	128	74	59	54	568	231
3	38	3	26	14	9	8	98	40
4	8	—	5	5	5	—	23	9
5	1	—	—	—	1	—	2	1
Total	984	108	555	299	280	241	2,467	
Average number	1·33	1·32	1·35	1·39	1·33	1·29	1·34	
	(C, as if prose, hyphened words separated as in (I) on p.)							
1	720	78	397	207	209	179	1,790	721
2	227	28	131	77	66	54	583	235
3	37	3	25	13	9	8	95	38
4	5	—	4	4	2	—	15	6
5	1	—	—	—	—	—	1	·4
Total	990	109	557	301	286	241	2,484	
Average number	1·32	1·31	1·35	1·38	1·32	1·29	1·33	

TABLE 3

Number of syllables (1)	*Words in sample of Gray's letters having the syllables of Col.* (1): Number (2)	Per mille (2a)
1	3,987	761
2	831	159
3	281	54
4	121	23
5	15	3 (5 and 6)
6	2	
Total	5,237	
Average number	1·35	

TABLE 4

Words in samples from Carlyle's works

Number of syllables	*French Revolution*	*Past and Present*	*Heroes and Hero Worship*	*Total*	*Per mille*
(1)	(2)	(3)	(4)	(5)	(6)
1	713	692	811	2,216	696
2	180	193	208	581	182
3	92	73	88	253	80
4	32	34	39	105	32
5	8	12	5	25	8
6	2	3	—	5	2
Total	1,027	1,007	1,151	3,185	
Average number	1·49	1·51	1·46	1·48	

TABLE 5

Words in samples from Macaulay's Essays

Number of syllables	*Clive*	*Hastings and Milton*	*Total*	*Per mille*
(1)	(2)	(3)	(4)	(5)
1	1,260	723	1,983	662
2	375	189	564	189
3	201	95	296	99
4	67	42	109	37
5	14	17	31	10
6	5	2	7	3
7	—	1	1	
Total	1,922	1,069	2,991	
Average number	1·55	1·55	1·55	

It is interesting to notice that, though Macaulay used in his essays more long words than Gray used in his letters, the poem "Horatius" contains under 4 per cent. of words of 3 or more syllables. We make three comments. It is, I think, sometimes a little doubtful as to how Macaulay meant his syllables to be counted; many of the longer words are the names of individuals or places; the repetition of a line or part of a line has a definite effect on the count, e.g. "In the brave days of old" occurs half-a-dozen times (*cf.* Yule).

I came across one rather surprising use of long words in Scott's *Tales of a Grandfather*, where, judging from rough counts over many pages, about 15 per cent. of the words used have 3 or more syllables; as he was writing for children it might have been expected that he would have used long words as little as possible.

An interesting case of word lengths arises with Swinburne. His blank verse dramas have nearly 90 per cent. of one-syllable words, and in "Chastelard" and "Bothwell" sets of 250 consecutive words can be found containing nearly 95 per cent. of such short words. After noticing this I tried his "Tristram of Lyonesse," which is written in ten-syllable rhymed couplets, and found a high proportion of short words—a much higher proportion than in Tennyson's "Idylls of the King"; the comparison with the Idylls seems natural owing to the subject, though the Idylls are in blank verse. In many of his poems Swinburne uses a fair proportion of two-syllable words because they fit into his metres or his rhymes, but even in the poems he seldom uses much less

than 80 per cent. of words of one syllable. Considering his reputation this seemed a little surprising, and I wondered if his prose was constructed on similar lines. So far, however, as his statements on the life and character of Mary, Queen of Scots, are concerned, he uses a fairly large number of long words. I did not make detailed counts, but contented myself with counting the proportion of one-syllable words in the plays and some of the poems; I did this over so many pages that I do not think a further analysis would modify the statements made above.

We may add here (see Table 6) two more authors showing a very high proportion of long words. The statistics relating to them will be dealt with from a different point of view at a later stage.

TABLE 6

Number of syllables	*Number of words in samples from*		
	Johnson	*Gibbon*	*Gibbon*
(1)	(2)	(3)	(4)
1	1,268	1,243	617
2	423	408	206
3	195	224	118
4	77	103	50
5	29	21	6
6	8	1	3
Total	2,000	2,000	1,000
Average number	1·60	1·63	1·63

Two sets of counts were made from Gibbon's *Decline and Fall.* The one in col. 3 was obtained from ten separate pieces of 200 words each. The one in col. (4) was obtained by taking the first complete word in the third line from the bottom of 1,000 pages. This second count was made because I thought it possible that 200 consecutive words might exaggerate the repetition of a particular word because the author would not be varying his subject-matter much within that range. But judging from the figures the differences are unimportant, especially if we group the words of 5 and 6 syllables together.

It will be seen from Table 6 that, judging by these samples, both Johnson and Gibbon used under 65 per cent. of one-syllable words, that Gibbon used 17 per cent. of words of 3 or more syllables, and that though Johnson only slightly exceeded 15 per cent. of words of this category as a whole, he used larger numbers of words of 5 and 6 syllables.

Near the other end of the scale we find that in the authorized version of the Bible a miscellaneous number of counts of one-syllable words showed that they were about 80 per cent. of the whole, and this figure is slightly exaggerated as compared with our other examples because the

TABLE 7

Number of syllables	*Number of words from Genesis (mainly) and Exodus*			
	Including all words	*Per mille*	*Excluding names, etc.*	*Per mille*
(1)	(2)	(2a)	(3)	(3a)
1	2,943	815	2,939	832
2	579	160	520	147
3	77	21	61	17
4	12	3	11	3
5	2	·6	2	·6
Total	3,613		3,533	
Average number	1·21		1·19	

retention of the -th terminal, as in giveth, creates a two-syllable word. In making these counts proper names and names of places were not eliminated; they occur fairly frequently and are, on average, longer than ordinary words in the Bible. I give in Table 7 details of subsequent counts from Genesis and Exodus.

This shows to a certain extent the effect of names of people, places, etc. Naturally I did not want counts involving a large number of proper names, and have not included in my counts the genealogical lists that occur in the Gospels or a page which was obviously full of names. The book used was the Eversley Bible; each page has about 260 words.

A tabulation of the lengths of words in Shakespeare's *Henry IV* (Part 2) separating the prose from the verse is shown in Table 8. Proper names were not excluded in counting. Approximate counts that had previously been made from Scenes of a few other plays gave similar results so far as the proportion of words of one syllable can be taken as a guide.

TABLE 8

Number of syllables	*Number of words in* Henry IV, *Part* 2			
	In prose passages	*Per mille*	*In verse passages*	*Per mille*
(1)	(2)	(2a)	(3)	(3a)
1	10,965	800	9,076	784
2	2,177	159	1,918	165
3	430	32	476	41
4	99	7	108	
5	23		4	10
6	2	2	—	
7	2		—	
Total	13,698		11,582	
Average number	1·25		1·28	

As an example of Elizabethan prose, Table 9 gives the distribution of words in eleven of Bacon's Essays. Within those eleven, the percentage of one-syllable words lay between 70 and 77 and some rougher counts of other Essays did not give anything outside that range.

TABLE 9

Number of syllables	*Words in eleven of Bacon's Essays*	
	Number	*Per mille*
(1)	(2)	(2a)
1	4,640	730
2	1,080	170
3	420	66
4	167	27
5	41	
6	3	7
7	1	
Total	6,352	
Average number	1·41	

Before we leave this part of the subject a few comments may be made. The first is that the samples are very small compared with the works from which they are taken except in the case of Gray's Poems, for his total output of poetry was small. The second that several people have made counts of one kind or another; in A. C. Bradley's volume of essays, *A Miscellany*, is one entitled "Monosyllabic Lines and Words," in which he gives some counts of, apparently, 100 words, and records the number of monosyllables, and he also gives the number of lines in certain poetical works that are completely monosyllabic. Though the essay is interesting he has, in some cases, selected passages that he regarded as typical of an author, and this is scarcely a practice that a statistician would care to follow. Far more valuable counts were made by Yule, and he gives many counts of nouns; among them he discusses the number of occurrences for nouns of one syllable in Bunyan and Macaulay (see his Chapter 6), and remarks that the proportions among occurrences are only 34·5 per cent. for Macaulay and 55·3 for Bunyan. The figure given for Macaulay can be compared with Table 5, col. (5); we have statistical evidence for what, I think, most people would guess, namely, that prepositions, conjunctions, pronouns, articles are nearly all short and are used over and over again by any author. The remarks on Yule's counts of one-syllable nouns brings me to my third comment—that anyone who makes a rough estimate of the occurrence of words of different lengths will appreciate the wealth and importance of words of one syllable in English speech and writing, and anyone who doubts the statistical evidence might try to write 1,000 words on any ordinary subject allowing himself only half the words as monosyllables. It may be possible, but I think the result would be poor as English.* This brings me to my fourth comment: I have taken all the examples from general literature; if I had resorted to technical or scientific works I could have found many long words and they would possibly have been repeated many times, but I do not think such works are suitable tests of vocabulary. I sometimes wonder if the language used is English! A fifth comment is to agree with Yule that accurate counting is hard to achieve: it is easy to adopt a rule and overlook it later, and it is also easy to make slips. I hope we have made no important errors. My next comment is that the distributions (except perhaps Johnson) are not sufficiently near to geometrical progressions to be so described. It seems almost unnecessary to say that the reader can get the first term of a geometrical progression (i.e. the proportionate number of words of one syllable) by taking the reciprocal of the mean. The common ratio is the difference between this and unity. We then reach the proportionate frequencies quickly. Thus taking the frequencies per mille from earlier tables the following (Table 10) gives comparisons with geometrical progressions.

TABLE 10.—*Comparisons with Geometrical Progressions*

	Gray's letters		*Carlyle*		*Macaulay*		*Johnson*		*Gibbon*	
Number of syllables	*Table* 3, *col.* (2*a*)	*g.p.*	*Table* 4, *col.* (6)	*g.p.*	*Table* 5, *col.* (5)	*g.p.*	*Table* 5, *col.* (2) *halved*	*g.p.*	*Table* 5 *col.* (3) × *col.* (3*a*) ÷ 3	*g.p.*
1	761	741	696	676	662	645	634	625	620	613
2	159	192	182	219	189	229	212	234	205	237
3	54	54	80	71	99	81	97	88	114	91
4	23	17	32	23	37	29	39	33	51	36
5	3	1	8	8	10	10	14	12	9	14
6 and over	·3	—	2	3	3	·6	4	8	1	9

Because of the repetition of particular words by an author, deviations from a graduation by a simple formula might be—I think would be—greater than those to which we are accustomed in text-book examples. The deviations seem to be similar to those that occur in a mortality experience based on policies of assurance where there are varying numbers of policies on the lives assured (see H. L. Seal, *Skandinavisk Aktuarietidskrift*, vol. xxx, pp. 18–43).

* In the *Times Lit. Supp.*, September 2nd, 1949, the following is quoted in a review: "Deteriorationism began to supplant perfectibilitarianism." Even the author who wrote this used just under 60 per cent. of one-syllable words if two quotations (about 230 words) afford a fair measure.

My final comment is that the distributions are more usual in kind than those, found by Yule for the occurrences of nouns, which descend quickly at the start and then become flat and continue to cases where there are hundreds of occurrences. And if we imagine a sufficiently voluminous work no noun need have been used only once, and the nouns most frequently used may occur thousands of times. This is no criticism of Yule's work. I think the counting of words in terms of syllables avoids the sampling difficulties which Yule overcame, but I should not think it could give so refined a test of authorship as Yule evolved. I do not, however, think that anyone could fail to distinguish between a count of 500 from, say, Gray's letters, or the authorized version of the Bible on the one hand and Carlyle, Macaulay, Johnson or Gibbon on the other. If we could confirm the proportionate lengths of words given in our tables by counts of 10,000 or 20,000 we might be able to fit authors to some sort of scale, but such labour, if it is worth while, would be beyond my capacity.

We may now turn to the other object I had in view, which was to see if skew correlation distributions of some of the kinds mentioned in my paper on cricket scores could be obtained from literature.

As the distribution of syllables is J-shaped there seemed a possibility of finding a double J-distribution by correlating the number of syllables with the number of vowels, and of finding distributions which are J-shaped for one variable and of the skew binomial type for the other by correlating syllables with either the number of letters in a word or the number of consonants. But if tables were made from Gray's letters, for instance, we should have to make a very large count to obtain a fair number of words of many syllables, and it seemed preferable to base the tables on the work of an author who used a high proportion of long words: the tails of the distribution will then be seen more easily.

I give two sets of tables (Tables 11 and 12) each relating to 2,000 words, one from Johnson and the other from Gibbon. The former were built up from *Preface to Shakespeare*, *Notes on Henry IV* and *Lives of the Poets* (*Milton*). The latter, as already mentioned, was built up from ten separate pieces of 200 words each. There cannot be a syllable without a vowel, so that a section of the table must be empty just as it will be in some theoretical distributions, as, for instance, when we correlate the "First sequence" of reds in *n* cards with the number of diamonds within the "first sequence," or if we correlate the total cricket score made by a side with the sum of the scores of, say, the first two batsmen.*

I may add by way of comment (1) that none of the means of the columns or the rows in Tables 11 and 12 are in a straight line; (2) that the distribution of vowels is not a geometrical progression; (3) that no measure of correlation has been attempted; (4) that such 2-variable tables seem unlikely to be of value in the study of literary vocabulary, and are given as mere examples of skew distributions on which theorists may be interested to work; (5) the distributions differ considerably from the contingency tables given by Yule (e.g. on his p. 253), where he is

TABLE 11.—*Samuel Johnson—2,000 Words*

Vowels	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
1	1,021	..	..	..	..	..	1,021	1·00
2	229	240	..	..	..	..	469	1·51
3	18	153	102	..	..	..	273	2·31
4	..	29	84	51	..	..	164	3·14
5	..	1	8	23	20	..	52	4·19
6	..	..	1	3	9	3	16	4·94
7	..	..	..	..	..	5	5	6·00
Total	1,268	423	195	77	29	8	2,000	
Mean	1·21	2·50	3·53	4·38	5·31	6·62		

* The total number of letters = total vowels + total consonants. It can be seen that in Table 9 there are two differences of 1. They could only have been traced by going through all the counts again, and this seemed unnecessary, but I apologize and regret.

TABLE 11—*contd*

Consonants	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
0	26	..	..	..	..	..	26	1·00
1	442	6	..	..	..	..	448	1·01
2	467	75	3	..	..	..	545	1·17
3	241	89	15	2	..	..	347	1·36
4	85	129	51	11	2	..	278	1·98
5	5	100	56	26	7	1	195	2·66
6	2	20	40	26	9	4	101	3·32
7	..	2	24	9	6	1	42	3·52
8	..	2	6	2	2	1	13	3·54
9	..	..	..	1	3	..	4	4·75
10	..	..	..	..	..	1	1	6·00
Total	1,268	423	195	77	29	8	2,000	
Mean	1·95	3·76	5·08	5·50	6·37	6·75		

Letters	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
1	26	..	..	..	..	..	26	1·00
2	388	..	..	..	..	..	388	1·00
3	443	5	..	..	..	..	448	1·01
4	231	62	..	..	..	..	293	1·21
5	136	55	..	..	..	..	191	1·28
6	37	108	11	..	..	..	156	1·83
7	5	109	33	2	..	..	149	2·21
8	2	63	49	8	..	..	122	2·51
9	..	16	53	21	2	..	92	3·10
10	..	3	32	25	4	..	64	3·47
11	..	2	11	12	9	1	35	3·89
12	..	..	6	7	9	1	23	4·21
13	..	..	..	1	1	3	5	5·40
14	..	..	..	1	2	1	4	5·00
15	..	..	..	..	2	1	3	5·33
16	..	..	..	..	..	1	1	6·00
Total	1,268	423	195	77	29	8	2,000	
Mean	3·16	4·27	8·61	9·87	11·58	13·37		

TABLE 12.—*Gibbon—*2,000 *Words from* Decline and Fall

Vowels	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
1	971	..	..	..	..	..	971	1·00
2	257	205	..	..	..	..	462	1·44
3	15	176	106	..	..	..	297	2·31
4	..	27	105	61	..	..	193	3·17
5	..	..	13	42	13	..	68	4·00
6	..	..	..	..	8	1	9	5·11
Total	1,243	4 08	224	103	21	1	2,000	
Mean	1·23	2·57	3·58	4·40	5·38	6·00		

TABLE 12—*contd*

Consonants	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
0	38	..	..	..	..	..	38	1·00
1	390	8	..	..	..	..	398	1·02
2	513	47	3	..	..	..	563	1·09
3	211	95	21	6	..	..	333	1·53
4	78	145	55	21	1	..	300	2·07
5	11	79	59	32	9	..	190	2·73
6	2	26	55	14	2	..	99	2·88
7	..	7	24	18	4	1	54	3·41
8	..	1	7	9	4	..	21	3·76
9	..	..	..	3	1	..	4	4·25
Total	1,243	408	224	103	21	1	2,000	
Mean	1·95	3·86	5·08	5·54	6·19	7·00		

Letters	*Syllables* 1	2	3	4	5	6	*Total*	*Mean*
1	37	..	..	..	..	..	37	1·00
2	358	..	..	..	..	..	358	1·00
3	435	3	..	..	..	..	438	1·01
4	220	37	..	..	..	..	257	1·14
5	149	56	2	..	..	..	207	1·28
6	34	122	9	..	..	..	165	1·85
7	9	100	36	4	..	..	149	2·23
8	1	65	58	17	..	..	141	2·67
9	..	18	52	22	1	..	93	3·07
10	..	7	50	25	6	..	88	3·34
11	..	..	12	14	5	..	31	3·78
12	..	..	4	14	4	..	22	4·00
13	..	..	1	7	..	1	9	4·11
14	..	..	..	..	4	..	4	5·00
15	..	..	..	..	1	..	1	5·00
Total	1,243	408	224	103	21	1	2,000	
Mean	3·18	6·42	8·66	9·94	11·57	13·00		

comparing the number of occurrences of nouns in the Imitatio with samples from à Kempis and with samples from Gerson.

In addition to the names already mentioned I must express my thanks to Mr. Robert E. F. Green, who did some of the counts and made many helpful suggestions.

The Distribution of Wars in Time

(Comments on L. F. Richardson's paper, *J. R. Statist. Soc.*, **107** (1944), 242)

By J. E. Moyal

In a recent paper L. F. Richardson (1) analysed Quincy Wright's data (2) on the number of outbreaks of war per year between A.D. 1482 and A.D. 1939; he found that a Poisson law gave a good representation of the distribution of these numbers in intervals of one and three years, with the mean proportional to the interval, but not for intervals of nine years and above (except in the case of two intervals of 216 years each). It is well known that a Poisson distribution may arise in a time series of this type if there is a probability $\lambda\delta t$ of a single outbreak in a small interval of time δt, where λ is a constant, and a probability of order $(\delta t)^2$ for more than one outbreak, the probability $\lambda\delta t$ being independent of the number of outbreaks previous to the interval δt. Such a model may form a useful first approximation in the present case, the deviations from it arising, e.g., from small internal correlations in the data, the number of outbreaks in the year $n+1$ not being quite independent of the number in the year n; or alternatively, there may be no internal correlation, but λ may not be quite constant, but vary slowly with time. I propose here to re-examine Wright's data in order to decide whether one of these two alternative hypotheses can explain Richardson's results.

It may perhaps be useful (though this is a well-known result: *cf.* Bartlett (3)) to show how the Poisson distribution arises in the above model. Let $p_n(t)$ be the probability of n outbreaks in the interval from 0 to t; then obviously

$$p_n(t+\delta t) = p_{n-1}(t)\,\lambda\delta t + p_n(t)(1-\lambda\delta t) + O(\delta t) \quad . \quad . \quad . \quad . \quad (1)$$

We form the probability generating function

$$\varphi(Z, t) = \sum_0^\infty Z^n p_n(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It then follows from (1) that

$$\varphi(Z, t+\delta t) = \sum_0^\infty p_n(t+\delta t)\,Z^n = \lambda\delta t \sum_1^\infty p_{n-1}(t)\,Z^n + (1-\lambda\delta t)\sum_0^\infty p_n(t)\,Z^n + o(\delta t)$$

$$= (1-\lambda\delta t + Z\lambda\delta t)\,\varphi(Z, t) + o(\delta t).$$

Hence, making δt tend to 0, we find

$$\frac{d\varphi}{dt} = \lambda(Z-1)\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If we take as initial condition that there are no outbreaks at $t = 0$, i.e. $p_0(0) = 1$ and $p_n(0) = 0$ for $n > 0$, then $\varphi(Z, O) = 1$ and the solution of (3) is simply

$$\varphi(Z, t) = e^{\lambda t(Z-1)} = e^{-\lambda t}\sum_0^\infty \frac{(\lambda t)^n}{n!} Z^n \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence the Poisson law for the probability of n outbreaks in time t

$$p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Richardson's analysis for the distribution in yearly intervals is repeated in Table 1 for the whole period of $N = 458$ years covered by Wright's data. The observed frequencies of years with 0, 1, 2, . . . outbreaks are compared with the theoretical frequencies derived from the Poisson law (5): with the year as unit of time, the maximum likelihood estimate of λ is the mean

number of outbreaks per annum which comes out as 0·670. The computation of χ^2 from Table 1 with the last three classes grouped gives 2·798, the 5 per cent. level with two degrees of freedom being 5·991. The variance $v = 0{\cdot}7383$, and the statistic $g = v/\lambda = 1{\cdot}1019$, the expectation value of g being 1, and its standard error $t = \sqrt{2/N} = \sqrt{2/458} = 0{\cdot}066$. Thus both the χ^2 and the g test indicate a good fit for the Poisson law.

In order to study the possibility of internal correlations between the number of outbreaks in successive years, the frequencies n_{rs} of pairs of years with s outbreaks succeeding r outbreaks were computed: the results are shown in Table 2. The autocorrelation coefficient computed from these data comes out

$$\rho = -0{\cdot}0066.$$

In the assumption of independence, ρ is approximately normally distributed with standard error $1/\sqrt{N} = 0{\cdot}0467$, i.e. more than five times the observed value of $|\rho|$. As an additional test, the observed frequencies n_{rs} were compared with theoretical frequencies Np_rp_s calculated in the assumption of independence, with p_r, p_s given by the Poisson law (5), using the previously estimated value of $\lambda = 0{\cdot}670$. The results are shown in Table 3. Estimation of χ^2 gives

$$\chi^2 = 9{\cdot}2048$$

with 10 degrees of freedom (taking into account the fact that λ was estimated from the data), the 5 per cent. level being 18·307. These two tests, therefore, give no evidence of any dependence between numbers of outbreaks in successive years.

Autocorrelations were also computed for various intervals ranging from 2 to 25 years. The results are given in Table 4. They seem to indicate some small positive correlation for intervals greater than one year, though only the values for 5 and 15 years are distinctly significant. This result may possibly be spurious, and be due to a secular variation of the mean rate as will appear below.

The alternative hypothesis is a slow secular variation in λ. In order to test for this, the means and variances were calculated for 9 successive periods of 50 years each, from 1490 to 1939; the results are shown in Table 4. The standard error of the g statistic ($g = v/\lambda$) in any such period is $\varepsilon = \sqrt{2/50} = 0{\cdot}2$. The largest value of $|g - 1|$, which occurred for the third period (1590–1639), was 0·344; all other values were $\leqslant 0{\cdot}2$. These results seem, therefore, consistent with the hypothesis of a Poisson distribution of type (5), with a slowly varying rate of outbreaks λ.

In order to study the form of this variation, a 50-year moving average of the number of outbreaks was computed over the whole period of 458 years. These are shown plotted as ordinates on a graph (Fig. 1), the abscissae being the centres of the 50-year intervals, taken every 5 years. The points are seen to lie without excessive scattering on a smooth curve with two marked maxima at about A.D. 1625 and 1830, a marked minimum at A.D. 1740, and a less marked maximum and minimum at A.D. 1555 and 1570 respectively.

In conclusion, the distribution of numbers of outbreaks of wars in time appears to be consistent with the hypothesis of a probability $\lambda\delta t$ of outbreak in any small interval of time δt, independent of the number of previous outbreaks, and with a rate λ that varies in a roughly periodic fashion, the ½ period being approximately 100 years.

Mr. Richardson has, however, pointed out to me that a certain arbitrariness is introduced in Quincy Wright's data through his definition of what constitutes a "war," involving as it does his judgment of its "importance," a point about which opinions may differ. This makes it difficult to compare events separated by large intervals of time, and renders somewhat dubious the conclusions reached above about a periodic variation in the rate of outbreaks of wars.

I am indebted to Mr. B. J. Prendiville for his suggestions, and to Mr. Prendiville and Mr. D. F. Ferguson for help in the computations.

References

(1) Richardson, L. F. (1944), *J. Roy. Statist. Soc.*, **107**, 242.
(2) Wright, Q. (1942), *A Study of War*. Chicago.
(3) Bartlett, M. S. (1946), "Stochastic Processes." Notes of a lecture course given at the University of North Carolina (Fall Quarter, 1946).

TABLE 1

Number of outbreaks per year	0	1	2	3	4	5	*Total*
Observed frequencies	242	145	49	15	4	0	458
Theoretical frequencies	234·4	157·0	52·6	11·7	2·0	0·3	458

$\lambda = \frac{307}{458} = 0\cdot670$ $\chi^2 = 2\cdot798$

$\nu = 0\cdot7383$ $g = \nu/\lambda = 1\cdot1019$ st. error of $g = 0\cdot0661$

$\rho = -0\cdot0066$ st. error $= 0\cdot0467$

TABLE 2

						Total
	n_{00} 131	n_{01} 73	n_{02} 25	n_{03} 11	n_{04} 2	242
	n_{10} 79	n_{11} 48	n_{12} 17	n_{13} 2	n_{14} 2	148
	n_{20} 23	n_{21} 18	n_{22} 6	n_{23} 1	n_{24} 0	48
	n_{30} 7	n_{31} 6	n_{32} 1	n_{33} 1	n_{34} 0	15
	n_{40} 2	n_{41} 2	n_{42} 0	n_{43} 0	n_{44} 0	4
Total	242	147	49	15	4	457

The corresponding thoretical values are:

TABLE 3

119·704	80·178	26·864	5·973	1·172
80·178	53·703	17·994	4·000	0·785
26·864	17·994	6·029	1·340	0·263
5·973	4·000	1·340	0·298	0·058
1·172	0·785	0·263	0·058	0·011

which gives, after grouping, a value for $\chi^2 = 9\cdot2048$ with 10 degrees of freedom.

TABLE 4

Interval in years	1	2	3	4	5	10	15	25
Autocorrelation	−0·00066	0·1167	0·0856	0·0095	0·1455	0·1115	0·1459	0·1083

TABLE 5

Dates	*Number of outbreaks*•	*Mean*	$49v$	$g=v/\lambda$	$\mid g-1 \mid$
1490–1539	25	·50	22·50	0·9184	0·0816
1540–1589	33	·66	35·22	1·0891	0·0891
1590–1639	38	·76	51·12	1·3728	0·3728
1640–1689	41	·82	33·38	0·8412	0·1588
1690–1739	16	·32	12·88	0·8216	0·1784
1740–1789	20	·40	16·00	0·8163	0·1837
1790–1839	51	1·02	46·98	0·9216	0·0784
1840–1889	45	·90	46·50	1·0544	0·0544
1890–1939	37	·74	39·62	1·0927	0·0927

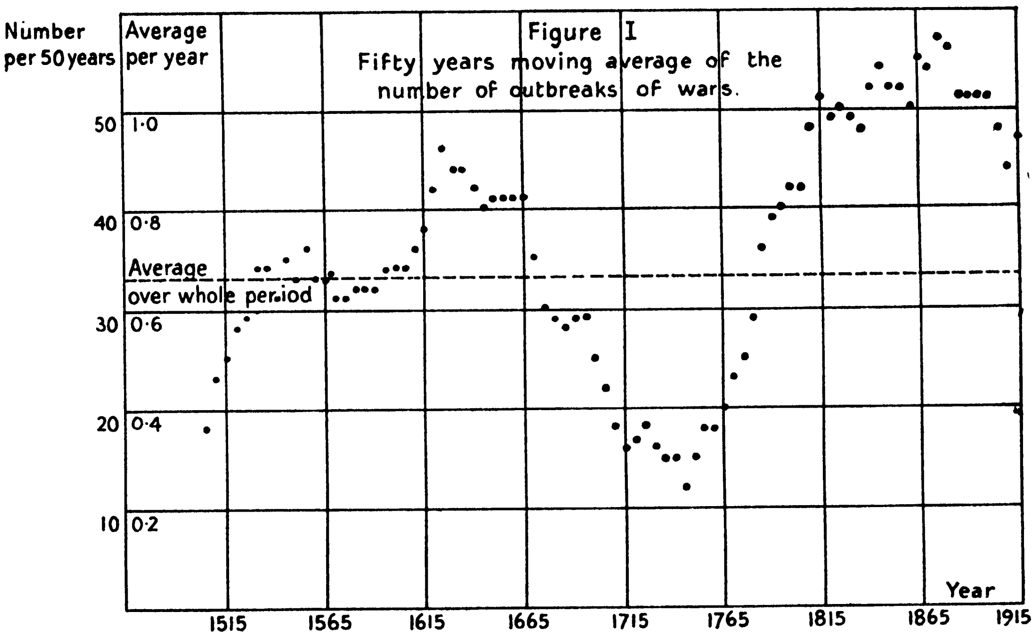


FIG. 1.

Wholesale Prices in 1948

By the Editor of "The Statist"

Table I

The Statist's *Annual Index Numbers* (*in continuation of Sauerbeck's figures*) (1867–77 = 100)

Year	Average No.	Year	Average No.	Year	Average No.	Year	Average No.	Year	Average No.
1948	260	1927	122	1906	77	1885	72	1864	105
'47	230	'26	125	'05	72	'84	76	'63	103
'46	186	'25	136	'04	70	'83	82	'62	101
'45	164	'24	139	'03	69	'82	84	'61	98
'44	160	'23	128	'02	69	'81	85	'60	99
'43	155	'22	131	'01	70	'80	88	'59	94
'42	151	'21	155	'00	75	'79	83	'58	91
'41	142	'20	251	1899	68	'78	87	'57	105
'40	128	'19	206	'98	64	'77	94	'56	101
'39	94	'18	192	'97	62	'76	95	'55	101
'38	90	'17	175	'96	61	'75	96	'54	102
'37	102	'16	136	'95	62	'74	102	'53	95
'36	88	'15	108	'94	63	'73	111	'52	78
'35	83	'14	85	'93	68	'72	109	'51	75
'34	81	'13	85	'92	68	'71	100	'50	77
'33	78	'12	85	'91	72	'70	96	'49	74
'32	79	'11	80	'90	72	'69	98	'48	78
'31	82	'10	78	'89	72	'68	99	'47	95
'30	96	'09	74	'88	70	'67	100	'46	89
'29	114	'08	73	'87	68	'66	102	'18	159*
'28	119	'07	80	'86	69	'65	101	'10	171*

* Jevons's numbers adjusted.

Table II

The Statist's *Annual Index Numbers—ten-year averages* (1867–77)

1848–1857 = 89	1899–1908 = 72	1913–1922 = 153	1927–1936 = 95
'58– '67 = 99	1900– '09 = 73	'14– '23 = 157	'28– '37 = 93
'68– '77 = 100	'01– '10 = 73	'15– '24 = 162	'29– '38 = 90
'78– '87 = 79	'02– '11 = 74	'16– '25 = 165	'30– '39 = 88
'88– '97 = 67	'03– '12 = 76	'17– '26 = 164	'31– '40 = 91
'90– '99 = 66	'04– '13 = 77	'18– '27 = 159	'32– '41 = 97
'91–1900 = 66	'05– '14 = 79	'19– '28 = 152	'33– '42 = 104
'92– '01 = 66	'06– '15 = 82	'20– '29 = 142	'34– '43 = 111
'93– '02 = 66	'07– '16 = 88	'21– '30 = 127	'35– '44 = 119
'94– '03 = 66	'08– '17 = 98	'22– '31 = 120	'36– '45 = 127
'95– '04 = 67	'09– '18 = 110	'23– '32 = 115	'37– '46 = 137
'96– '05 = 68	'10– '19 = 123	'24– '33 = 110	'38– '47 = 150
'97– '06 = 70	'11– '20 = 146	'25– '34 = 104	'39– '48 = 167
'98– '07 = 71	'12– '21 = 148	'26– '35 = 99	

Monthly Fluctuations of the Index Numbers * *of* 45 *Commodities*, 1867–77 = 100

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1898	62·8	63·4	63·0	65·5	66·4	64·7	64·3	64·0	63·9	63·6	63·9	63·8	64
1901	72·2	71·7	71·0	70·6	70·5	69·8	69·5	69·8	69·6	69·6	69·0	68·4	70
'03	69·5	70·2	70·4	69·4	69·6	69·5	69·5	70·0	69·1	69·0	69·0	70·0	69
1904	70·4	70·8	70·8	70·5	69·9	69·4	69·9	70·4	70·7	71·0	71·2	70·9	70
'05	71·2	71·4	71·8	72·0	71·7	72·0	72·5	72·3	72·4	73·2	74·2	74·9	72
'06	75·2	75·0	75·7	76·5	77·0	76·9	76·4	76·7	77·5	78·5	78·6	79·7	77
'07	80·0	80·7	80·0	80·7	82·4	82·0	81·1	79·4	79·1	78·8	76·7	76·2	80
'08	76·0	74·5	74·1	73·8	73·6	72·9	73·1	72·2	72·5	72·2	72·2	72·3	73
1909	72·0	71·9	72·4	74·3	75·4	75·1	75·2	74·9	74·7	75·2	75·5	76·3	74
'10	77·1	78·1	79·1	78·5	78·2	76·9	78·1	78·2	77·6	77·2	77·8	77·9	78
'11	78·5	78·6	78·9	80·0	80·3	80·0	78·9	79·5	80·3	80·7	80·6	80·9	80
'12	81·8	82·9	84·4	85·0	85·3	85·5	86·5	85·9	86·7	85·8	85·3	86·4	85
'13	86·4	86·1	86·7	86·2	85·7	84·1	84·2	85·0	85·7	84·5	83·3	83·8	85
1914	83·5	83·8	82·8	82·3	82·3	81·2	82·4	87·9	89·3	89·8	88·8	91·6	85
'15	96·4	100·9	103·7	105·9	107·2	106·4	106·4	107·0	107·8	110·0	113·1	118·4	108
'16	123·6	127·0	130·4	134·2	135·4	131·0	130·5	134·5	134·4	141·5	150·8	154·3	136
'17	159·3	164·0	169·0	173·0	175·0	180·4	176·9	175·7	176·4	180·6	182·9	185·1	175
'18	186·2	187·3	188·0	189·8	191·1	192·3	192·9	195·9	197·1	197·8	195·3	196·0	192
1919	192·1	187·5	184·7	184·6	194·6	199·4	206·4	212·7	214·8	224·3	231·0	235·2	206
'20	245·3	260·4	261·8	266·1	260·0	255·7	254·6	253·5	248·7	239·9	223·8	207·2	251
'21	197·2	183·0	177·2	169·8	162·2	155·8	158·2	154·3	149·4	138·4	136·7	133·6	155
'22	132·5	132·2	133·3	134·8	135·5	135·6	134·0	129·6	127·9	130·1	130·6	129·1	131
'23	130·2	131·9	132·7	134·0	132·2	127·9	124·8	125·0	127·8	127·7	132·4	133·2	128
1924	137·2	138·8	137·0	136·8	136·4	136·3	138·4	138·0	141·3	146·1	145·5	147·7	139
'25	144·8	143·1	140·1	137·5	135·7	131·2	134·3	134·3	132·7	130·2	132·9	130·4	136
'26	129·3	127·9	126·1	125·5	125·7	124·9	126·0	127·0	128·0	131·0	130·8	123·9	125
'27	123·1	124·1	123·6	123·3	123·8	123·1	122·0	122·8	121·5	120·6	121·5	121·4	122
'28	120·9	121·1	123·6	125·6	126·2	122·6	120·3	118·0	116·8	116·8	117·9	117·9	119
1929	117·0	120·1	120·5	116·5	113·0	113·1	115·2	113·9	112·6	111·1	108·3	108·8	114
'30	106·6	104·8	103·0	101·5	98·8	95·8	94·4	92·2	90·8	90·4	88·6	86·9	96
'31	85·7	85·5	85·5	84·4	82·2	82·6	80·2	79·1	80·7	82·3	83·0	85·4	82
'32	84·7	86·7	84·1	82·5	80·2	77·0	78·9	80·7	80·4	77·8	77·9	77·7	79
'33	77·8	77·0	77·0	78·5	80·9	81·3	81·7	81·2	80·7	80·5	79·3	80·0	78
1934	82·5	82·5	82·2	81·0	81·1	80·7	82·4	83·4	82·1	81·1	81·0	82·8	81
'35	83·6	83·4	82·9	84·1	85·2	83·7	84·3	84·1	85·1	85·8	86·3	86·7	83
'36	87·1	87·1	86·7	86·2	85·6	84·8	87·1	89·0	90·4	91·7	94·5	98·9	88
'37	99·6	102·1	107·3	104·7	106·2	104·7	105·9	104·4	103·3	100·8	96·7	97·3	102
'38	96·5	96·4	94·2	93·5	91·4	91·4	91·1	88·6	88·6	88·8	87·4	89·1	90
1939	88·7	88·6	89·0	90·5	90·6	90·6	88·7	90·4	99·7	105·8	112·2	120·1	94
'40	124·1	124·5	123·4	126·0	128·0	130·0	129·5	131·5	131·6	131·8	132·7	134·5	128
'41	134·9	136·3	138·0	141·1	143·5	144·4	145·3	145·1	145·5	143·7	145·5	146·5	142
'42	148·6	153·4	153·5	154·5	156·6	154·4	150·0	149·4	149·7	150·4	151·3	152·2	151
'43	153·3	153·2	154·0	154·9	155·6	155·4	156·0	154·4	154·6	153·7	153·9	153·9	155
1944	154·9	155·1	155·7	157·9	159·5	160·0	161·9	161·5	160·1	159·6	160·3	160·7	160
'45	161·4	161·5	162·1	162·4	164·0	166·7	165·2	163·3	162·7	162·7	162·7	163·2	164
'46	165·6	166·2	166·6	168·4	169·3	169·7	174·4	175·7	180·5	196·2	198·1	200·5	186
'47	206·7	209·6	212·1	215·9	216·5	218·1	223·4	225·9	228·9	236·0	240·8	246·5	230
'48	250·5	253·4	256·0	256·4	260·0	263·9	260·4	260·1	258·1	261·5	263·8	266·0	260
'49	267·6	266·8	265·4	272·0	269·2	267·2	263·3	262·5	285·7				

* The average of the twelve monthly figures of each year does not necessarily coincide with the annual figures, as the latter are calculated mostly from the average of 52 weekly quotations, while the former are based on end-of-the-month prices.

Summary of Index Numbers. Groups of Articles, 1867–77 = 100

	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver*	Wheat Harvest †	Average Price of Consols ‡	Average Bank of England Rate ‡
												£	%
1873 .	106	109	106	107	141	103	106	114	**111**	97·4	80	$92\frac{1}{2}$	4·750
1896 .	53	73	59	62	63	54	63	60	**61**	50·5	112	$110\frac{3}{4}$	2·483
1911 .	70	90	61	75	93	76	81	83	**80**	40·4	110	$79\frac{3}{16}$	3·467
1919 .	179	213	147	185	220	228	219	222	**206**	85·3	98	$54\frac{1}{12}$	5·166
'21 .	143	218	83	158	181	140	145	153	**155**	48·1	118	$47\frac{29}{30}$	6·092
1922 .	107	184	82	130	142	134	122	131	**131**	51·6	105	$56\frac{15}{32}$	3·692
'23 .	98	162	101	122	155	140	115	133	**128**	49·4	105	$57\frac{63}{64}$	3·496
'24 .	119	158	105	130	158	170	119	145	**139**	50·7	107	$56\frac{63}{64}$	4·0
'25 .	118	162	89	128	154	165	117	142	**136**	52·5	114	$56\frac{3}{8}$	4·575
'26 .	108	150	88	119	154	133	112	130	**125**	47·1	99	$54\frac{29}{30}$	5·0
1927 .	108	138	83	114	141	131	116	127	**122**	42·8	109	$54\frac{39}{48}$	4·650
'28 .	107	142	78	114	123	136	114	123	**119**	44·0	109	$55\frac{13}{16}$	4·5
'29 .	99	146	72	110	126	122	108	117	**114**	40·2	114	$54\frac{5}{16}$	5·508
'30 .	77	142	54	96	112	84	94	96	**96**	29·0	99	56	3·4
'31 .	68	119	50	83	100	63	81	80	**82**	20·4	99	$55\frac{3}{16}$	3·975
1932 .	72	105	50	79	99	64	78	80	**79**	19·5	105	$66\frac{25}{32}$	3·017
'33 .	60	106	47	74	107	67	76	82	**78**	18·7	114	$73\frac{22}{32}$	2·0
'34 .	63	108	50	77	109	72	76	84	**81**	20·0	120	$80\frac{21}{32}$	2·0
'35 .	66	107	42	76	112	80	79	88	**83**	26·4	112	$86\frac{1}{2}$	2·0
'36 .	76	109	41	81	118	83	84	93	**88**	18·5	100	$85\frac{11}{64}$	2·0
1937 .	93	117	49	93	142	93	98	109	**102**	18·4	99	$76\frac{9}{32}$	2·0
'38 .	81	111	43	84	136	75	83	95	**90**	17·6	122	$74\frac{5}{32}$	2·0
'39 .	74	115	47	83	137	93	88	102	**94**	17·1	112	$66\frac{21}{64}$	2·5
'40 .	112	141	58	111	167	149	117	140	**128**	17·1	108	$72\frac{9}{16}$	2·0
'41 .	140	142	65	125	181	162	131	154	**142**	18·0	109	$79\frac{13}{16}$	2·0
1942 .	170	148	66	140	184	163	142	160	**151**	18·1	123	$82\frac{5}{16}$	2·0
'43 .	156	156	72	138	187	166	156	167	**155**	18·1	123	$80\frac{3}{4}$	2·0
'44 .	152	156	73	137	197	182	161	178	**160**	18·1	121	$80\frac{15}{32}$	2·0
'45 .	155	156	78	139	209	189	159	182	**164**	23·1	—	$87\frac{3}{16}$	2·0
'46 .	155	154	88	140	239	231	198	219	**186**	36·7	—	$95\frac{13}{32}$	2·0
'47 .	191	149	100	156	304	295	263	284	**230**	33·3	—	$89\frac{1}{8}$	2·0
'48 .	217	155	107	171	368	348	279	324	**260**	34·1	—	—	2·0
Average 1904–13	68	91	53	73	95	74	76	81	**77**	44·1	106	$82\frac{23}{32}$	3·733
1890–99	61	80	63	68	71	56	66	64	**66**	55·8	103	$103\frac{9}{16}$	2·958
'78–87	79	95	76	84	73	71	81	76	**79**	82·1	97	$99\frac{9}{16}$	3·264
1818–27	109	90	151	111	128	105	106	112	**111**	98·0	—	—	3·692

* Silver (see note on p. 386), parity of 1 gold to $15\frac{1}{2}$ silver = 100.
† Wheat harvest in U.K. to 1895: 29 bushels = 100; from 1896: 30 bushels = 100.
‡ Average price of Consols and the average Bank of England rate of discount are actual figures, not index-numbers; Consols 3% to 1888, $2\frac{3}{4}$% from 1889, $2\frac{1}{2}$% from April, 1903.

THE STATIST'S *Index Numbers—monthly average by groups,* 1867–77 = 100

	Vegetable Food	Animal Food	Sugar, Tea, and Coffee	Food-stuffs	Minerals	Textiles	Sundry Materials	Total Materials	All Commodities
1946									
Jan.	155·4	155·7	77·8	**139·2**	225·8	190·4	155·0	**184·6**	**165·6**
Feb.	155·6	155·7	79·6	**139·6**	225·5	192·0	155·4	**185·5**	**166·2**
Mar.	156·1	155·7	78·5	**139·6**	225·3	192·0	157·2	**186·3**	**166·6**
April	157·9	155·7	78·5	**140·4**	230·6	192·0	160·1	**188·8**	**168·4**
May	156·7	153·6	82·3	**139·9**	230·6	196·1	161·4	**190·7**	**169·3**
June	157·0	153·6	82·1	**140·0**	231·6	197·2	161·4	**191·4**	**169·7**
July	160·7	153·6	84·2	**141·9**	241·3	207·7	163·7	**198·1**	**174·4**
Aug.	153·1	154·7	85·2	**139·4**	247·5	209·1	167·7	**201·9**	**175·7**
Sept.	154·2	154·7	84·0	**139·6**	256·7	218·1	175·3	**210·3**	**180·5**
Oct.	154·4	154·7	85·9	**140·1**	257·8	269·5	200·6	**237·2**	**196·2**
Nov.	151·6	154·7	85·6	**138·8**	261·7	271·7	206·4	**241·4**	**198·1**
Dec.	152·9	154·7	89·2	**140·1**	263·1	275·7	210·6	**244·2**	**200·5**
1947									
Jan.	156·0	154·7	91·1	**141·9**	277·5	287·6	215·1	**254·2**	**206·7**
Feb.	156·0	154·7	92·0	**142·0**	280·1	287·6	224·8	**258·9**	**209·6**
Mar.	156·1	154·7	91·2	**141·9**	291·0	284·9	230·4	**263·5**	**212·1**
April	157·3	154·7	93·7	**143·0**	289·7	284·1	245·5	**269·2**	**215·9**
May	158·1	148·1	101·0	**142·4**	288·4	285·1	249·0	**270·7**	**216·5**
June	173·5	148·1	99·4	**148·6**	288·7	284·2	245·1	**268·9**	**218·1**
July	182·0	148·1	100·9	**152·3**	293·1	292·9	251·0	**275·2**	**223·4**
Aug.	188·5	148·1	102·4	**155·5**	293·3	297·1	253·0	**277·4**	**225·9**
Sept.	190·9	144·0	103·2	**155·1**	299·9	303·9	256·8	**282·9**	**228·9**
Oct.	196·5	144·0	102·9	**157·5**	311·2	320·7	262·6	**293·6**	**236·0**
Nov.	201·8	144·0	104·1	**160·0**	315·9	328·2	269·2	**299·9**	**240·8**
Dec.	204·2	144·0	102·1	**160·5**	340·9	333·2	271·6	**309·2**	**246·5**
1948									
Jan.	207·5	146·1	100·0	**162·3**	350·8	334·5	278·2	**315·0**	**250·5**
Feb.	207·7	155·3	101·5	**166·0**	356·4	335·3	279·4	**317·3**	**253·4**
Mar.	210·4	155·3	108·5	**168·7**	359·5	338·7	280·7	**319·8**	**256·0**
April	210·0	155·3	105·7	**168·0**	360·2	342·6	280·6	**321·1**	**256·4**
May	211·4	155·3	103·4	**168·0**	359·4	364·4	279·6	**327·2**	**260·0**
June	225·1	155·3	105·8	**174·3**	367·8	363·4	280·2	**329·4**	**263·9**
July	214·6	155·3	106·9	**170·1**	367·6	355·1	279·1	**326·4**	**260·4**
Aug.	217·2	155·3	107·8	**171·4**	366·9	350·6	279·7	**325·0**	**260·1**
Sept.	217·1	155·3	105·5	**170·9**	367·3	345·0	275·9	**321·8**	**258·1**
Oct.	217·7	155·3	110·6	**172·2**	385·1	342·9	278·1	**326·8**	**261·5**
Nov.	222·5	155·3	112·2	**174·5**	384·9	348·4	279·3	**329·0**	**263·8**
Dec.	221·8	155.3	114.7	**174.8**	384.7	357.0	281.8	**332.6**	**266·0**
1949									
Jan.	223·6	155·3	117·2	**176·1**	392·4	358·1	280·4	**334·4**	**267·6**
Feb.	224·7	155·3	119·0	**177·0**	392·3	353·8	278·7	**332·4**	**266·8**
Mar.	226·3	155·3	114·7	**176·7**	391·4	347·1	278·9	**330·2**	**265·4**
April	242·7	195·4	115·0	**198·4**	380·0	344·1	278·1	**325·9**	**272·0**
May	243·7	197·4	117·1	**200·0**	370·9	338·3	273·6	**319·7**	**269·2**
June	251·0	197·4	121·0	**203·9**	359·0	333·1	270·3	**313·5**	**267·2**
July	246·5	197·4	123·1	**202·4**	358·7	319·1	267·3	**307·8**	**263·3**
Aug.	238·7	197·4	123·8	**199·3**	360·7	321·1	266·6	**308·7**	**262·5**
Sept.	248·9	197·4	152·8	**209·7**	414·7	372·3	271·9	**341·2**	**285·7**

Quarterly Movements of Prices *

Summary of Index Numbers, 1867–77 = 100

Years	Quarters	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver †
'37	I	93·1	112·7	47·1	90·6	144·3	97·4	102·3	112·0	**103·0**	18·6
	II	94·3	121·0	49·6	94·7	143·5	99·9	102·9	112·8	**105·2**	18·7
	III	93·3	121·4	50·2	94·6	147·7	94·6	101·4	111·8	**104·5**	18·4
	IV	95·5	116·3	47·2	93·0	136·3	81·2	95·6	102·1	**98·3**	18·0
'38	I	92·3	116·4	43·1	90·8	134·8	77·8	92·4	99·3	**95·7**	18·5
	II	89·2	114·1	42·2	88·5	132·0	73·5	86·5	94·8	**92·1**	17·4
	III	78·0	107·4	42·9	81·4	135·9	73·9	85·0	95·3	**89·4**	17·3
	IV	68·8	105·8	43·3	77·1	140·6	74·3	85·1	96·7	**88·4**	17·4
'39	I	68·0	110·9	43·5	78·7	134·2	79·6	84·0	96·2	**88·8**	17·7
	II	66·7	112·9	46·9	79·6	134·4	85·7	85·3	98·7	**90·6**	17·3
	III	65·2	117·0	48·1	80·7	135·8	91·1	88·0	101·8	**92·9**	15·7
	IV	93·2	130·3	57·4	99·3	146·7	123·5	106·4	122·5	**112·7**	17·8
'40	I	104·2	141·2	57·4	108·0	160·1	141·2	116·1	135·6	**124·0**	16·3
	II	108·0	140·5	56·7	109·2	165·2	148·4	122·2	141·8	**128·0**	16·7
	III	117·3	140·5	58·6	113·5	171·1	148·6	122·2	143·4	**130·9**	17·6
	IV	127·5	140·5	58·8	117·8	173·5	151·9	119·8	144·1	**133·0**	17·9
'41	I	129·8	141·3	61·9	119·8	179·0	154·8	124·7	148·6	**136·4**	18·0
	II	137·9	142·8	65·3	124·4	181·2	161·6	137·2	156·6	**143·0**	18·0
	III	145·0	142·8	65·4	127·4	181·0	165·6	138·5	158·3	**145·3**	18·0
	IV	151·0	142·8	68·9	130·7	181·6	163·1	134·2	155·8	**145·2**	18·1
'42	I	179·6	144·3	69·3	143·4	181·6	161·1	140·6	157·9	**151·8**	18·1
	II	188·6	147·3	69·5	148·3	182·2	160·4	146·1	160·2	**155·2**	18·1
	III	158·4	147·3	71·4	136·0	185·8	158·3	144·1	159·7	**149·7**	18·1
	IV	156·6	151·1	71·4	136·7	186·8	164·9	145·0	162·0	**151·3**	18·1
'43	I	159·1	155·7	72·8	139·7	188·0	163·9	148·4	163·8	**153·5**	18·1
	II	159·9	155·7	73·1	140·1	188·3	165·1	153·5	166·4	**155·3**	18·1
	III	156·1	155·7	74·1	138·7	188·2	163·4	156·0	166·9	**155·0**	18·1
	IV	150·1	155·7	74·2	136·2	187·8	160·8	157·4	166·6	**153·8**	18·1
'44	I	151·6	155·7	73·7	136·7	191·4	161·1	159·6	168·6	**155·2**	18·1
	II	151·8	155·7	74·5	137·0	193·9	180·0	160·2	175·3	**159·1**	18·1
	III	152·2	155·7	75·8	137·4	199·3	183·7	161·4	178·5	**161·2**	18·1
	IV	151·4	155·7	73·7	136·7	203·9	184·6	155·4	177·5	**160·2**	18·1
'45	I	154·8	155·7	73·7	138·0	204·8	186·0	157·3	178·9	**161·7**	19·6
	II	160·5	155·7	74·6	140·6	209·4	189·8	158·3	181·7	**164·4**	19·5
	III	157·7	155·7	73·8	139·2	212·5	189·7	156·2	181·6	**163·7**	20·2
	IV	154·7	155·7	74·4	138·2	212·1	190·0	154·3	180·8	**162·9**	31·1
'46	I	155·7	155·7	78·6	139·4	225·5	191·5	155·9	185·5	**166·1**	31·1
	II	157·2	154·3	81·0	140·1	230·9	195·1	160·9	190·3	**169·1**	31·1
	III	156·0	154·3	84·5	140·3	248·5	211·6	168·9	203·4	**176·9**	38·2
	IV	153·0	154·7	86·6	139·7	260·9	272·3	205·9	240·9	**198·3**	41·6
'47	I	156·0	154·7	91·4	141·9	282·9	286·7	223·4	258·9	**209·5**	36·7
	II	162·9	150·3	98·0	144·7	288·9	284·5	246·5	269·6	**216·8**	32·8
	III	187·1	146·7	102·2	154·3	295·4	298·0	253·6	278·5	**222·5**	30·4
	IV	200·8	144·0	103·0	159·3	322·7	330·0	267·8	300·9	**241·1**	33·4
'48	I	208·5	152·2	103·3	165·7	355·6	336·2	279·4	317·4	**253·3**	33·7
	II	215·5	155·3	105·0	170·1	362·5	356·8	280·1	325·9	**260·1**	33·7
	III	216·3	155·3	106·7	170·8	367·3	350·2	278·2	324·4	**259·5**	33·9
	IV	220·7	155·3	112·5	173·8	384·9	349·4	279·7	329·5	**263·8**	33·6

* The averages of the four quarterly figures to each year do not necessarily coincide with the annual averages, as the latter are based as far as possible on average weekly prices. See also the *Journal*, 1893, p. 221; 1895, p. 144; 1901, p. 90; and 1909, p. 70.

† Silver, parity of 1 gold to 15½ silver = 100.

Construction of the Tabular Statements

The following table illustrates the method of construction of the index numbers. The index numbers here given are based on the average prices for the eleven years 1867–77. Take, for instance, the *Gazette* price of English wheat:

	s.	d.	
Average, 1867–77	54	6	= 100, average point.
,, 1914	35	0	= 64, or 36 per cent. *below* the average point.
,, 1930	80	7	= 148, ,, 48 ,, *above* ,, ,,
,, 1936	53	3	= 98, ,, 2 ,, *below* ,, ,,

The individual index numbers, therefore, represent simple percentages of the average point. The articles are grouped in six categories:

	Index Nos.	1867–77 Total Numbers	Example for 1948 Total Numbers	Example for 1948 Average
1. Vegetable food, corn, etc. (wheat flour, barley, oats, maize, potatoes, and rice)	8	800	1,736	217
2. Animal food (beef, mutton, pork, bacon, and butter)	7	700	1,082	155
3. Sugar, coffee, and tea	4	400	430	107
1–3. *Food*	19	1,900	3,248	171
4. Minerals (iron, copper, tin, lead, and coal) . .	7	700	2,574	368
5. Textiles (cotton, flax, hemp, jute, wool, and silk) .	8	800	2,787	348
6. Sundry materials (hides, leather, tallow, oils, soda, nitrate, indigo, and timber)	11	1,100	3,074	279
4–6. *Materials*	26	2,600	8,435	324
General Average	45	4,500	11,683	260

The general average is drawn from all forty-five descriptions which are treated as of equal value, and is the simple arithmetic mean as shown above.

Index of Silver Prices

The base of the index numbers given below is 60·84*d.* per standard oz. = 100, this being a parity of 1 fine oz. of gold to $15\frac{1}{2}$ standard ozs. of silver.*

	Price per oz. standard	Index number		Price per oz. standard	Index number
	d.			*d.*	
Average 1873	$59\frac{1}{4}$	=97·4	*Lowest* Nov., 1902	$21\frac{11}{16}$	=35·6
,, '90–99	34	=55·8	End Dec., 1906	$32\frac{5}{16}$	=53·1
,, 1917–26	$40\frac{1}{2}$	=66·6	,, Dec., '08	$23\frac{3}{16}$	=38·1
,, 1893	$35\frac{5}{8}$	=58·6	,, Dec., '12	29	=47·7
,, '96	$30\frac{3}{4}$	=50·5	,, Dec., '13	$26\frac{9}{16}$	=43·7
,, 1914	$25\frac{5}{16}$	=41·6	,, Dec., '14	$22\frac{11}{16}$	=37·3
,, '15	$23\frac{11}{16}$	=38·9	,, Dec., '15	$26\frac{1}{4}$	=43·1
,, '16	$31\frac{5}{16}$	=50·4	,, Dec., '16	$36\frac{1}{2}$	=58·7
,, '17	$40\frac{7}{8}$	=65·8	,, Dec., '17	$43\frac{1}{2}$	=70·0
,, '18	$47\frac{1}{2}$	=76·4	,, Dec., '18	$48\frac{7}{16}$	=77·9
,, '19	57	=85·3	,, Dec., '19	$77\frac{1}{4}$	=98·3
,, '20	$61\frac{9}{16}$	=76·1	,, Dec., '20	$40\frac{7}{8}$	=49·2
,, '21	$36\frac{7}{8}$	=48·1	,, Dec., '21	$34\frac{5}{8}$	=49·3
,, '22	$34\frac{7}{16}$	=51·6	,, Dec., '22	$31\frac{9}{16}$	=49·6
,, '23	$31\frac{15}{16}$	=49·4	,, Dec., '23	$33\frac{7}{16}$	=49·0
,, '24	34	=50·7	,, Dec., '24	$31\frac{13}{16}$	=50·4
,, '25	$32\frac{1}{8}$	=52·5	,, Dec., '25	$31\frac{11}{16}$	=52·1
,, '26	$28\frac{11}{16}$	=47·1	,, Dec., '26	25	=41·1
,, '27	$26\frac{1}{32}$	=42·8	,, Dec., '27	$26\frac{1}{2}$	=43·6
,, '28	$26\frac{3}{4}$	=44·0	,, Dec., '28	$26\frac{3}{8}$	=43·3
,, '29	$24\frac{7}{16}$	=40·2	,, Dec., '29	$21\frac{7}{16}$	=35·2
,, '30	$17\frac{11}{16}$	=29·0	,, Dec., '30	$14\frac{7}{16}$	=23·7
,, '31	$14\frac{3}{8}$	=20·4	,, Dec., '31	$20\frac{7}{16}$	=21·6
,, '32	$17\frac{7}{8}$	=19·5	,, Dec., '32	$16\frac{1}{2}$	=17·2
,, '33	$18\frac{1}{8}$	=18·7	,, Dec., '33	$19\frac{1}{16}$	=19·5
,, '34	$21\frac{7}{32}$	=20·0	,, Dec., '34	$24\frac{5}{8}$	=22·6
,, '35	29	=26·4	,, Dec., '35	$22\frac{1}{2}$	=20·6
,, '36	$20\frac{1}{16}$	=18·5	,, Dec., '36	$21\frac{5}{16}$	=19·4
,, '37	$20\frac{1}{16}$	=18·4	,, Dec., '37	$19\frac{1}{16}$	=17·7
,, '38	$19\frac{17}{32}$	=17·6	,, Dec., '38	$20\frac{1}{16}$	=17·3
,, '39	$20\frac{13}{32}$	=17·1	,, Dec., '39	$22\frac{1}{2}$	=17·3
,, '40	$22\frac{19}{64}$	=17·1	,, Dec., '40	$23\frac{5}{16}$	=17·9
,, '41	$23\frac{7}{16}$	=18·0	,, Dec., '41	$23\frac{1}{2}$	=18·1
,, '42	$23\frac{1}{2}$	=18·1	,, Dec., '42	$23\frac{1}{2}$	=18·1
,, '43	$23\frac{1}{2}$	=18·1	,, Dec., '43	$23\frac{1}{2}$	=18·1
,, '44	$23\frac{1}{2}$	=18·1	,, Dec., '44	$23\frac{1}{2}$	=18·1
,, '45	$30\frac{1}{2}$	=23·1	,, Dec., '45	44	=31·1
,, '46	$48\frac{5}{8}$	=36·7	,, Dec., '46	$55\frac{1}{2}$	=41·6
,, '47	$44\frac{7}{16}$	=33·3	,, Dec., '47	45	=33·8
,, '48	45	=34·1	,, Dec., '48	$2\frac{1}{2}$	=31·9

* All the index numbers in the table from 1916 to 1925 inclusive and from 1931 to date are calculated on the basis of the gold prices of silvei instead of the sterling prices, though the latter are the price quotations given in the table. In arriving at the index numbers for these dates the prices of gold are taken as follows: For 1916, 1917 and 1918 the price is taken as 86*s.* $9\frac{1}{2}$*d.* per fine oz., derived from the "pegged" New York rate of \$4·76$\frac{1}{2}$ to the £. For 1919 the average price of gold is taken as 93*s.* $4\frac{1}{4}$*d.*, this being the parity price with the U.S. dollar, the average New York exchange in that year being \$4·429. For the other dates the index numbers are based on the quotations in the London market for exportable gold.

World's Production of Silver (in millions of ounces)

	United States	Mexico	Canada	Australia	Other Countries	Total
1905	56·1	65·0	5·9	15·0	30·3	172·3
'06	56·5	55·2	8·5	14·2	30·6	165·0
'07	56·5	61·0	12·8	19·0	34·8	184·2
'08	52·4	73·6	22·1	17·2	37·8	203·1
'09	54·7	73·9	27·5	16·3	39·7	212·1
'10	57·1	71·4	32·9	21·5	38·8	221·7
'11	60·4	79·0	32·7	16·6	37·5	226·2
'12	63·8	74·6	31·6	18·1	36·2	224·3
'13	66·8	70·7	31·5	3·5	51·4	223·9
'14	72·4	27·5	28·4	3·6	36·5	168·4
'15	74·9	39·5	28·4	4·1	37·3	184·2
'16	74·4	38·2	25·4	4·2	26·6	168·8
'17	71·7	35·0	22·2	10·0	35·3	174·2
'18	67·8	62·5	21·2	10·0	35·9	197·4
'19	56·7	62·7	15·7	7·4	32·0	174·5
'20	55·5	66·8	12·6	7·5	33·0	175·4
'21	53·1	64·5	13·1	4·9	35·7	171·3
'22	56·2	81·1	18·6	11·3	46·3	213·5
'23	73·3	90·9	17·8	13·3	50·7	246·0
'24	65·3	91·5	19·7	10·8	52·2	239·5
'25	66·1	92·9	20·2	11·1	54·8	245·1
'26	62·7	98·3	22·4	11·2	59·0	253·6
'27	60·4	104·6	22·7	9·0	57·3	254·0
'28	58·4	108·5	21·9	9·0	59·5	257·3
'29	61·2	108·7	23·1	9·0	59·7	261·7
'30	51·0	105·0	26·0	8·9	57·1	248·0
'31	31·0	86·0	21·0	7·6	50·4	196·0
'32	24·0	69·0	18·0	6·5	47·5	165·0
'33	22·8	68·1	15·2	11·0	52·0	169·1
'34	32·5	74·1	16·4	10·8	56·6	190·4
'35	45·6	75·6	16·6	11·4	71·5	220·7
'36	63·4	77·5	18·3	12·7	81·8	253·7
'37	71·3	84·7	22·7	14·3	81·5	274·5
'38	61·7	81·0	23·8	101·3		267·8
'39	63·9	75·9	24·5	101·6		265·9
'40	68·3	82·6	25·3	96·3		272·5
'41	71·1	78·4	23·4	90·1		263·0
'42	55·9	84·9	21·8	—		—
'43	44·8	71·2	18·5	—		—
'44	37·3	63·0	14·7	—		—
'45	29·3	61·0	14·0	—		—
'46	21·4	48·3	13·7	—		—
'47	36·1	49·2	12·5	—		—
'48*	37·0	46·0	16·0	—		—

* Provisional. (Estimate by Messrs. Samuel Montagu & Co.)

(000's omitted)

Year	Value of output £	Year	Value of output £	Year	Value of output £
1851	17,200	1884	20,830	1917	87,236
'52	26,550	'85	21,250	'18	78,605
'53	31,090	'86	21,430	'19	73,078
'54	25,490	'87	21,735	'20	68,522
'55	27,015	'88	22,644	'21	67,848
'56	29,520	'89	25,375	'22	66,723
'57	26,655	'90	24,421	'23	77,888
'58	24,930	'91	26,846	'24	81,807
'59	24,970	'92	30,134	'25	82,267
'60	23,850	'93	32,363	'26	82,211
'61	22,760	'94	37,229	'27	82,582
'62	21,550	'95	40,843	'28	82,400
'63	21,390	'96	41,559	'29	84,500
'64	22,600	'97	48,509	'30	88,500
'65	24,040	'98	58,949	'31	95,100
'66	24,220	'99	63,027	'32	103,400
'67	22,805	1900	52,312	'33	107,700
'68	21,945	'01	53,630	'34	116,000
'69	21,245	'02	60,975	'35	125,700
'70	21,370	'03	67,337	'36	140,900
'71	25,400	'04	71,380	'37	148,700
'72	24,200	'05	78,143	'38	159,000
'73	23,600	'06	82,707	'39	165,900
'74	22,950	'07	84,857	'40	174,000*
'75	22,700	'08	90,995	'41	167,200*
'76	22,540	'09	93,302	'42	146,300*
'77	23,830	'10	93,544	'43	112,600*
'78	22,020	'11	94,930	'44	101,000*
'79	21,400	'12	95,783	'45	98,100*
'80	22,130	'13	97,481	'46	99,200*
'81	21,150	'14	92,709	'47	100,700*
'82	20,500	'15	97,114	'48	102,900*
'83	20,640	'16	92,597		

Gold.—The table shows the world's annual gold production since 1851. Before 1911 the estimates are those of the Bureau of the U.S. Mint and other authorities. The estimates since 1926 are those of the Union Corporation, Limited. The value is taken throughout at £4·25 per fine oz.

* Estimated or provisional figure.

*Average Prices of Commodities**

No. of Article	0	1	2	3	4	5	6	7	8	1–8	9	10
		Wheat		Flour	Barley	Oats	Maize §	Potatoes*	Rice	Vegetable Food	Beef ‡	
Year	Silver †	English *Gazette*	American	Town Made white (now "G.R.")	English *Gazette*	English *Gazette*	American Mixed	Good English	Rangoon Cargoes to Arrive	Total	Prime	Middling
	d. per oz.	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per sack, (280 lbs.)	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per qr.	*s.* per ton	*s.* and *d.* per cwt.		*d.* per 8 lbs.	*d.* per 8 lbs.
1922 ..	$34\frac{7}{16}$	47·10	52·11	$45\frac{3}{4}$	40·1	29·1	$31\frac{1}{4}$	130	14·10	—	$88\frac{5}{8}$	82
'27 ..	$26\frac{1}{32}$	49·3	58·3	$44\frac{9}{16}$	42·0	25·4	$30\frac{9}{10}$	136	15·11	—	70	62
'28 ..	$26\frac{3}{4}$	44·8	50·10	$40\frac{1}{6}$	39·0	29·0	$38\frac{15}{32}$	133	15·0	—	74	$66\frac{1}{2}$
'29 ..	$24\frac{7}{16}$	42·2	51·3	$38\frac{1}{2}$	35·5	24·7	$36\frac{1}{4}$	111	14·3	—	71	66
'30 ..	$17\frac{11}{16}$	34·3	36·10	$33\frac{1}{12}$	28·3	17·2	23	93	13·0	—	73	68
'31 ..	$14\frac{3}{8}$	24·0	25·1	$22\frac{7}{8}$	28·0	17·8	$15\frac{9}{16}$	146	9·8	—	67	61
'32 ..	$17\frac{27}{32}$	25·0	27·5	$24\frac{7}{32}$	27·1	19·3	$18\frac{13}{16}$	152	9·8	—	65	59
'33 ..	$18\frac{1}{8}$	22·10	25·7	$23\frac{29}{32}$	28·7	15·10	$17\frac{1}{16}$	86	7·9	—	61	52
'34 ..	$21\frac{7}{32}$	20·2	28·0	$23\frac{1}{8}$	30·11	17·5	$19\frac{19}{32}$	97	7·8	—	58	52
'35 ..	29	22·2	31·1	$25\frac{3}{4}$	28·7	18·9	$17\frac{1}{4}$	107	8·10	—	54	49
'36 ..	$20\frac{1}{16}$	30·9	35·1	$31\frac{17}{32}$	29·5	17·8	$19\frac{1}{2}$	146	9·0	—	54	50
'37 ..	$20\frac{1}{16}$	40·0	49·7	$40\frac{1}{3}$	39·0	23·11	$26\frac{9}{16}$	136	10·5	—	61	57
'38 ..	$19\frac{17}{32}$	28·11	39·3	$30\frac{21}{32}$	36·4	21·2	$28\frac{1}{16}$	111	10·7	—	62	58
'39 ..	$20\frac{13}{32}$	21·5	30·1	$22\frac{9}{32}$	31·7	19·3	$26\frac{1}{2}$	117	11·1	—	61	58
'40 ..	$22\frac{19}{64}$	42·10	33·6	$24\frac{5}{16}$	64·10	37·2	$39\frac{13}{16}$	143	15·5	—	72	68
'41 ..	$23\frac{7}{16}$	62·10	32·2	$27\frac{1}{2}$	85·8	40·10	43 ‖	164	23·8	—	72	68
'42 ..	$23\frac{1}{2}$	68·6	36·9	35	165·5	42·0	43 ‖	134	26·0	—	76	68
'43 ..	$23\frac{1}{2}$	69·8	45·5	$38\frac{1}{4}$	112·5	43·8	43 ‖	111	27·2	—	79	71
'44 ..	$23\frac{1}{2}$	63·11	56·0	$38\frac{3}{4}$	94·6	45·3	43 ‖	115	27·2	—	79	71
'45 ..	$30\frac{1}{2}$	61·10	63·7	40	89·2	45·9	43 ‖	139	27·2	—	79	71
'46 ..	$48\frac{5}{8}$	63·7	65·11	40	86·6	45·3	43 ‖	137	27·2	—	$79\frac{3}{4}$	72
'47 ..	$44\frac{7}{16}$	71·9	68·10	40	88·7	50·9	43 ‖	162	49·6	—	80	72
'48 ..	45	90·0	73·4	40	95·8	58·0	43 ‖	207	57·6	—	79	72
Average 1904–13	$26\frac{3}{4}$	$31\frac{1}{2}$	36	30	$25\frac{1}{2}$	$18\frac{1}{2}$	$24\frac{1}{2}$	78	$7\frac{3}{4}$	—	51	$44\frac{1}{2}$
1890–99	34	$28\frac{1}{2}$	$31\frac{1}{2}$	$27\frac{1}{2}$	$25\frac{1}{2}$	$17\frac{1}{2}$	$19\frac{1}{2}$	72	$6\frac{1}{4}$	—	47	$37\frac{1}{2}$
'78–87	50	40	$43\frac{1}{2}$	$34\frac{1}{2}$	$31\frac{1}{2}$	21	25	102	8	—	$55\frac{1}{2}$	46
'67–77	$58\frac{1}{2}$	$54\frac{1}{2}$	56	46	39	26	$32\frac{1}{2}$	117	10	—	59	50

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	0	1	2	3	4	5	6	7	8	1–8	9	10
1922 ..	51·6	88	95	100	103	112	96	111	148	853	150	164
'27 ..	42·8	90	104	98	108	97	95	116	159	867	119	124
'28 ..	44·0	82	91	87	100	112	118	114	150	854	125	133
'29 ..	40·2	77	91	84	91	95	112	95	143	788	120	132
'30 ..	29·0	63	66	72	72	66	71	79	130	619	124	136
'31 ..	20·4	44	45	50	71	68	48	125	93	544	114	122
'32 ..	19·5	46	49	53	69	74	58	130	93	572	110	118
'33 ..	18·7	42	46	52	73	61	53	74	78	479	103	104
'34 ..	20·0	37	50	50	79	67	60	83	77	503	98	104
'35 ..	26·4	41	56	56	73	72	53	91	88	530	92	98
'36 ..	18·5	56	63	69	75	68	60	125	90	606	92	100
'37 ..	18·4	73	89	88	100	92	82	116	105	745	103	114
'38 ..	17·6	53	70	67	93	81	86	95	106	651	105	116
'39 ..	17·1	39	54	48	81	74	82	100	111	589	103	116
'40 ..	17·1	78	60	53	166	143	122	122	154	898	122	136
'41 ..	18·0	115	57	60	220	157	132	140	236	1,117	122	136
'42 ..	18·1	126	66	76	424	162	132	115	260	1,361	129	136
'43 ..	18·1	128	81	83	288	168	132	95	272	1,247	134	142
'44 ..	18·1	117	100	84	242	174	132	98	272	1,219	134	142
'45 ..	23·1	113	114	87	229	176	132	118	272	1 241	134	142
'46 ..	36·7	117	118	87	222	174	132	117	272	1,239	135	144
'47 ..	33·3	132	123	87	227	195	132	138	495	1,529	135	144
'48 ..	33·7	165	131	87	245	223	132	177	576	1,736	134	144

* The annual prices are the average monthly or weekly quotations, except potatoes, which are the average weekly quotations during the eight months January to April and September to December.
† Not included in the general average. ‡ Meat (9–13), by the carcase, in the London Central Meat Market.
§ La Plata from 1924. ‖ Argentine maize (Feeding Stuffs); £10 per ton fixed by Ministry of Food.

Average Prices of Commodities—Contd.

No. of Article	11	12	13	14	15	9-15	16A	16B	17	18A *	18B *	18
	Mutton		Pork	Bacon	Butter		Sugar			Coffee		
Year	Prime	Middling	Large and Small, average	Waterford	Friesland, Fine to Finest	Animal Food Total	British West Indian Refining‖	Beet, German, 88 p. c., f.o.b.	Java, Floating Cargoes §	Ceylon-Plantation. Low Middling†	Rio. Good	Mean of 18A and 18B
	d. per 8 lbs.	d. per 8 lbs.	d. per 8 lbs.	s. per cwt.	s. per cwt.		s. per cwt.	s. per cwt.	s. per cwt.	s. per cwt.	s. per cwt.	
1922 ..	125	121 3/5	101	145 1/3	202 1/2	—	15	14 1/8	15 5/16	120 2/3	74 1/3	—
'27 ..	86	79 3/5	85	102 1/4	178	—	16 7/16	12 1/2	13 9/16	143 1/2	71 1/2	—
'28 ..	92 1/4	87	77	101 3/4	185 7/8	—	13 1/2	10 11/16	11 1/2	143 9/32	81 11/32	—
'29 ..	89 1/2	83	91	116 13/32	180 1/8	—	11 7/32	8 5/16	8 7/8	141 1/8	74 27/32	—
'30 ..	92	86	89	105 5/8	146 13/16	—	8 5/8	5 27/32	6 13/32	106 23/32	42 11/32	—
'31 ..	79	73	65	83 3/8	130	—	7 13/16	5 11/16	6 7/16	101 1/4	33 15/16	—
'32 ..	63	55	54	77	126 3/8	—	7 1/2	5 5/16	5 19/32	105 3/32	54 5/16	—
'33 ..	69	63	60	81 1/2	105 9/16	—	7 5/32	4 23/32	5 1/4	86 25/32	42 1/8	—
'34 ..	74	70	65	90 9/32	79 5/8	—	6 5/32	4 5/32	4 11/16	87 7/32	42 13/16	—
'35 ..	75	70	62	89	92 5/8	—	6 3/16	3 21/32	4 3/4	67 9/16	29 21/32	—
'36 ..	73	68	65	93 27/32	98 21/32	—	6 13/32	3 1/32	4 3/4	58 1/2	30 31/32	—
'37 ..	78	74	68	94	108 3/32	—	7 31/32	5 1/4	6 1/2	75 3/4	36 3/4	—
'38 ..	62	56	69	97 3/16	114 9/16	—	7 1/32	4 5/16	5 15/32	75	19 3/4	—
'39 ..	68	64	70	97 3/4	122	—	9 3/32	6 1/32	7 7/32	73 3/4	22 1/2	—
'40 ..	85	76	96	114 7/8	143	—	9 3/4	—	8 1/8	86 5/16	28 3/32	—
'41 ..	85	76	96	123 11/16	142 11/16	—	9 3/4	—	8 1/8	137 1/2	30 1/4	—
'42 ..	90	78	101	130	143 1/2	—	9 3/4 ‡	—	8 1/8 ‡	130 ‡	28 3/4 ‡	—
'43 ..	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	37 ‡	—
'44 ..	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	37 ‡	—
'45 ..	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	59 ‡	—
'46 ..	100 3/4	81 1/2	97	142	139 3/4	—	—	—	23 1/4	150 ‡	77 3/4	—
'47 ..	101	81	98	124 1/4	126	—	—	—	27 1/2	150 ‡	73 3/8	—
'48 ..	100	81	99	154 3/4	122 3/4	—	—	—	24 1/8	169	79	—
Average												
1904–13	58 1/4	51 3/4	47 5/8	67	113	—	10 1/8	10 7/8	12	75 1/2	43 3/4	—
1890–99	54 1/2	41 1/2	42 3/4	59	100	—	11 1/2	11 1/2	13 3/4	98	62	—
'78–87	64 1/2	53	49	71	116	—	17	18	21 1/2	78	52	—
'67–77	63	55	52	74	125	—	23	24	28 1/2	87	64	—

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	11 Prime	12 Middling	13	14	15	9-15	16A–16B	17	18A	18B	18
1922 ..	199	221	194	196	162	1,286	62	54	140 *	116 *	128
'27 ..	136	145	163	138	142	967	62	47	165	112	139
'28 ..	146	158	148	137	149	996	51	40	165	127	146
'29 ..	142	151	175	157	144	1,021	42	31	162	117	140
'30 ..	146	155	171	143	117	992	31	22	123	66	95
'31 ..	125	133	125	113	104	836	29	23	120	53	87
'32 ..	100	100	104	104	101	737	27	20	121	85	103
'33 ..	110	114	115	110	84	740	25	18	100	66	83
'34 ..	117	127	125	122	64	757	22	16	100	67	84
'35 ..	119	127	119	120	74	749	21	17	78	46	62
'36 ..	116	124	131	127	79	763	21	17	67	48	58
'37 ..	124	135	125	127	86	820	28	23	87	57	72
'38 ..	98	102	133	131	92	777	24	19	86	31	59
'39 ..	108	116	135	132	98	808	32	26	85	35	60
'40 ..	135	138	185	155	114	985	42	29	99	44	72
'41 ..	135	138	185	167	114	997	42	29	158	47	102
'42 ..	143	142	194	176	115	1,035	42	29	150	45	98
'43 ..	152	153	196	192	121	1,090	42	29	161	58	109
'44 ..	152	153	196	192	121	1,090	42	29	161	58	109
'45 ..	152	153	196	192	121	1,090	42	29	161	93	127
'46 ..	160	148	187	192	112	1,078	—	82	172	121	146
'47 ..	160	147	188	168	101	1,043	—	96	172	115	143
'48 ..	159	147	190	209	99	1,082	—	85	194	123	158

* Index numbers not included in general average. ‡ Nominal.
† E. India good middling from 1908–1947. Kenya in 1948. § Raw Centrifugals, 96% Pol., from 1924.
‖ White Javas, C.I.F., from 1924.

Average Prices of Commodities—Contd.

No. of Article	19A *	19B *	19	16–19	1–19	20A	20B	21	22	23	24	25
	Tea			Sugar, Coffee, and Tea	Food	Iron			Copper	Tin	Lead	Coal
Year	Congou, Common	Average Import Price	Mean of 19A and 19B	Total	Total *	Scottish Pig	Cleveland (Middlesbrough) Pig	Bars, Common	Standard	Straits	English Pig	Wallsend Hetton in London†
	d. per lb.	d. per lb.				s. and d. per ton	s. and d. per ton	per ton	£ per ton	£ per ton	£ per ton	s. per ton
1922 ..	$8\frac{5}{8}$	14·9	—	—	—	99·10	90·7	$11\frac{1}{5}$	$63\frac{1}{3}$	162	$25\frac{2}{5}$	$34\frac{2}{5}$
'27 ..	$6\frac{3}{4}$	18·58	—	—	—	80·5	73·0	$11\frac{1}{4}$	$55\frac{3}{4}$	$303\frac{7}{8}$	$25\frac{9}{16}$	$23\frac{1}{16}$
'28 ..	$6\frac{5}{16}$	16·84	—	—	—	69·9	65·9	$9\frac{15}{16}$	$63\frac{3}{4}$	$229\frac{7}{16}$	$22\frac{5}{16}$	$21\frac{7}{16}$
'29 ..	$6\frac{1}{16}$	16·11	—	—	—	74·0	70·3	$9\frac{3}{4}$	$75\frac{9}{16}$	$207\frac{7}{16}$	$24\frac{1}{32}$	$23\frac{3}{8}$
'30 ..	$5\frac{7}{32}$	15·12	—	—	—	76·0	67·0	$9\frac{31}{32}$	$54\frac{3}{4}$	$144\frac{29}{32}$	$19\frac{1}{2}$	$24\frac{25}{32}$
'31 ..	$4\frac{13}{16}$	13·29	—	—	—	71·0	58·6	$10\frac{3}{32}$	$38\frac{1}{8}$	$121\frac{1}{8}$	$14\frac{9}{32}$	$24\frac{5}{8}$
'32 ..	$4\frac{9}{32}$	10·75	—	—	—	68·2	58·6	10	$31\frac{21}{32}$	140	$13\frac{5}{16}$	$23\frac{13}{32}$
'33 ..	$6\frac{21}{32}$	11·87	—	—	—	66	62·3	$9\frac{11}{16}$	$32\frac{21}{32}$	$202\frac{1}{2}$	$13\frac{1}{4}$	$22\frac{3}{8}$
'34 ..	$8\frac{21}{32}$	13·20	—	—	—	69·6	66·11	$9\frac{17}{32}$	$30\frac{9}{32}$	$232\frac{1}{4}$	$12\frac{13}{32}$	$20\frac{1}{4}$
'35 ..	$6\frac{23}{32}$	13·06	—	—	—	70·6	67·10	$9\frac{5}{8}$	$32\frac{15}{32}$	$230\frac{15}{32}$	16	$20\frac{9}{32}$
'36 ..	$6\frac{1}{4}$	13·19	—	—	—	78·6	73·2	$10\frac{3}{16}$	$37\frac{13}{32}$	$207\frac{21}{32}$	$19\frac{15}{32}$	$23\frac{3}{32}$
'37 ..	$6\frac{9}{16}$	14·58	—	—	—	104·6	94·4	$12\frac{5}{16}$	$54\frac{21}{32}$	$246\frac{3}{4}$	$24\frac{29}{32}$	$24\frac{1}{2}$
'38 ..	$6\frac{15}{32}$	14·04	—	—	—	118·0	109	$13\frac{1}{4}$	$41\frac{21}{32}$	$193\frac{31}{32}$	$17\frac{1}{16}$	$25\frac{11}{16}$
'39 ..	$6\frac{1}{4}$††	14·18	—	—	—	104·3	100·7	$12\frac{3}{8}$	$44\frac{1}{4}$	$232\frac{17}{32}$	$17\frac{3}{8}$	$25\frac{11}{32}$
'40 ..	—	15·33	—	—	—	114·10	116·4	$14\frac{5}{16}$	62	$273\frac{5}{16}$	$26\frac{1}{2}$	$28\frac{1}{16}$
'41 ..	—	15·13	—	—	—	123	128	$15\frac{5}{8}$	62	$284\frac{5}{16}$	$26\frac{1}{2}$	$30\frac{27}{32}$
'42 ..	—	16·25	—	—	—	123	128	$15\frac{5}{8}$	62	275	$26\frac{1}{2}$	$32\frac{1}{2}$
'43 ..	—	18·44	—	—	—	123	128	$15\frac{5}{8}$	62	275	$26\frac{1}{2}$	$34\frac{15}{16}$
'44 ..	—	19·48	—	—	—	123	128	$16\frac{1}{2}$	62	300	$26\frac{1}{2}$	$38\frac{5}{8}$
'45 ..	—	20·06	—	—	—	$139\frac{15}{16}$	$140\frac{1}{2}$	18	62	300	$29\frac{1}{5}$	$42\frac{2}{3}$
'46 ..	—	21·31	—	—	—	167	$165\frac{1}{2}$	$19\frac{5}{8}$	$77\frac{1}{5}$	$321\frac{5}{8}$	$47\frac{3}{4}$	$45\frac{1}{6}$
'47 ..	—	27·65	—	—	—	$176\frac{3}{4}$	$175\frac{7}{16}$	$20\frac{7}{16}$	$130\frac{7}{8}$	$426\frac{3}{4}$	$86\frac{7}{8}$	$48\frac{1}{8}$
'48 ..	—	32·21	—	—	—	$194\frac{1}{2}$	$193\frac{1}{2}$	$21\frac{1}{4}$	134	$547\frac{5}{8}$	97	$54\frac{3}{4}$
Average 1904–13	$7\frac{7}{8}$	$8\frac{1}{8}$	—	—	—	$57\frac{3}{4}$	$51\frac{1}{2}$	$6\frac{7}{8}$	$67\frac{3}{4}$	$164\frac{7}{8}$	$15\frac{5}{8}$	$18\frac{1}{8}$
1890–99	$4\frac{5}{8}$	$9\frac{3}{4}$	—	—	—	47	$41\frac{1}{2}$	$5\frac{1}{2}$	50	81	12	$17\frac{1}{2}$
'78–87	$6\frac{3}{4}$	$12\frac{3}{4}$	—	—	—	46	38	$5\frac{1}{2}$	55	89	14	$16\frac{3}{4}$
'67–77	$11\frac{1}{4}$	$17\frac{1}{4}$	—	—	—	69	60	$8\frac{1}{4}$	75	105	$20\frac{1}{2}$	22
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
1922 ..	77*	86*	82	326	2,465	148		136	84	154	123	156
'27 ..	60	108	84	332	2,166	119		136	74	289	125	105
'28 ..	56	98	77	314	2,164	105		120	85	219	109	97
'29 ..	54	93	74	287	2,096	112		118	101	198	117	106
'30 ..	46	88	67	215	1,826	111		121	73	138	95	113
'31 ..	42	78	60	199	1,579	100		123	52	115	71	112
'32 ..	38	62	50	200	1,509	98		121	43	131	65	106
'33 ..	58	68	63	189	1,408	99		117	44	193	65	103
'34 ..	77	77	77	199	1,459	106		116	40	221	61	92
'35 ..	60	76	68	168	1,447	107		117	43	219	78	92
'36 ..	56	76	66	162	1,531	118		123	50	198	95	105
'37 .	58	85	72	195	1,760	154		149	73	235	121	111
'38 ..	58	81	70	172	1,600	176		161	56	185	83	117
'39 ..	56	82	69	187	1,584	159		150	59	221	85	115
'40 ..	—	89	89	232	2,115	179		173	83	260	129	127
'41 .	—	88	88	261	2,375	195		189	83	271	129	141
'42 ..	—	94	94	263	2,659	195		189	83	262	129	148
'43 ..	—	107	107	287	2,624	195		189	83	262	129	159
'44 ..	—	113	113	293	2,602	195		200	83	286	129	176
'45 ..	—	116	116	314	2,645	217		218	83	286	142	194
'46 ..	—	124	124	352	2,669	258		238	103	306	233	205
'47 ..	—	160	160	399	2,971	273		248	175	406	424	219
'48 ..	—	187	187	430	3,248	300		258	179	521	473	249

* Index numbers not included in the general average.
† Best Yorkshire house after 1916.
† First 9 months only.

Average Prices of Commodities—Contd.

No. of Article	26	20–26	27	28	29A	29B	30A	30B	31	32A	32B	33
	Coal	Minerals	Cotton		Flax		Hemp		Jute	Wool		
Year	Average Export Price	Total	Middling American	Fair Dholerah ‖	Petrograd ¶	Russian Average Import Price	Manila Fair Roping	Petrograd Clean (a)	Good Medium††	Merino, Port Philip, Average Fleece	Merino, Adelaide, Average Greasy	English Lincoln Half Hogs
	s. per ton		d. per lb.	d. per lb.	£ per ton	£ per ton	£ per ton	£ per ton	£ per ton	d.* per lb.	d. per lb.	d. per lb.
1922 ..	24·16	—	12·10	8	95	84 7/8	33 1/8	57 ‡	30 3/4	39	17 1/8	9 3/4
'27 ..	17·80	—	9·54	8·27	95 13/16	74 1/32	43 3/8	66 5/16	32 3/4	38 1/16	17 7/16	15 13/32
'28 ..	15·67	—	10·92	8·66	98 1/2	91 15/16	37 11/16	63 13/32	33 13/16	37	17 5/16	17 29/32
'29 ..	16·13	—	10·26	7·73	76 7/16	71 7/32	37 7/32	61	32	35 3/8	13 3/32	16 1/16
'30 ..	16·64	—	7·49	5·12	53 1/8	60 7/32	26 1/2	48 7/32	20	18 9/32	8 13/32	10 3/4
'31 ..	15·98	—	5·90	4·60	36	35 9/16	18 3/4	27 1/2	15 15/16	14·7	7·1	8 1/2
'32 ..	16·27	—	5·24	4·85	45 7/32	42 5/32	18 23/32	36	16 3/32	15·0	7·2	5 25/32
'33 ..	16·08	—	5·54	4·53	51 13/32	48 15/32	15 7/8	37	14 7/8	19·9	9·3	5 29/32
'34 ..	16·08	—	6·70	4·80	60 11/16	50 29/32	14 29/32	42 5/16	14 3/32	21 1/4	10·4	7
'35 ..	16·30	—	6·71	5·42	79 3/8	72 5/16	19 11/32	43 1/4	16 15/16	20·1	9·5	7 11/32
'36 ..	16·98	—	6·71	5·12	63 25/32	60 15/16	28 11/32	42 27/32	17 19/32	24·7	12·2	10 5/32
'37 ..	19·05	—	6·21	4·80	78 13/32	70 3/32	34 5/16	38 3/8	19 11/16	26·9	12·7	16·9
'38 ..	21·32	—	4·93	3·67	66 29/32	63 1/2	21 25/32	38 5/16	17 21/32	18·6	8·9	11·9
'39 ..	21·12	—	5·95	4·41	90 21/32	72 9/16	22 5/16	48 5/16	26 19/32	17·9	9·0	12·2
'40 ..	27·23	—	8·10	6·26	177 11/32	183 5/32	26 3/8	100 13/32	27 3/4	29·6	14·9	19·4
'41 ..	32·22	—	9·14	7·65	200 ‡	—	31 13/16	126 ‡	26 13/32	32	16 1/8	21
'42 ..	34·87	—	8·83	7·37	200 ‡	201·6	33 ‡	130 ‡	24 3/4	32	16 1/8	21
'43 ..	36·91	—	7·83	6·38	200 ‡	205 ‡	33 ‡	130 ‡	33 1/8	32	16 1/8	21
'44 ..	39·19	—	11·32	9·50	200 ‡	205 ‡	40 1/2 ‡	130 ‡	40 1/4	32	16 1/8	21
'45 ..	40·27	—	12·75	10·80	200 ‡	230 (b)	40 1/2 ‡	130 ‡	39 1/4	31·6	15·9	21·1
'46 ..	40·94	—	14·87	12·14	252	244 3/8	51	144 1/2	49 1/6	35 3/16	16 3/4	20·7
'47 ..	47·61	—	21·21	16·28	262	248	92 1/3	208 1/6	80	58·2	30	22·7
'48 ..	74·07	—	23·23	17·80	247	281·3	88 1/5	220 7/16	96 9/10	92 1/8	61 1/8	33
Average 1904–13	11 7/8	—	6 1/2	5	32 1/2	36 7/8	30 3/8	31 3/8	18 7/8	17 3/8	9	10 7/8
1890–99	10 3/8	—	4 1/4	3	27	27	26 1/2	25	12 1/2	13 1/2	6 1/2	10
'78–87	9	—	6	4 1/4	33	34	35 1/2	26 1/2	15	18 1/2	8 3/8	11 3/4
'67–77	12 1/2	—	9	6 3/4	46	48	43	35	19	21 1/4	9 7/8	19 3/4

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	26	20–26	27	28	29A–29B	30A–30B	31	32A–32B	33
1922 ..	193	994	134	118	191	116	162	180	49
'27 ..	142	990	106	123	181	141	172	177	78
'28 ..	125	860	121	128	203	130	178	174	91
'29 ..	129	881	114	114	157	126	168	156	81
'30 ..	133	784	83	76	121	96	105	86	54
'31 ..	127	700	66	68	76	58	84	70	43
'32 ..	130	694	58	72	93	70	85	71	29
'33 ..	129	750	62	67	106	68	78	94	30
'34 ..	129	765	74	71	119	73	74	102	35
'35 ..	130	786	74	80	161	80	89	96	37
'36 ..	136	825	74	76	133	91	93	119	53
'37 ..	152	995	69	71	158	93	104	127	86
'38 ..	171	949	55	54	139	77	93	88	60
'39 ..	169	958	66	65	174	91	140	86	62
'40 ..	218	1,169	90	93	383	163	146	142	98
'41 ..	258	1,266	101	113	416	202	139	155	106
'42 ..	279	1,285	98	109	427	209	130	155	106
'43 ..	295	1,312	87	95	431	209	174	155	106
'44 ..	314	1,383	126	141	431	219	212	155	106
'45 ..	322	1,462	142	160	457	219	207	153	107
'46 ..	328	1,671	165	180	528	251	259	167	105
'47 ..	381	2,126	236	241	543	385	421	283	114
'48 ..	594	2,574	258	264	598	396	510	492	167

* Port Philip fleece washed nominal since 1895, exactly in proportion with the value of clean wool.
‡ Nominal. ‖ Now No. 1 Oomra, Fine.
¶ Livonian Z.K. from 1921–45. Medium grade continental w/retted from 1946.
†† Lightnings from 1931 to 1942. Daisee 3/4 from 1943.
(a) Russian Siretz Group 1, Sort 1 from 1931–33; Jugo-Slav Peasant from 1934–45. Italian S.B. from 1946.
(b) Average of 4 months only. Belgian from 1946.

Average Prices of Commodities—Contd.

No. of Article	34	27–34	35A	35B	35C	36A	36B	37	38	39	40A	40B
	Silk		Hides			Leather		Tallow	Oil			Seeds
Year	Tsatlee *	Textiles Total	River Plate, Dry	River Plate, Salted	Average Import Price	Dressing Hides	Average Import Price	Town	Palm	Olive	Linseed	Linseed
	s. per lb.		*d.* per lb.	*d.* per lb.	*d.* per lb.	*d.* per lb.	*d.* per lb.	*s.* per cwt.	£ per ton	£ per ton	£ per ton	*s.* per qr.
1922 ..	28 3/4	—	9 1/8	8 7/8	8·06	24 1/3	36	34 5/8	34 3/5	75 1/2 †	39 1/5	75 1/2
'27 ..	15 7/16	—	12 5/16	10 7/16	9·85	22 15/16	36 15/16	33 7/8	34 1/4	102 5/8	31 3/4	64 5/16
'28 ..	14	—	15 7/16	11 29/32	12·09	23 13/16	37 7/16	36 15/16	35 3/4	80 7/16	29 7/8	66 15/16
'29 ..	13 7/8	—	10 9/16	8 1/4	10·80	19 1/4	38 1/4	36 13/32	34 21/32	72	35 15/32	74 1/8
'30 ..	10 15/16	—	6 25/32	6 1/2	7·80	18 1/2	33 1/2	28 23/32	25 1/4	52 1/8	36 5/8	61 15/16
'31 ..	8 15/16	—	5 1/2	5 9/10	6·12	17 3/4	32 1/2	19 1/3	19 7/16	53 9/10	18 3/16	38 7/8
'32 ..	8 5/32	--	4 9/10	4 3/4	5·47	17 3/32	28 7/8	21 1/16	17 5/8	57 3/16	17	38 5/16
'33 ..	6 3/4	—	5 1/4	4 11/16	5·65	17 11/32	26 29/32	19 27/32	15 11/16	53 15/16	20 3/4	39 3/4
'34 ..	5 7/16	—	4 13/16	4 7/8	5·71	17 1/2	25 5/16	17 3/4	13 17/32	62 3/16	21 13/32	42 3/8
'35 ..	5 13/32	—	5 5/32	5 1/8	5·51	17 1/2	25 13/16	24 5/8	19 3/8	61 13/32	24 13/32	43 1/4
'36 ..	5 25/32	—	6 1/4	6	6·47	17 1/2	27 11/32	23 9/32	19 31/32	70 3/16	28 1/2	48 5/32
'37 ..	8 7/16	—	8 21/32	7 9/16	8·62	18 9/32	28 5/16	23 15/32	22 9/16	95 23/32	31 1/8	54 11/32
'38 ..	7 31/32	—	6 21/32	5 9/16	6·35	14 5/16	24 1/32	17 13/32	14 7/8	68 19/32	26 7/16	46 27/32
'39 ..	13 3/32	—	6 3/4	6 1/8	6·39	18 5/32	23 1/16	16 9/32	14 5/16	73 15/16	29 1/8	49 25/32
'40 ..	17 1/2	—	8	8 1/2	8·50	25 7/32	23 3/4	22 3/32	19	114 1/4	44 3/32	66 1/8
'41 ..	15 21/32	—	7 1/2	8 1/2	8·45	24 3/8	24 1/4	22 1/2	20 3/4	118	41 1/2	70 1/4
'42 ..	16 †	—	8 3/8	9	9·25	26	26 1/8	24 3/16	23	118	44 13/16	78 9/16
'43 ..	16 †	—	8 7/8	9 1/16	9·49	27	24 15/16	34 1/8	36	118 †	48 1/2	109 1/2
'44 ..	16 †	—	10 11/16	9 3/8	10·14	27	28 7/8	43 1/2	42 1/4	118 †	60 3/4	107 3/4
'45 ..	16 †	—	10 11/16	9 3/8	9·42	25	31 1/8	43 1/2	42 1/4	150 †	62	109 1/2
'46 ..	44 1/2	—	17 7/8	15 5/8	11·73	29	38 1/5	46 1/4	43 3/8	270	87 7/8	130 3/4
'47 ..	31 3/8	—	22 17/32	19 7/32	21·71	40	58 5/8	57 7/8	89 5/8	330	193 3/4	211 3/4
'48 ..	23 9/16	—	20 31/32	20 7/16	20·72	43 3/8	64 5/8	60	99 1/4	329	193 15/16	216 3/4
Average 1904–13	11 3/4	—	9 5/8	7 1/2	6 7/8	16	17	31 1/2	31 1/4	43 3/4	26 5/8	49 1/2
1890–99	11 1/2	—	6 1/4	5 1/2	5	13 1/4	13 3/4	25	24 1/2	35	19 1/4	38
'78–87	15	—	8 5/8	6 3/4	6 1/2	15	17	35 1/2	32 1/2	40	23	46
'67–77	23	—	9	7	6 7/8	16	18 3/4	45	39	50	30	60

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	Silk (34)	Textiles Total (27–34)	Hides (35A–35C)	Leather (36A–36B)	Tallow (37)	Palm (38)	Olive (39)	Linseed (40A–40B)
1922 ..	125	1,075	114	174	77	89	151	127
'27 ..	67	1,045	142	172	82	88	205	107
'28 ..	61	1,086	172	176	81	92	161	108
'29 ..	60	976	129	166	64	89	144	122
'30 ..	48	669	92	150	43	65	104	110
'31 ..	39	504	77	146	47	51	108	63
'32 ..	35	513	66	132	44	45	114	61
'33 ..	29	534	68	127	39	40	108	67
'34 ..	24	572	67	123	55	35	124	71
'35 ..	24	641	69	125	52	50	123	75
'36 ..	25	646	82	129	52	51	140	85
'37 ..	37	745	109	134	39	58	192	95
'38 ..	35	601	81	110	36	38	137	81
'39 ..	57	741	84	119	49	37	148	88
'40 ..	76	1,191	109	141	50	49	228	122
'41 ..	68	1,300	107	135	54	53	236	124
'42 ..	70	1,304	116	150	76	59	236	137
'43 ..	70	1,327	120	149	81	92	236	176
'44 ..	70	1,460	132	161	97	108	236	187
'45 ..	70	1,515	129	161	97	108	300	190
'46 ..	193	1,848	199	194	103	111	540	243
'47 ..	136	2,359	277	284	129	230	660	450
'48 ..	102	2,787	271	311	133	254	658	456

* Common New Style from 1921 to 1936. China, Extra "A" from 1937–46; Italian in 1947. † Nominal.

Average Prices of Commodities—Contd.

No. of Article	41A ‡	41B ‡	41C ‡	41	42	43	44	45A	45B	35–45	20–45	1–45
	Petroleum *				Soda		Indigo	Timber				
Year	Motor Spirit c.i.f.	Kerosene (Burning Oil) c.i.f.	Gas Oil c.i.f.	Mean of 41A, 41B and 41C	Crystals	Nitrate of Soda	Bengal, Good Consuming	Hewn, Average Import Price	Sawn or Split, Average Import Price	Sundry Materials Total	Materials Total	Grand Total
	d. per imp. gall.	*d.* per imp. gall.	*d.* per imp. gall.		*s.* per ton	*s.* per cwt.	*s.* per lb.	*s.* per load	*s.* per load			
1922 ..	15·89	6·32	3·98	—	123	$14\frac{1}{3}$	$9\frac{5}{8}$	$46\frac{5}{8}$	$117\frac{1}{2}$	—	—	—
'27 ..	7·25	5·15	4·23	—	100	$12\frac{9}{16}$	$5\frac{1}{2}$	$45\frac{7}{16}$	$107\frac{9}{16}$	—	—	—
'28 ..	5·96	4·48	3·32	—	100	$10\frac{31}{32}$	$5\frac{1}{2}$	$45\frac{7}{8}$	$111\frac{5}{32}$	—	—	—
'29 ..	6·75	4·87	3·21	—	100	$10\frac{1}{4}$	$5\frac{1}{2}$	$44\frac{3}{4}$	$107\frac{21}{32}$	—	—	—
'30 ..	6·52	4·54	3·21	—	100	$9\frac{27}{32}$	$5\frac{1}{2}$	$44\frac{1}{2}$	$102\frac{3}{16}$	—	—	—
'31 ..	3·63	3·10	2·33	—	100	$9\frac{1}{8}$	$5\frac{1}{2}$	$37\frac{19}{32}$	$83\frac{3}{8}$	—	—	—
'32 ..	3·97	3·34	2·39	—	100	$8\frac{5}{8}$	$5\frac{1}{2}$	$35\frac{1}{2}$	$75\frac{1}{16}$	—	—	—
'33 ..	3·43	2·94	2·48	—	100	$8\frac{11}{32}$	$5\frac{1}{2}$	$31\frac{7}{8}$	$75\frac{19}{32}$	—	—	—
'34 ..	3·28	2·65	2·45	—	100	$7\frac{13}{16}$	$5\frac{1}{2}$	$31\frac{1}{2}$	$79\frac{13}{32}$	—	—	—
'35 ..	3·51	2·88	2·32	—	100	$7\frac{5}{8}$	$5\frac{1}{2}$	$32\frac{1}{4}$	$73\frac{5}{32}$	—	—	—
'36 ..	3·77	2·76	2·34	—	100	$7\frac{5}{8}$	$5\frac{3}{4}$	$37\frac{9}{16}$	$78\frac{27}{32}$	—	—	—
'37 ..	4·52	3·71	3·14	—	100	$7\frac{25}{32}$	$5\frac{3}{4}$	$58\frac{3}{8}$	$103\frac{7}{32}$	—	—	—
'38 ..	4·08	3·49	3·22	—	100	8	$5\frac{3}{4}$	$61\frac{3}{32}$	$94\frac{5}{16}$	—	—	—
'39 ..	5·00	3·71	3·61	—	100	$8\frac{1}{8}$	$5\frac{3}{4}$	$57\frac{9}{16}$	$107\frac{9}{32}$	—	—	—
'40 ..	5·79	5·68	5·31	—	100	$9\frac{7}{8}$	$5\frac{3}{4}$ †	$106\frac{29}{32}$	$169\frac{15}{16}$	—	—	—
'41 ..	6·97	5·95	5·93	—	100	$13\frac{1}{32}$	$5\frac{3}{4}$ †	$191\frac{1}{16}$	$214\frac{7}{16}$	—	—	—
'42 ..	7·77	7·00	7·07	—	$102\frac{1}{2}$	$13\frac{1}{2}$	$5\frac{3}{4}$ †	$235\frac{1}{8}$	$236\frac{1}{4}$	—	—	—
'43 ..	8·93	7·51	7·61	—	$107\frac{1}{2}$	$13\frac{1}{2}$	$5\frac{3}{4}$ †	269	$246\frac{1}{4}$	—	—	—
'44 ..	8·31	6·78	6·83	—	$107\frac{1}{2}$	$15\frac{1}{4}$	6 †	$222\frac{15}{16}$	$273\frac{15}{16}$	—	—	—
'45 ..	6·23	5·33	5·38	—	$107\frac{1}{2}$	$15\frac{1}{4}$	6 †	$190\frac{1}{2}$	$235\frac{1}{4}$	—	—	—
'46 ..	5·81	5·21	5·20	—	$107\frac{1}{2}$	$16\frac{1}{8}$	6 †	$184\frac{7}{8}$	$258\frac{1}{2}$	—	—	—
'47 ..	7·69	6·83	6·52	—	$107\frac{1}{2}$	$17\frac{7}{8}$	7 †	187·43	289·20	—	—	—
'48 ..	9·38	8·74	8·69	—	$107\frac{1}{2}$	$19\frac{3}{4}$	7 †	215·02	352·82	—	—	—
Average 1904–13				—	60	$10\frac{5}{8}$	3	38	56	—	—	—
1890–99		Base, 1922		—	53	$8\frac{1}{4}$	$4\frac{3}{8}$	40	45	—	—	—
'78–87				—	62	$12\frac{1}{2}$	6	47	47	—	—	—
'67–77				—	92	14	$7\frac{1}{4}$	60	54	—	—	—

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

Year	41A	41B	41C	41	42	43	44	45A–45B	35–45	20–45	1–45
	‡	‡	‡								
1922 ..	100	100	100	100	134	102	128	143	1,339	3,408	5,873
'27 ..	46	82	106	78	109	90	76	134	1,276	3,311	5,477
'28 ..	37	71	83	64	109	78	76	138	1,256	3,202	5,366
'29 ..	42	77	81	67	109	73	76	134	1,190	3,047	5,143
'30 ..	41	72	81	65	109	70	76	129	1,034	2,487	4,313
'31 ..	23	50	60	44	109	65	76	106	890	2,094	3,673
'32 ..	25	53	60	46	109	62	76	97	855	2,062	3,571
'33 ..	22	47	62	44	109	60	76	94	837	2,121	3,529
'34 ..	21	42	62	42	109	56	76	97	839	2,176	3,635
'35 ..	22	46	60	43	109	54	76	92	871	2,298	3,745
'36 ..	24	42	60	42	109	54	79	102	925	2,414	3,945
'37 ..	28	59	79	55	109	56	79	142	1,081	2,821	4,581
'38 ..	26	55	81	54	109	57	79	136	921	2,471	4,071
'39 ..	31	60	91	61	109	58	79	145	964	2,663	4,247
'40 ..	36	90	133	86	109	70	79	243	1,285	3,645	5,760
'41 ..	44	94	150	96	109	93	79	356	1,438	4,004	6,379
'42 ..	49	111	178	113	111	96	79	413	1,564	4,153	6,812
'43 ..	56	119	199	122	117	96	79	452	1,715	4,354	6,978
'44 ..	52	107	172	110	117	109	83	436	1,776	4,619	7,221
'45 ..	39	84	135	86	117	109	83	373	1,753	4,730	7,375
'46 ..	36	82	131	83	117	115	83	389	2,177	5,696	8,365
'47 ..	48	108	164	107	117	128	97	418	2,897	7,382	10,353
'48 ..	59	138	218	138	117	141	97	498	3,074	8,435	11,683

* Prior to 1922, Kerosene Burning Oil (in barrels), base period 1873–77; from 1922 c.i.f. values per imperial gallon of Motor Spirit, Kerosene (Burning Oil) and Gas Oil, base period 1922. † Nominal.

‡ Index-numbers not included in the general average.

REPORT OF THE COUNCIL

For the FINANCIAL YEAR *ended December* 31*st*, 1948, *and for the* SESSIONAL YEAR *ending June* 29*th*, 1949, *presented at the* ONE HUNDRED AND FIFTEENTH ANNUAL GENERAL MEETING *of the* ROYAL STATISTICAL SOCIETY, *held at the London School of Hygiene and Tropical Medicine, W.C.* 1, *on June* 29*th*, 1949.

THE Council has the honour to submit its One Hundred and Fifteenth Report.

Number of Fellows

During the calendar year 1948 the number of new Fellows elected into the Society (including a few former Fellows restored to the roll) was 238. This figure is slightly less than the previous year's total (263), and the table below clearly shows that the peak of entry was reached in 1946. During the current year exits numbered 96. Though this is the highest absolute number yet reached, and a few resignations may have been due to the increase in the annual subscription, it is apparent that as a percentage of the total Fellowship (4·9) it is not very different from the rate prevailing before the war (4·2 per cent. in 1937 and 5·2 per cent. in 1938). The net increase in the number of ordinary Fellows, as a result of these movements in and out, was 142 compared with 188 and 231 in the two preceding years.

	Number of Fellows (excluding Honorary Fellows)		
Calendar Year	Lost by death, withdrawal or default	Elected, or restored to the roll	On the roll at December 31st
1938	56	76	1,083
1939	39	64	1,108
1940	76	47	1,079
1941	65	60	1,074
1942	69	74	1,079
1943	61	121	1,139
1944	29	159	1,269
1945	29	177	1,417
1946	56	287	1,648
1947	75	263	1,836
1948	96	238	1,978

In addition to the numbers given in the table above there were, at December 31st, 1948, 15 Honorary Fellows, of whom 7 were Presidents for the time being of other Societies concerned with the advancement of statistical knowledge. The ordinary Fellows at the end of 1948 included 103 representing corporate bodies or institutions which wished to associate themselves in this way with the work of the Society.

The total number of ordinary Fellows at the close of 1948 was, it will be seen, a fraction below the 2,000 mark, but elections and restorations in the present session (see below) have now brought the Society just above it. While this growth in the Society is certainly satisfactory, it should not be regarded as satisfying. If the Society is to expand its activities still further than it has in the post-war years, if it is to maintain the standard and to increase the volume of its publications, it will need an increase both in its staff and in its income. Such income must come at least mainly from a further increased Fellowship. Present Fellows, so many of whom are new, may, perhaps, be reminded that the Society is by no means limited to professional statisticians or to those technically qualified. Election is, and always has been, open to all persons who have a real interest in the subject, and with the advancement of statistical science and practice at heart. It is not too much to say that many persons with no technical qualifications, but with wide interests and knowledge as well as a love of statistics, have been a vital part of the Society in the past.

The Council has no doubt whatever that they will continue to be a vital part and it hopes that Fellows will encourage such persons to seek election and will themselves not hesitate to nominate them.

The Council again published a complete list of Fellows last year and hopes to be able to do so again in the present year. This list can serve its purposes effectively only if Fellows will take the trouble to notify the Assistant Secretary promptly of changes either of description or address. Every Fellow is asked on receipt of this Annual Report to check his own entry in the List of Fellows, and, if he has not previously done so, to inform the Assistant Secretary *immediately* of any correction that is required.

Elections and Restorations

From June, 1948, to May, 1949, inclusive, 208 Fellows were elected or restored to the roll, and on May 17th, 1949, the total number of Fellows was 2,014. The names of Fellows elected from June, 1948, to May, 1949, are listed in Appendix A.

Losses by Death

The Council regrets to report that during the year ending May 17th, 1949, the Society lost by death the undermentioned Fellows:—

	Date of election
Mitchell, Wesley C. (Honorary Fellow)	1924
*cp**Collet, Clara E., M.A.	1892
Kay, Harry Bevin, F.I.A.C.	1924
Knoop, Professor Douglas	1921
Piper, Henry Mansell, C.B.E.	1915
Rylands, Sir William Peter, Bt.	1920
*cp*Verdier, James William, O.B.E.	1921

* Life Fellow. *c* Served on Council.
p Contributed to Proceedings.

Wesley Clair Mitchell, who became an Honorary Fellow of the Society 25 years ago and died at the age of 74, was one of the most distinguished of American economists. His central interest, comprising both economics and statistics, lay in the problems of industrial fluctuations, and to this subject he contributed, in 1913, his book entitled *Business Cycles*, a book which rapidly became a classic. After the first world war he founded and directed for a quarter of a century the National Bureau of Economic Research in New York, where, with devoted pupils and colleagues, he expanded his early single-handed efforts into a comprehensive survey of the whole problem of cyclical fluctuations.

Miss Clara Collet was born in 1860 and at her death in 1948 had been a Fellow of the Society for over 50 years. She served on the Council, with a few intervals, from 1919 to 1935. In 1898 she read a paper before the Society on the available official statistics relating to the industrial employment of women. Her working life was, in fact, devoted to the improvement of women's social and economic status from the time that she became, in 1886, one of Charles Booth's Assistants on the London Survey, through her years of service as Chief Investigator for Women's Industries at the Board of Trade and subsequently on Trade Boards, down to her study on Domestic Service contributed to the new London Survey of 1930–35. A woman with a clear and brilliant mind and of a charming and ever kindly personality she will be remembered with affection by many of the older Fellows of the Society.

James William Verdier, who died at the age of 82, worked, in association with Sir Alfred Flux and other distinguished Fellows of the Society, at the Board of Trade for the greater part of his life. After his retirement in 1927, he became Secretary of the New Survey of London Life and Labour. He served two terms of office on the Council of the Society in 1931–32 and 1935–36, and in 1922 read a paper on the "Interpretation of Statistics relating to Shipping Casualties and Loss of Life at Sea."

Vice-Presidents

For the session 1948–49 the President, acting under Bye-law 28, appointed as Vice-Presidents of the Society Sir Henry Clay, Professor Bradford Hill, Dr. J. O. Irwin and Mr. M. G. Kendall.

Meetings of the Society

The Ordinary General Meetings of the Society have been held in each month from November, 1948, to May, 1949, at the London School of Hygiene and Tropical Medicine. The large attendances and animated discussions at these meetings suggest that the widely varying subject matters presented all have their interests to Fellows, and that such a variety of topics is therefore desirable. For maintaining this variety the Council is dependent upon the Fellows themselves and it would once more remind them not to be backward in submitting papers for reading. The procedure to be followed in submitting papers is printed with the List of Fellows, or can be obtained from the Assistant Secretary in leaflet form. The Council also welcomes suggestions on subjects to be discussed at meetings, although it must be understood that, in the absence of the offer of a paper, it may not always be possible to accept them.

The following papers have been read at Ordinary General Meetings during the session:—

1948	
November 11th .	SMEED, Dr. R. J. Some Statistical Aspects of Road Safety Research.
December 21st .	AINSWORTH, R. B. Earnings and Working Hours of Manual Wage Earners in the United Kingdom in October, 1938.
1949	
January 19th . .	CAMPION, H. International Statistics.
February 23rd .	BENJAMIN, B. Local Government Statistics.
March 16th . .	TINTNER, Prof. G. Foundations of Probability and Statistical Inference.
April 27th . .	BRONOWSKI, Dr., *et al.* An Investigation into the Erection Times of Nine Types of Non-traditional House.
May 17th . .	NICHOLSON, J. L. Variations in Working Class Family Expenditure.

The Oxford Conference

One meeting of the Society in a new form, and therefore of very particular interest, was the week-end conference held, under the chairmanship of the President, at St. Hugh's College, Oxford, on Saturday and Sunday, October 2nd and 3rd, 1948. The conference was divided into five sessions of two hours each, and 13 papers were read and discussed. As particulars of these papers have been published in the Journal (Series A, Part II, 1948, p. 161) it is unnecessary to give them in detail here. What the Council would, with satisfaction, report is the undoubted success of this venture. Staying in St. Hugh's College were 94 Fellows, while others resident or staying in Oxford brought the attendance up to at least 120. It is apparent that Fellows would welcome such conferences—at least at not too frequent intervals—and the Council hopes to arrange a second in the near future. It would be glad of any suggestions from Fellows. For the success of the Oxford Conference it is greatly indebted to Dr. D. J. Finney and Professor D. G. Champernowne, who took charge of all the local arrangements as well as the construction of the programmes. Such help at the centre at which the conference is to be held is fundamental to its success.

The Research Section

Dr. J. O. Irwin has for a second year served as Chairman of the Research Section, Mr. E. C. Fieller was elected Honorary Secretary, and the other members of the Section's Committee have been: Mr. F. J. Anscombe, Professor G. A. Barnard, Dr. H. E. Daniels, Dr. O. L. Davies, Dr. H. O. Hartley, Mr. D. G. Kendall, Dr. C. A. B. Smith, Dr. L. Solomon, with Mr. M. G. Kendall and Mr. L. H. C. Tippett nominated by the Council, and Mr. J. R. N. Stone, Dr. B. L. Welch and Dr. F. Yates ex-officio members as Editors of Series B of the Journal.

During the Session four meetings have been held at which the following papers were read:—

1948		
November 2nd	.	HALDANE, Prof. J. B. S., F.R.S. Some Statistical Problems in Genetics.
1949		
January 6th .	.	BOAG, J. W. Maximum Likelihood Estimates of the Proportion of Patients Cured by Cancer Therapy.
March 10th .	.	BARNARD, Prof. G. A. A New Theory of Statistical Inference.
June 9th .	.	(Symposium on Stochastic Processes) BARTLETT, Prof. M. S. Some Evolutionary Stochastic Processes. KENDALL, D. G. Stochastic Processes and Population Growth. MOYAL, Dr. J. E. Stochastic Processes and Statistical Physics.

These meetings have been well attended, with an average attendance of over 100 Fellows and visitors. It is hoped that the re-christening of the Supplement (see subsequent paragraph on Journal) will bring the Section's proceedings to the notice of a still wider circle of Fellows and other readers.

The Industrial Applications Section

The Section Committee, which co-ordinates the work of Local Groups and gives guidance in the functions of the Section as a whole, has consisted of five members appointed by the Council (Mr. D. J. Desmond, Dr. B. P. Dudding, Dr. J. O. Irwin, Mr. W. J. Jennett and Miss J. Keen), and two representatives of each Local Group. Mr. Desmond was elected Chairman of the Section and Mr. Jennett the Honorary Secretary. During the session the Officers of the Local Group Committees have been :—

Group	*Chairman*	*Honorary Secretary*
Birmingham	Mr. D. J. Desmond	Mr. B. J. A. Martin
London	Mr. D. Newman	Miss J. Keen
Sheffield	Mr. W. T. Hale	Mr. G. H. Jowett
North-Eastern	Mr. N. J. Squirrell	Mr. J. B. Nadauld
Tees-side (Sub-Group) . . .	Mr. H. Kenney	Mr. J. T. Richardson

A revised regulation will in future enable the retiring Section Committee to nominate three of its members to serve on the next Committee and thereby ensure continuity of action.

By the end of 1948 membership of the Section had increased slightly to over 500, this number including rather more than 200 Fellows. During the current year this number has been further increased by the formal incorporation, in April, 1949, of the Provisional South Wales Group which has held a full Session of meetings in Crumlin, Cardiff, Swansea and Aberystwyth under the chairmanship of Dr. T. V. Starkey, with Mr. E. H. Lloyd as Honorary Secretary.

During the Session the following 36 discussion meetings have been held :—

1948		*Birmingham and District*
September 24th	.	The Accuracy of Automatic Lathes—D. J. Desmond.
October 29th	.	Statistical Principles in Quality Specifications for Plastics—C. Wainwright.
November 26th	.	Factorial Regression Analysis—Philip Lyle.
1949		
January 28th	.	What the Engineer requires of Statistics—T. U. Matthew.
February 25th	.	Operational Research—A. W. Swan.
March 25th .	.	Application of the Planned Experiment—O. L. Davies.
April 29th .	.	Sequential Sampling—E. D. van Rest.
1948		*London Group*
October 8th	.	Quality Control Charts in Cost and Productivity—G. Todd.
November 12th	.	Control Charts and Associated Techniques—W. J. Jennett.
December 10th	.	A Statistical Department in the Steel Industry—A. W. Swan.
1949		
February 4th	.	The Statistician Looks at a Mechanical Transport Section—W. R. Buckland.
March 4th .	.	Statistical Methods in the Interpretation of Experimental Results—G. E. P. Box.
March 17th .	.	The Use of Statistical Methods in Road Research Problems—F. Garwood.
April 1st .	.	Statistical Principles in Quality Specifications for Plastics—C. Wainwright.
May 6th .	.	Forum—E. S. Pearson, B. P. Dudding, E. C. Fieller, Philip Lyle.

Sheffield Group

1948	
September 30th .	The Limitation of Statistics—Symposium—J. Hebden, R. Padgett, J. Ridley Thompson, J. R. Widdowson.
October 28th .	The Activities of the Mathematics Division of the National Physical Laboratory—E. C. Fieller.
November 25th .	Some Uses for Elementary Statistics in the Refractories Industry—K. Cowling.
1949	
January 27th .	Industrial Philosophy for the Statistician—R. J. Sarjant.
February 24th .	Simple Foundry Sand Investigations—C. M. Handley.
March 31st . .	We have Applied Statistics—Symposium—M. Dolan, H. C. Harrison, J. R. Widdowson, J. Murdoch.
April 28th . .	Wartime Experience of Operational Research—T. A. Evans.

North-Eastern Group

1948	
October 20th .	A Statistical Contribution to Production Economy—B. P. Dudding.
November 17th .	Incentives and the Young Worker—L. T. Wilkins.
December 15th .	Some Problems in Initiating Statistical Control—W. J. Ross.
1949	
February 16th .	Probability and the Experimenter—E. D. van Rest.
March 16th . .	The Integration of Elementary and More Advanced Statistical Methods in the Control of Processes—W. J. Jennett.
April 27th . .	Statistical Tables in Quality Control—N. J. Squirrell.
May 25th . .	The Organization and Operation of a Statistical Section in Heavy Industry—A. W Swan.

Tees-side Sub-Group

1948	
October 14th .	Sampling and Analysis of a Product in Bags—J. Richardson.
November 16th .	Incentives and the Young Worker—L. T. Wilkins.
December 9th .	Making Decisions from Numerical Data—G. E. P. Box.
1949	
January 11th .	Elementary Regression Analysis—W. T. Hale.
February 17th .	Probability and the Experimenter—E. D. van Rest.
March 10th . .	Statistics Applied to a Continuous Process: The Blast Furnace—G. H. Jowett.
April 21st . .	Sequential Testing—F. R. Himsworth.

In addition, at the British Association meeting held in September, 1948, papers on the Application of Statistics to Engineering were presented by the following members of the Section: Professor Barnard, Mr. Desmond, Dr. Dudding, Mr. Jennett, Mr. Swan.

Attendances at the Group meetings listed above have been substantially the same as in the previous session except for a welcome increase in the North-Eastern Group. Such a programme of meetings clearly involves much preparation, and the Council is indebted both to the officers of the Section and to those of its local groups for their work on its behalf and for making the Section so active and successful a component of the Society.

The Study Section

The Study Section Committee for the session has been: Chairman, Mr. W. R. Buckland; Secretary, Mr. L. T. Wilkins; Council representatives, Miss J. I. Douglas and Mr. R. F. George; other members, Miss J. R. Weatherburn, Mr. A. Blackwell, Mr. J. I. Mason, and Mr. D. Newman.

During the Session the Committee arranged the following eight meetings:—

1948	
October 13th .	Financial Sources—M. S. Rix.
November 12th .	Joint Meeting with the Industrial Applications Section (London Group): Control Charts and Associated Techniques—W. J. Jennett.
December 8th .	Machine Methods—H. Gearing.

1949			
January 12th .	Pictorial Presentation—P. Redmayne.		
February 9th .	Joint Meeting with the Market Research Society—		
	Teaching of Statistics (*b*) Teaching for Market Research	Open Forum:	A. P. McAnally. A. H. Elliott. W. F. F. Kemsley. L. T. Wilkins.
March 9th . .	Organization of a Statistics Department	Open Forum:	G. S. Browne. J. Stafford. A. W. Swan. B. Benjamin.
April 6th .	Applications of Discriminant Functions in Biology—P. Armitage.		
May 11th . .	Annual General Meeting of the Section.		

These meetings fall, it will be seen, into three type-groups: (*a*) the conventional form of a paper followed by discussion, (*b*) Open Forums, and (*c*) Joint Meetings with other bodies. The last has been very successful and it is hoped to arrange further meetings of this kind, particularly in association with local groups of the Society's Industrial Applications Section.

The Open Forum with the Market Research Society (February 9th) was the second of a series on the teaching of statistics. This form of presentation has also proved popular.

The attendances at these meetings have increased during the 1948–49 session by about 50 per cent. on the previous year to an average of 54. The level of interest in the Section's proceedings may also be judged by the lively discussions at meetings and the fact that the "guillotine" has had to be applied on more than one occasion. The Council is glad to report this rapid and successful resuscitation of its pre-war Study Group.

Both the Study Section and the London Group of the Industrial Applications Section have been extremely fortunate in being able, once again, to hold their meetings at the Lighting Service Bureau of the E.L.M.A. The Council is greatly indebted to the E.L.M.A. for their kindness in placing such admirable accommodation at the disposal of its Sections.

Guy Medal

The Council has pleasure in reporting that, acting on the recommendations of the Committee of the Industrial Applications Section and of its own Executive Committee, it has awarded a Guy Medal in bronze to Mr. W. J. Jennett for his various papers on the applications of statistics to industry read over the last seven years to the Society's Industrial Applications Section. These papers, coupled with Mr. Jennett's work as honorary secretary of the Section, have played a considerable part in the development of the Industrial Applications Section and, in a wider field, in the extension of statistical techniques through industry. The Council is glad to take this opportunity of acknowledging Mr. Jennett's services in this field and, indirectly, the importance it attaches to the work of the Society's Industrial Applications Section.

Frances Wood Memorial Prize

The Council offered the Frances Wood Memorial Prize for competition in 1948 for, as customary, the best investigation, on statistical lines, of any problem bearing directly or indirectly upon the economic or social conditions of the people. Eleven essays were received and the Council has awarded two prizes:—

To Mr. J. L. Nicholson for an essay on "Variations in Working Class Expenditure."

To Mr. L. T. Wilkins for an essay on "The Prevalence of Deafness in England, Scotland and Wales.

Mr. Nicholson's essay was presented to the Society at its meeting on May 17th, 1949.

The Society's Examinations

The second Examination for the Society's Certificate was held in September, 1948. Twenty-seven candidates presented themselves for all or part of the examination and as a result three

Fellows satisfied the examiners in part of the examination and the following 13 Fellows were awarded Certificates:—

Allinson, Vanessa Adele.
Aston, Arthur Harold.
Brown, Marjorie Elizabeth.
Carruthers, Nellie.
Deeks, Herbert William George.
Edge, Charles Geoffrey.
Kensit, George Robinson.
Komlosy, Graham Frederick.
Lloyd, David.
Poulton, Emily Penelope.
Richards, Dennis Leslie.
Skinner, John Cecil.
Whitehouse, Arnold Martyn.

The third Certificate Examination was held in April, 1949, and 25 candidates sat this examination (in whole or part). 10 Fellows satisfied the examiners in part of the examination and the following eight Fellows of the Society were awarded its Certificate.

Curwen, Michael Patrick.
Dryden, John Aungle.
Freeman, Peter Wilfred.
Murray, Andrew Digby.
Pritchard, Arthur Alan.
Reece, John Arnold.
Steer, Edward Thomas.
Wilkins, Leslie Thomas.

The first Diploma Examination was also held in April, 1949. There were seven candidates and the following Fellows were awarded the Diploma of the Society, the specialised field on which one section of the examination was based being given in brackets:—

Spicer, Clive Colquhoun (Medical statistics).
Taylor, John (Biological and agricultural research).

It is hoped that the next examination for both Certificate and Diploma will be held in April, 1950. Particulars will be announced in due course.

All the examinations reported above were held in the Department of Medical Statistics at the London School of Hygiene and Tropical Medicine and the Council acknowledges very gratefully the granting of these facilities and the help in conducting the examinations given by Professor Bradford Hill and his colleagues. It also extends its thanks to its Examinations Committee and others who have assisted it in carrying out this task. The composition of the Committee has been: The President (Chairman), Professor R. G. D. Allen, Dr. D. J. Finney, Mr. R. F. George, Professor E. S. Pearson, Mr. J. R. N. Stone, Dr. F. Yates.

The Journal

During the session the Council considered very closely the position of the Society's two publications—the Journal and its Supplement. As reported in the Journal (Part IV, 1947, p. 366), it concluded that the title of "Supplement" did injustice to an issue that had grown from modest beginnings to a scientific journal of high repute. It decided, therefore, that from 1948 the two publications would *both* be issued under the main title *Journal of the Royal Statistical Society*, the original Journal being distinguished by the sub-title "Series A (General)" and the Supplement by the sub-title "Series B (Methodological)." The volume numbers continue as in the past. At present Series A is sent to all Fellows and Series B to those who make request for it. To meet these demands and the rising costs of printing the Council would like to extend the sales of Series B to non-Fellows, and hopes that Fellows will assist by bringing it to the attention of likely purchasers (University Departments, Libraries, etc.). Fellows will also notice that advertisements can be accepted for publication in the Journal. For advice in this development the Council is indebted to a Fellow of the Society, Mr. J. R. Clementson.

During the year the Society has begun to publish a series of articles on the statistics of various branches of trade, industry, commerce, and other sociological subjects. The first of these articles, by Dr. M. G. Kendall on the U.K. Mercantile Marine, appeared in Vol. CXI, Series A, Part II; the second, by Professor Major Greenwood on Medical Statistics, appeared in Part III of the same volume; the third, by Mr. H. Leak on Censuses of production and distribution, will appear in Vol. CXII, Series A, Part I, and thereafter it is hoped to publish at least one in each number. Articles already promised cover Agriculture, Coalmining, Civil Aviation, Banking, Inland Transport and the Rubber Industry and the list is being extended. It is hoped ultimately to publish these articles (revised where necessary) in a single volume. In the meantime copies

of the individual articles will be on sale at the Society's offices. The Council feels that Fellows will welcome this new venture and hopes that they will help towards its success by bringing it to the notice of persons likely to be interested.

The Library

The Library Committee of the Council (Mr. R. F. George,. Chairman, Professor Bradford Hill amd Dr. J. O. Irwin) reports that during the calendar year 1948 the services given to Fellows and non-Fellows authorized to use the Library can be summarized statistically as follows, the figures in brackets showing the corresponding values for the two previous years, 1947 and 1946 (in that order):—366 (385,399) Fellows, or a fraction under 20 per cent. of the total number, borrowed books. These Fellows made by post or in person 1,229 (1,534, 1,587) effective applications (i.e. their requests to borrow a book or books could be met), and between them they borrowed 2,314 (2,691, 2,939) volumes. These figures reveal a decline in the borrowing of books in the last three years though not so much, it will be seen, in the number of Fellows concerned as in the number of books they borrowed. A partial explanation may well be that standard works on statistics can now be purchased more easily by their would-be readers. The demand for some of these books in previous years had, indeed, been so great that the Library Committee thought it right to provide additional copies to meet it.

Much use, of course, is made of the Library's facilities which does not result in a book being borrowed—the consulting of books in the Reading Room, requests for information by post and telephone, etc. Such services cannot be measured satisfactorily. The number of signatures of Fellows and visitors using the Reading Room in 1948 was 603 (667, 1,953), but these figures certainly understate the real numbers. Non-serial works added to the Library during the year numbered 429 (317, 454). By all measurements the use made of the Library is at a much higher absolute level than in pre-war years (though not necessarily relatively to the number of Fellows). Thus the number of volumes borrowed in 1938 was 1,683 compared with the 2,314 ten years later.

A suggestion has been made that the Library be kept open to a later hour, e.g. 9 p.m., one night a week. With the Society's small staff it might well be that such a service could be given only if the Library were closed at some other time to compensate for it. Before coming to any decision the Council would be glad to have the views of Fellows on this matter so that it may judge whether there is likely to be an effective demand for the additional service.

The use by Fellows of the suggestion book is of great service to the Council's Library Committee in considering the purchase of books, and the Council would remind Fellows of it and also that items for inclusion can be sent by post to the Librarian. In the coming year the Library Committee hopes to collect statistics, not previously available, on the nature of those requests for books made by Fellows which could not be met. These figures should show whether deficiencies exist in the Library in certain fields of work or whether the books asked for largely fall outside the Society's legitimate interests.

Housing

References having been made in the Press to a scheme for housing Scientific Societies on some central site in London it was learnt that the development of any such scheme would be under consideration by the Scientific Societies' Accommodation Committee of the Royal Society. Upon this a letter, signed by the seven Fellows of the Royal Society who are Fellows of the Royal Statistical Society, and countersigned, on behalf of the Council, by the President and Senior Honorary Secretary, was directed to the President of the Royal Society. This letter set out in detail the claims of the Royal Statistical Society to inclusion and asked that they should be fully considered under this scheme. The President of the Royal Society in acknowledgment has given an assurance that the Society's case will be considered by the Scientific Societies' Accommodation Committee.

The International Statistical Institute

Under the revised Statutes of the International Statistical Institute it was laid down that national statistical societies may become affiliated to the Institute, the chief purpose of such a relationship being the coalescence of international and national professional statistical interests, without, of

course, affecting the autonomy of the national societies. The Council unanimously resolved to seek such affiliation, and its proposal has been officially accepted by the Institute. Under the Institute's statutes "the Secretary-General or other designated representative of each affiliated organization" is entitled to ex-officio membership in the Institute and the Council has appointed Mr. R. F. George to represent it. It notes with pleasure that since the Washington meeting of the Institute the following Fellows have been elected members: Professor R. G. D. Allen, Mr. H. Campion, Professor Bradford Hill, Mr. M. G. Kendall and Mr. J. R. N. Stone (Great Britain), Professor M. H. Belz (Australia), Dr. W. E. Deming, Mr. J. Marschak and Professor J. Neyman (U.S.A.) and Dr. H. Wold (Sweden). Mr. Udny Yule has been elected to Honorary Membership of the Institute, and the Council offers its congratulations on that honour.

Finance

Appendices B and C consist of the abstracts of the Honorary Treasurer's Accounts, viz. the statement of Income and Expenditure for the year 1948 and the Balance Sheet as at December 31st, 1948, together with the report thereon of the Auditor.

Income, other than Life Composition Fees, increased from £6,445 in 1947 to £7,806 in 1948, while expenditure decreased from £8,018 to £7,234. A deficit of £1,573 was thus converted into a surplus of £572.

There was an increase of £1,427 in the total of annual Fellowship subscriptions, on account partly of the increase in the number of Fellows, but largely of the increase in the rate of annual subscription. Receipts from sales of the Journal, Series A, decreased by £98, but receipts from sales of the Journal, Series B (formerly known as the Supplement), increased by £27.

The reduction in expenditure was due partly to the publication, in 1947, of three parts of the Supplement, one of which, in normal circumstances, would have been issued in 1946. Only two parts of the Journal, Series B, were issued in 1948, and total expenditure on publication was reduced by £815.

Expenditure relating to the activities of the Sections was also lower by £180, while a special payment of £250 in 1947, in respect of a meeting of the International Statistical Institute, constituted a non-recurring item in that year. On the other hand, expenditure upon Salaries and Wages increased by approximately £300, and Stationery and Sundry Printing by £120.

Composition Fees fell from £735 in 1947 to £429 in 1948, and at the end of the year the Life Composition Fee Fund, which was maintained at the total of the Composition Fees received from Fellows still living, amounted to £6,772. The Composition Fees (£105) of Compounders who died during the year, together with the surplus of income over expenditure of £572, was added to the Accumulated Fund, which amounted to £6,863 at the end of the year.

The Council and Officers

Early in the 1948–49 session Mr. H. L. Seal resigned from the Council on taking an appointment abroad. The Council invited Mr. B. Benjamin to take his place.

Acting under the Bye-laws the Council gave opportunity to Fellows to make suggestions for the composition of the Council for the Session 1949–50. It took closely into account all the suggestions received and, indeed, adopted the majority of them. The Council's recommendations were then circulated to all Fellows. No alternative nominations were received and therefore the Fellows named below will be announced at the Annual General Meeting on June 29th, 1949, as having been elected as President, other officers and other members of Council for the session 1949–50. The Council takes very great pleasure, which it knows will be shared by all Fellows, in welcoming Sir Geoffrey Heyworth as the Society's new President.

It cannot conclude its Report without an endeavour to acknowledge, even if inadequately, how much it, and all Fellows, owe to the retiring President. It is difficult to convey to those not in close contact with the workings of the Society how much of his time, energy and wisdom Dr. David Heron has given to its affairs. Though it has, doubtless, been a labour of love, not many have served the Society so unsparingly and so ably.

The Council would also take the present opportunity of reminding Fellows how much in these difficult times the Society owes to its Assistant Secretary. The doubling of the Fellowship, the expansion of Sections, the development of the examinations, the Oxford conference and many

other changes have all thrown heavy additional burdens upon Miss Croker and her small staff. The Council is greatly indebted to her for her ready acceptance of all these duties and her skilful discharge of them.

President

Sir Geoffrey Heyworth.

Council

R. G. D. Allen.
*M. S. Bartlett.
B. Benjamin.
†W. R. Buckland.
H. Campion.
D. G. Champernowne.
Sir Henry Clay.
W. Manning Dacey
H. E. Daniels.
D. J. Desmond.
Iris Douglas.
B. P. Dudding.
E. C. Fieller.
D. J. Finney.
*R. A. Fisher.
R. Glenday.
H. O. Hartley.
W. J. Jennett.
P. Lyle.
Sir George Maddex.
†A. H. Marshall.
E. S. Pearson.
†J. H. Richardson.
†L. G. K. Starke.
P. Stocks.
L. H. C. Tippett.

Honorary Treasurer

J. H. Jones.

Honorary Secretaries

A. Bradford Hill. R. F. George. J. R. N. Stone.

Honorary Foreign Secretary

A. Bradford Hill.

On behalf of the Council,
D. Heron,
President.

A. Bradford Hill
R. F. George
J. R. N. Stone
Honorary Secretaries,
June, 1949.

Those marked * were not Members of Council during the preceding session, and those marked † have never previously served on the Council.

APPENDIX A

From June, 1948, to May, 1949, inclusive, the candidates named below were elected Fellows of the Society:—

Abdulcader, Mohamad Hassenmohideen Mohamad.
Adamson, John Ernest.
Alexander, Paul Herbert Walter.
Andrew, Leslie Thomas.
Apfel, Artur.

Back, Frederick Charles Douglas.
Baines, Albert.
Barber, Patricia Edna.
Baron, Raymond Vincent.
Baxendale, Stanley.
Beech, Donald Gordon.
Bentley, William Frederick Frere.
Berkson, Joseph.
Bevington, Anne.
Bhattacharya, Nripendra.
Bonner, Basil Bradlaugh.
Borch, Karl Henrik.
Boyne, Alexander William.
Brass, William.
Brazier, Alan Frank.
Brech, Ronald.
Brickman, Frank.
Bronowski, Jacob.
Brown, William Alexander.
Burgess, Cecil Gilbert.

Caldwell, Robert Laurence.
Carpenter, Kenneth John.
Carreyette, Jack.
Chadwick, Frances Mary.
Clapp, Margaret Anne.
Coll, Jose Augusto.
Cooper, Ronald Arthur.
Coote, Brian Charles.
Copland, Brian Dalton.
Cork, George.
Corrin, Edward.
Coventry, William Frederick.
Crossland, Harry.
Crump, Samuel Lee.
Curwen, Michael Patrick.

Dale-Green, John Daniel.
Davies, Daniel Gordon.
Davies, James Cedric.
Davis, Edwin John.
Denison-Pender, The Hon. Richard Ernest.
Devenport, Christopher Horace.
Dewsbury, George Guthrie.
Dixon, Mervin George.
Doll, William Richard Shaboe.
Drover, Eileen.
Duff, John Leonard.

Eddison, Roger Tatham.
Edwards, F. E.
Egermayer, František.
Ellis, Alec William Tyler.
Ellis, Charles Arthur.

Faulding, Geoffrey Hudson.
Fazakerley, Thomas Wood.
Fletcher, William.
Floyd, Joan Winifred Margaret.
Forshaw, Harry.
Forward, Horace Grey.
Fowler, Brian Gerald Nepean.
Fraser, Alexander George MacGregor.
Freeman, Peter Wilfred.

Gillett, Francis Richard.
Goodman, Nancy.
Grailcourt, Donald Charles Raymond.
Granger, William.

Hatley, George Ashton.
Heard, Victor William John.
Hewitt, Jack Donald.
Heyworth, Sir Geoffrey.
Hinks, Raymond.
Hobbs, John Charles.
Hodges, Alex Lovell.
Hole-Baker, Edwin.
Huggins, Hastings Dudley.
Humphreys, Thomas Alfred.
Hunter, Charles Balfour Clephan.
Hustwick, Ian Alexander Marshall.
Hyman, Doris.

Inkson, Robert Henry Ewen.

Jackson, Ronald Arthur.
James, Geoffrey Sargood.
Janes, Eric Gordon.

Jayasundara, Layward.
Jernė, Niels Kay.
John, Peter William.
Jones, Daniel.
Jones, Ivor Rhys.
Jones, William Owen.

Kesteven, William Leofric.
Knight, Edward Walter Gordon.
Knowles, Kenneth Guy Jack Charles.
Kuznets, George Michael.

Lampkin, Harold.
Leggatt, Sidney.
Lesford, Joan Margaret.
Lieversz, Eardley Greville Lyon.
Lindsell, Donald Albert.
Lloyd, Frederick John.
Logan, William Philip Dowie.
van Lottom, Laurens.

McErlain, James Joseph.
Marriott, Francis Henry Charles.
Mason, William Henry.
Massey, William Joseph.
May, Joan Doris.
Mellone, Margaret Anne.
Millar, David Wallace Giles Purvis.
Mitchell, James Ferguson.
Morley, Eric.
Mostowy, George F.
Mulready-Jones, Montague William.
Murdoch, John.
Murray, Andrew Digby.

Newsam, John Oswald.
North, John Dudley.

Odell, Kenneth George.
Oggier, Raymond Emile Frederick.
Oldmeadow, Ernest James Francis.
Orson, Rasin Ward.

Paradine, Charles George.
Parker, Ruth Florence.
Parkinson, Anna Elizabeth.
Pedersen, Ivan.
Pegler, James Basil Holmes.
Plane, Ronald Fife.
Poulton, Emily Penelope.
Prior, Ronald Sidney.

Qureshi, Dil Mohammad.

Raghavachari, S.
Ratcliffe, John Gold.
Rau, A. A.
Read, William Arthur.
Reiners, William Joseph.
Rhee, H. Albert.
le Riche, William Harding.
Rizkalla, Sabet Kiddis.
Robertson, Alan.
Rogansky, Jean Dorothy.
Rosenbaum, Sidney.
Ross, William John.
Rowe, James Charles Lacy.
Royde, Chaim Alexander.

Sachs, David.
Saltmarsh, Joseph Arthur.
Sandall, Ray StClair.
Scheffé, Henry.
Schiller, Paul.
Sears, Geoffrey Wadsworth.
Seddon, Patrick Walsh.
Sen, Amarendra Nath.
Senior, Albert.
Sichel, Herbert Simon.
Smith, Norman Llewellyn.
Smith, Walter Laws.
Somasundarum, Samuel Jeyasingam.
Srivastava, Narain Prakash.
Srivastava, Rama Singh.
Stebbing, Charles Arthur.
Stephen, Margaret Rennie Duguid.
Stevenson, Alan Carruth.
Stonell, Anthony Charles.
Stott, Kenneth Ainsworth.
Stuvel, Gerhard.

Tanner, John Curnow.
Thomas, Gordon Glyn.
Thompson, John Francis.
Thornley, Ernest Rayner.
Thornton, Edward Harold.
Tunnell, George Alfred.
Twinn, Peter Frank George.

Umarji, Raghavendra Ramacharya.

Vane, Walter Brian.

Walters, John.
Weaver, Denis.

Webster, Leslie Edward.
Weightman, James Stanley.
Weinstein, Marcus Emanuel.
Westgarth, David Robson.
Wheeler, Frank Leslie Vicars.
Whillock, Edward Victor.
Whitell, Joseph Douglas.
Whiteside, George Denby.
Wijesekera, Nandadeva.
Wilcox, Douglas George.
Williams, John Laurence.
Williams, Kennedy.
Wilson, Kenneth James.
Wood, Sam.
Woolger, Leonard Percy.
Wray, George.
Wright, Conrad Payling.

Corporate Representatives

Clapham, John Charles Richard, *representing* The British Boot, Shoe & Allied Trades Research Association.
Cheyney, Leonard Frank, *representing* The Institute of Municipal Treasurers and Accountants (Inc.).
Deneen, William, *representing* The National Coal Board (S.W. Division).
Goodenough, Sir William, *representing* Barclays Bank, Ltd.
Little, Leo Thomas Frank, *representing* The Incorporated Accountants Research Committee.
Newcombe, Benjamin G. C., *representing* The Bowater Paper Corporation, Ltd.
Roberts, Eric Varcoe, *representing* Leeds College of Commerce.
Thomas, Mark Hartland, *representing* The Council of Industrial Design.

APPEN

STATEMENT OF INCOME AND EXPENDITURE

EXPENDITURE

1947 £	1947 £		1948 £	1948 £
	380	Rent		380
	317	House expenses		341
	1,432	Salaries and wages (including contribution to staff superannuation scheme)		1,733
	239	Pension and allowance		239
	15	Insurance		29
	117	Office equipment		36
	128	Postage and telephone		190
	552	Stationery and miscellaneous printing		672
		Publication and distribution expenses:		
2,034		Journal, Series A (General) and reprints	2,013	
1,702		Journal, Series B (Methodological) and reprints	908	
——	*3,736*		——	2,921
		Library:		
109		Books	85	
50		Binding	76	
——	*159*		——	161
	186	General Meetings—ordinary and annual		200
	42	Council and committee travelling expenses		34
		Expenses of sections:		
63		Research	52	
229		Industrial applications	90	
44		Study	14	
——	*336*		——	156
		Examination expenses:		
41		Printing, stationery and postage	45	
31		Examiners' fees	31	
——	*72*		——	76
	37	Auditor's fee (1947)		37
	20	Miscellaneous expenses		29
	250	*Expenses of Society's representative attending meetings of International Statistical Institute in U.S.A.*		—
	8,018			7,234
	—	Balance carried to Accumulated Fund, being excess of income over expenditure for the year 1948		572
	8,018			7,806
	735	Amount carried to Life Composition Fee Fund		429
	£8,753			£8,235

DIX B

FOR THE YEAR ENDED 31ST DECEMBER, 1948

INCOME

1947 £	1947 £		1948 £	1948 £
		Annual subscriptions:		
2,923		Fellowship	4,350	
1		Research Section	2	
114		Industrial Applications Section	96	
1		Study Section...	2	
——	*3,039*		——	4,450
	50	Contribution from Royal Economic Society		50
	565	Dividends and interest (gross)		578
		Sales of Journal:		
2,233		Journal, Series A (General) and reprints	2,135	
444		Journal, Series B (Methodological) and reprints	471	
——	*2,677*		——	2,606
	2	Sales of other publications		3
		Examination receipts:		
102		Fees	92	
—		Sale of papers	11	
——	*102*		——	103
	10	Special donations		1
	—	Miscellaneous receipts		15
	6,445			7,806
	1,573	*Balance carried to Accumulated Fund, being excess of expenditure over income for the year 1947*		—
	8,018			7,806
	735			429
	£8,753			£8,235

APPEN

BALANCE SHEET AT

FUNDS AND LIABILITIES

1947 £	£		1948 £	£
		Accumulated Fund:		
7,654		Balance at December 31st, 1947	6,186	
105		*Add:* Amount transferred from Life Composition Fee Fund ...	105	
—		Excess of income over expenditure for the year per annexed account	572	
7,759			6,863	
1,573	*6,186*	*Less: Excess of expenditure over income for the year 1947* ...	—	6,863
		Life Composition Fee Fund:		
5,818		Balance at December 31st, 1947	6,448	
735		*Add:* Life Composition Fees received during year	429	
6,553			6,877	
105	*6,448*	*Less:* Contributions of compounders who died during year transferred to Accumulated Fund	105	6,772
		Building Fund (per contra):		
1,149		Balance at December 31st, 1947	1,192	
43	*1,192*	*Add:* Income for year	45	1,237
		Liabilities and Income held in suspense:		
2,139		Sundry Creditors	2,023	
		Amounts received in advance:		
360		Annual subscriptions	225	
870		Payments for Journals	1,155	
8	*3,377*	Examination fees	68	3,471
	17,203			18,343
		Frances Wood Memorial Fund (per contra):		
542		Balance at December 31st, 1947	512	
30		*Add:* Income for year	30	
572			542	
60	*512*	*Less: Prizes awarded in* 1947	—	542
	£17,715			£18,885

DAVID HERON, *President.*

J. H. JONES, *Honorary Treasurer.*

DIX C

31ST DECEMBER, 1948

ASSETS

1947			1948	
£	£	*Investments* at cost or under:	£	£
5,580		£10,527 12*s*. 3*d*. 2½% Consols (Guy Bequest)	5,580	
1,185		£2,236 11*s*. 3*d*. 2½% Consols	1,185	
1,324		£1,864 14*s*. 1*d*. 3½% Conversion Loan	1,324	
490		£500 3½% War Loan	490	
2,000		£2,970 1*s*. 10*d*. 3% Savings Bonds 1965–75	2,992	
2,000		£2,000 3% Defence Bonds	2,000	
100		*£666 4% 2nd Prefd. Stock, L. & N.E. Railway*	—	
—		£194 16*s*. 1*d*. 3% British Transport Guaranteed Stock 1978–88	100	
12,679			13,671	
		(Market Value December 31st, 1948, *less* interest accrued, £17,893.)		
		Investment (*Building Fund*):		
1,192		£1,305 11*s*. 3*d*. 3½% Conversion Loan	1,237	
		(Market Value December 31st, 1948, £1,390.)		
	13,871			14,908
		Current Assets:		
466		Debtors and amounts paid in advance	600	
84		Arrears of subscriptions recoverable (estimated)	84	
141		Interest accrued on investments (gross)	155	
2,641		Cash at banks and in hand	2,596	
	3,332			3,435
	17,203			18,343
		Frances Wood Memorial Fund:		
300		*£600 4% Prefce. Stock, L.M. & S. Railway*	—	
—		£512 12*s*. 6*d* 3% British Transport Guaranteed Stock 1978–88	300	
		(Market Value December 31st, 1948, £515.)		
189		Post Office Savings Bank Deposit	228	
23		Income tax refund due	14	
	512			542
	£17,715			£18,885

NOTE.—No value is placed in the Accounts on—

(1) Journals and other publications in stock.
(2) Books in library.
(3) Pictures, furniture and equipment.

REPORT OF THE AUDITOR

I have obtained all the information and explanations which to the best of my knowledge and belief were necessary for the purposes of my audit. In my opinion proper books of account have been kept by the Society so far as appears from my examination of those books.

I have examined the above balance sheet and annexed statement of income and expenditure which are in agreement with the books of account. In my opinion and to the best of my information and according to the explanations given me the Balance Sheet gives a true and fair view of the state of the Society's affairs as at 31st December, 1948, and the statement of income and expenditure gives a true and fair view of the Income and Expenditure for the year ended on that date.

5, London Wall Buildings,
E.C.2;
31*st*, *May*, 1949.

A. RAE SMITH, Auditor.
Chartered Accountant.

PROCEEDINGS OF THE ONE HUNDRED AND FIFTEENTH ANNUAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY, HELD AT THE LONDON SCHOOL OF HYGIENE AND TROPICAL MEDICINE, ON WEDNESDAY, JUNE 29TH, 1949.

The Chair was taken by the PRESIDENT, Dr. DAVID HERON.

The PRESIDENT announced that as no alternative nominations had been received, the President, Council, and Officers for the session 1949–50, nominated as shown on the list already circulated, were duly elected.

On the proposal of Mr. R. F. George, seconded by Sir WILLIAM ELDERTON, Sir Alan Rae Smith was re-elected Auditor of the Society for the session 1949–50 at a fee of thirty-five guineas.

The PRESIDENT presented the Report of the Council for the financial year 1948 and the session 1948–49, and moved that it be adopted.

During his submission of the Report he presented a Guy Medal in Bronze to Mr. W. J. Jennett and Frances Wood Memorial Prizes to Mr. J. L. Nicholson and Mr. L. T. Wilkins. He also presented Diplomas and Certificates to those successful candidates in the 1949 Diploma examination and the 1948 and 1949 Certificate examinations who were able to be present at the meeting.

The HONORARY TREASURER, Professor J. H. Jones, presented the accounts and seconded the motion for the adoption of the Report.

After discussion of the various items the Report was unanimously adopted.

The meeting then terminated.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Sampling Methods for Censuses and Surveys*. By F. Yates. London: Griffin, 1949. xiv + 318 pp. 9″. 24*s*.

It has been apparent for some time that the theory and practice of sampling were developing in scope and complication, but only on reading this book does one realize how highly developed they had become. All statisticians will welcome this synoptic treatment of an important branch of their subject—the first of its kind; and those who have to undertake censuses and surveys by sample will find it essential.

The book is offered to "those who have little or no previous training in mathematical statistics, but who have some training or experience in the presentation and handling of statistical data." Consequently the subject is not developed mathematically and no proofs are given; nevertheless the development is systematic and clear.

After some general remarks on sampling, random error and bias, there is a chapter describing the various types of sample used in survey work, starting with the random sample and proceeding to various kinds of samples in strata, systematic samples, and so on. Then there are two chapters on practical problems arising in planning and executing a survey, and analysing and presenting the results; these chapters, although they do not entirely avoid the platitude, are full of advice and worldly wisdom that is well worth presenting; for example:

> "Accurate information can only be obtained in such surveys [i.e. of the questionnaire type] if full and willing co-operation of those providing the information is obtained. The survey must therefore, have a clear purpose which can be explained to the respondents, and the questions asked must be relevant to this purpose. If additional questions dealing with unrelated subjects are included, or if the questions relating to the main enquiry seem trivial, and do not cover aspects which appear of importance to those providing the information, the survey will cease to appear as a serious enquiry into a particular subject, and will meet with unfavourable reactions, summed up in such terms as "snooping'."

So far the exposition is verbal and descriptive. There are no formulae in the first half of the book. The second half contains the formulae and instructions for estimating population values and sampling errors, and for evaluating the relative efficiencies and economies of different types of samples in various circumstances. The reader will be unlikely to make much of these chapters unless he is fairly familiar with at least the statistical theory of simple random sampling and with the analysis of variance.

The book deals ostensibly with application to agricultural, social and population surveys, but only the chapters on practical problems have this limitation, and the more general parts have a wide application, which, however, remains to be illustrated for other fields.

Although the book is comprehensive it has the merit of being relatively short. This is due partly to the wholly admirable disciplined and systematic presentation, and partly to a condensed style of writing. For example, stratification is introduced in the following terms:

> "In a stratified sample the population of sample units is subdivided into groups or 'strata' before selection of the sample. These strata may all contain the same number of units, or differing numbers of units. If a uniform sampling fraction is used, the same fraction of the units of each stratum is included in the sample, the units selected being chosen at random from all the units within each stratum. A stratified sample is thus equivalent to a set of random samples on a number of sub-populations, each equivalent to one stratum."

The book abounds in well described examples, fully worked out, but when essential steps are introduced in general terms, as in the passage quoted, concentrated attention is required in

reading. Punched cards and sorting machines, and their use, are described in eleven pages or so; and a statement of the theory of sampling errors occupying only a few pages is a *tour de force* of condensed and yet coherent exposition. These are useful as summaries to the reader who already knows about these matters, but it is doubtful if they will do much to inform the uninstructed reader. Such parts of the book require an entirely different degree of sophistication in the reader from that required in other parts. Dr. Yates states that he had to prepare the book in considerable haste, and he scarcely hopes that perfect balánce has been achieved; perhaps he had this kind of thing in mind when making this statement.

All this is intended as comment rather than criticism. In practice all readers do not need to understand all parts of the book with equal ease; everyone who has to do with sampling will find in it enough of clear exposition, example and instruction to carry him a very long way; and the statistician with a mathematical turn of mind will find enough of theoretical importance to interest him. L. H. C. T

2.—*Statistics and their Application to Commerce*. By A. Lester Boddington. 9th ed. London: H. F. L. (Publishers), Ltd., 1949. xvi + 359 pp. 8½". 18*s*.

This is one of the most readable and most disappointing books on elementary statistics. First published in 1921, it has now reached its ninth edition, which includes two chapters on index numbers written by Mr. A. J. H. Morrell in place of the very unsatisfactory chapter in previous editions.

The book is intended for business men. It covers the various averages used in statistical work, the measurement of dispersion and skewness, correlation, methods of presentation and index numbers. Whether this is all the business man wants to know is debatable; it is certainly not all that the business statistician needs to learn. Methods for analysing time series and the application of sampling to market research and quality control are not dealt with at all.

There are two main defects in this work: first, that much of what is covered is too sketchy to be of any value in practical work, and secondly, that it is out-of-date. In the chapter on measures of dispersion, for example, the standard deviation is shown without relating it anywhere in the book to the normal curve, yet in dealing with the measurement of error in the correlation coefficient the author gives the formula for the probable error, which was dropped long ago by statisticians. This sort of treatment is dangerous for students, who will learn by rote only, whereas they should be led to correlation through a study of regression and be taught measures of dispersion in terms of the normal curve. And it can only puzzle the business man, who may justifiably ask why anybody should want to square all the deviations from the average before adding them up when the average deviation is quite clear. For some reason the author also gives the formulae for the third moment and the modulus.

The normal curve is discussed in the chapter on "The Graphic Method" and the result is unfortunate. The normal curve is primarily of interest for its mathematical properties, and although this type of diagram should certainly be described, it would have been better, in the reviewer's opinion, to have done so in terms of the functional relationship of two variables with the normal curve as an important probability distribution. The examples given coúld be improved; they are not assymptotic to the abscissa and are all flat-topped (platykurtic).

The two chapters on index numbers written by Mr. Morrell cover the subject fairly adequately. The first deals with the general question of construction, and provides a better introduction than will be found in most British elementary text-books on statistics. Objection may be raised, however, to the order of the sections. In discussing the various methods of averaging the author introduces the concept of weighting without making it clear what the term really means until later in the chapter. There is much to be said for discussing the aggregative and weighted relatives methods together, for in this way the student gets a clearer idea of the meaning of the resultant index number. The geometric average can then be dealt with as an alternative, leading to a discussion of different bases, the time reversal and factor reversal tests, and Fisher's ideal index number. The outcome of the order in this chapter is that factor reversal is not discussed, and the reader is left wondering why the author, writing about Fisher's index, states that "theoretically it is almost perfect." The two sections on "The Chain Base Method" and "Changing the Base Year" could appropriately have been taken together, for a chain base is really only a method of splicing index numbers, whereas two other sections on weighting and the aggregative method come between them.

The other chapter summarizes very well sixteen well-known index numbers and will be welcome to all students. Mr. Morrell's two chapters are available separately under the title, *An Introduction to Index Numbers* (H. F. L. (Publishers) Ltd., 4*s*. 6*d*.). Apart from these two chapters, most of the remainder of the book should be thoroughly revised and expanded. F. A. F.

3.—*Die Geschichte der Lage der Arbeiter in Deutschland von* 1800 *bis in die Gegenwart.* Band II, 1933 bis 1948, 2nd ed. By J. Kuczynski. Berlin: Die Freie Gewerkschaft, 1948. 8″ × 5½″. 308 pp.

Those who are acquainted with the author's previous books, published in English, e.g. *Hunger and Work,* and *The Condition of the Workers in Great Britain, Germany and Soviet Union* 1932–1938, will know what to expect in taking up the author's latest publication, published in the Soviet Sector of Berlin. Mr. Kuczynski does not hesitate to lash out against what he describes as "German Fascism." The title of the present book is misleading: it is not primarily a history of German labour conditions from 1933 to 1948. The first third of the book is taken up with a description and violent denunciation of the "structure and general economics of German Fascism." This is followed by a statistical survey of 40 pages on the situation of the German worker from 1933 to 1937, much of which has been given in the author's previous books. There follows a chapter called "Lebensgeschichte des deutschen Arbeiter 1933 bis 1937," but which consists largely of a series of selected extracts from the German labour inspectors' reports for the year 1935 and the year 1936. A short chapter of about 40 pages on conditions from 1938 to May, 1945, for which statistical data are much less complete, is followed by another of equal length, specially added to this second edition, on conditions in the Western zones of Germany from 1945 to 1948. This chapter is necessarily somewhat scrappy and, in fact, except for one figure on p. 265, stops at 1947. Moreover, it is limited to the British and U.S. zones only. It is significant that no information is given on conditions in the Soviet zone of Germany. A large part of the book is therefore not devoted to the history of German labour conditions from 1933 to 1948, and the addition of a short chapter on conditions in the British and U.S. zones in 1945 to 1947 hardly justifies the change in title in this second edition to a history of German labour conditions "from 1933 to 1948."

The relevant chapters of the book are hardly an objective account of the situation of the German worker. While most of the statistics are drawn from official publications, and the sources are usually indicated, the author does not hesitate to make estimates of his own when necessary (not always fully explaining them), to attack the statistics as misleading and to interpret them so as to support his own thesis. No one can object to his attacking official statistics, if full and proper arguments are given. Does not the Oxford University Institute of Statistics publish side by side two cost-of-living index numbers, one headed "official," i.e. calculated by the Ministry of Labour, and the other headed "actual," i.e. compiled by the Institute? But the author in this book goes much further, and interprets the statistics sometimes incorrectly. One example must be given in detail. On p. 132 he quotes, correctly, the official statistics of the average gross earnings of the German worker in 1937, viz. RM. 26·50 per week, and estimates the net earnings (also correctly) after deduction of taxes. social insurance, etc., at about RM. 22·50. He then adds that "the official expenditure budget on which the cost of living index is based amounts to about RM. 40 per week" and draws the following conclusion, which he prints in italics: "This means that, even according to the official fascist statistics, it would have been necessary to raise wages by about 80 per cent. in order to make possible the standard of life recognized as normal by the Fascists." The first figure of RM. 22·50 on which this estimate of 80 per cent. is based is a general average wage *per head* (men, women and children, town and country), and excludes certain deductions from wages; the second figure of RM. 40 is the actual average weekly *family* expenditure of those, mostly urban, families who kept accounts in the family budget inquiry of 1937, and includes all expenditures. Further comment is superfluous. On p. 130 the author confuses an index of prices with an index of expenditure, in order to prove that prices have risen much more than the official index shows. J. W. N.

CURRENT NOTES

The Library of the Society has recently been enriched by a gift of great personal interest and historical value. This is the manuscript of Professor W. S. Jevons's paper on the Study of Periodic Commercial Fluctuations, read to Section F of the British Association in 1862 and reprinted with some alterations in the *Statistical Journal*, June, 1866. The manuscript itself is a delight to examine, with its legible handwriting and well-arranged tables. With it is a series of charts and diagrams, some dealing with financial data, some with prices and some with meteorological measurements; all are meticulously plotted with a needle on fine drawing-paper, then coloured and annotated.

Here we have concrete evidence of the enormous labour and diligence a pioneer in the early days of statistical analysis had to exercise in applying even simple methods; all calculations had to be made and checked without the aid of machines, all graphs produced by hand upon unlined paper. Study in these conditions was arduous, and Professor Jevons was evidently both indomitable and extraordinarily patient. Statisticians do not need to be told that his investigations established principles and methods the development of which has even now not been completed. They could not handle these early examples of his workmanship without an increased admiration for the man himself. The draughtsmanship of the charts is almost professional, and the lithographer for whom he prepared them should have had a straightforward task. In line, colour and lettering they remind one of William Blake, and their perfection of detail would be a credit to a mediaeval illuminator. His daughter may be assured that her gift will have an honoured place among the Society's treasures. We are most grateful to her for this addition to our collection.

The Trustees of the Houblon-Norman Fund invite applications for Fellowships or Grants in aid of research into the working of financial and business institutions in Great Britain and elsewhere and the economic conditions affecting them.

Apart from exceptional cases, awards will be confined to British-born subjects normally resident in the United Kingdom and will take effect from October 1st, 1950. Fellowships will be awarded for one year, renewable for a second year; and the amount of all awards will depend upon the circumstances of the candidate and the likely expenses of his work.

Applications should be made not later than March 31st, and forms of application may be obtained from the Secretary of the Fund (H. C. B. Mynors), c/o the Bank of England, London, E.C.2.

Fellows will have observed with pleasure the following inclusions in the Honours List of January, 1950:

Knight: Professor A. L. Bowley, C.B.E., Sc.D., D.Litt.; R. E. Yeabsley, C.B.E., F.C.A., F.S.A.A.

C.M.G.: A. K. Cairncross, Ph.D.

M.B.E.: J. Bamber.

OBITUARY

MAJOR GREENWOOD

In Major Greenwood, professor emeritus of epidemiology and vital statistics in the University of London, who died suddenly on October 5th, 1949, while attending a cancer research meeting in London, the Society has lost one of its most distinguished Fellows, and one who for nearly forty years worked continuously to serve and advance its interests. Born in 1880, the son of a general practitioner in north-east London, he was educated, on the classical side, at Merchant Taylors' School (then situated in the City). From there he passed to the London Hospital Medical College, where on qualifying M.R.C.S., L.R.C.P. in 1904 he was appointed demonstrator in physiology under Leonard Hill. Sir Leonard writes: "I was working on divers' sickness at the time and he joined in the experiments. We did a lot on dogs and then went into the chamber ourselves. He reached a pressure of 208 feet of water, a record then, and had a "bend" in the forearm afterwards for a short while. I was also editing a second volume of *Recent Advances in Physiology*, and asked him to deal with the Special Senses, which he did very well. By recognizing the ability of a student with nothing behind him to show his worth and appointing him my assistant I may claim to have started Greenwood upon his career."

Meanwhile, with a flair for mathematical reasoning and an early realization of the need for introducing measurement into medicine, he was drawn to University College to study statistics under Karl Pearson. "Some have thought," wrote one of his closest friends, "that he selected statistics as his life's work because that branch of medicine was most remote from the emotional. Perhaps they were right; he was a sensitive soul and often found refuge in doing sums, though the human meaning might still intrude." However that may be, it is certainly likely that he would have had little success in clinical medicine, and that he realized it, for he made no attempt to practise. In 1910 he left the London Hospital to take up the post of statistician at the Lister Institute of Preventive Medicine, and at this time his long association with the Society was begun. Elected a Fellow in 1909 (nominated by Mr. Udny Yule and Sir Arthur Newsholme) he read the first of his many papers in 1911—a joint study with R. H. Candy of hospital mortality rates following fractures and lobar pneumonia from 1751 to 1901. This was followed in 1913 by a piece of collaborative work with Udny Yule on the determination of size of family, and of the distribution of characters in order of birth from samples taken through members of the sibships. Outside the Society he published vital statistical papers over a wide field—studies of infant mortality, marital infection in tuberculosis, and so on. His next contribution to the *Journal*—"Problems of Industrial Organization"—is a reflection of his change of work during the first World War. He had taken charge of a medical research subsection of the Ministry of Munitions, and was thus introduced to the world of industry with its war-time problems of optimum hours of work, special occupational hazards, the incidence of tuberculosis, the feeding of the work-people in munition factories and so on. Once more he collaborated with Mr. Udny Yule, this time in their well-known work on the frequencies of repeated accidents among munition workers—and the literature that has grown from this study of "accident-proneness" is enormous.

At the end of the war he left the Lister Institute to take charge of medical statistics in the newly created Ministry of Health—though in fact he never worked in Whitehall, but had his small department stationed at the Medical Research Council's institute at Hampstead, where he was in close contact with John Brownlee, the Council's statistician, and his old chief Leonard Hill. On Brownlee's death he took over the direction of the Council's statistical staff. He had also, since its inception after the war, been Chairman of its Statistical Committee, a body which has long worked to promote the use of statistics in medicine and their proper presentation in official publications. He became a close friend and adviser to Sir Walter Fletcher, the first Secretary to the Medical Research Council, and also to Sir George Newman, the first Chief Medical Officer to the Ministry of Health. He had, too, much to do with the development of international medical statistics under the League of Nations. His scholarship and deep reading in historical epidemiology are apparent in many Ministry of Health reports of these years. His main contribution to the *Journal* at this time was an essay in international comparison—a contrasting of the death-rates of Sweden and England and Wales.

In 1928 he became the first professor of epidemiology and vital statistics at the London School of Hygiene and Tropical Medicine, a post which he held until his retirement under the age limit

in 1945. Here his researches continued to range over an astonishingly wide field—pioneer and long-term investigations in experimental epidemiology with Professor W. W. C. Topley, excursions into psychology, both historical and modern, with Dr. May Smith and Professor Millais Culpin, vital statistical studies with other colleagues, and historical essays on persons and subject-matter which, with an insight into past characters, he had made all his own. At the same time he took a close interest in the work of the University, serving—and always serving actively—on the Senate and many of its committees and on Boards of Studies. He had become an Honorary Secretary of the Society in 1919, and served as such until his election as President in 1934. (He had served on the Council since 1912, and almost uninterruptedly to the time of his death.) His opening presidential address he devoted to "University Education; its Recent History and Function" and his valedictory address (for in those more spacious days two addresses in two years were expected of a President) dealt with "English Death Rates, Past, Present and Future." The Society had awarded him its Guy Medal in silver in 1924 and that in gold—a jealously guarded distinction—in 1945. He had become a Fellow of the Royal College of Physicians in 1924 and of the Royal Society in 1928, and a year previously had been awarded its Buchanan medal. Other distinctions included the Milroy Lectureship of the Royal College of Physicians in 1922, the Herter Lectureship in Baltimore in 1931 and membership of the International Statistical Institute.

It is not difficult to list these manifest marks of success, to note the growing recognition and the building up of a national and international reputation, to record the books and scientific papers that flowed from a ready and accomplished pen. It is not so easy to assess the character that lay behind, and it was undoubtedly complex. To some he may have seemed distant and unapproachable, to some cynical and censorious. He was indeed not a person whom it was easy to get to know well, but his aloofness was due mainly to shyness and a difficulty in unbending to which in later life was added a melancholy he avoided laying upon others. Although a brilliant talker, he probably found it easier to unburden himself in writing upon matters really close to his heart. He was certainly censorious of pomposity and pretentiousness, and could be a devastating critic of the illogical and the stupid. In the medical journals his letters, epigrammatic and satirical, were widely read with enjoyment or pain—according to one's position. He disliked "cheap" and uncritical scientific work, and in his own work would spare no pains. Many Fellows of the Society will have listened to his thoughtful and lucid speeches during discussions of papers read at meetings. Few will have realized the amount of trouble which had preceded that short speech—always carefully within its allotted time limit. He would have regarded it as an insult to his audience to come unprepared, although he could, in fact, speak extemporarily without much difficulty. Equally in his lectures, sometimes "put across" almost conversationally, only the initiated knew the writing and re-writing and the study that had preceded them. Both lectures and speeches were interspersed with apt quotations, which his knowledge of literature, both English and classical, and his phenomenal memory allowed him to insert almost at will.

At this near point of time it is probably not possible to assess adequately Greenwood's contribution to science. To a contemporary an enduring fame would appear to be based either upon his contributions to historical epidemiology and his unique studies of the pioneers of public health, social medicine and epidemiologists, or upon his collaborative work with Topley in the then utterly untouched field of experimental epidemiology—the attempt to learn the laws governing the rise and fall of community diseases in the laboratory. Perhaps it is wrong to attempt to separate these two approaches, for Greenwood himself brought them together harmoniously in his Herter Lectures. And in the future it may indeed seem that one of his greatest contributions, if not *the* greatest, lay merely in his outlook, in his statistical approach to medicine, then a new approach and one long regarded with suspicion. And he fought this fight continuously and honestly—for *logic*, for *accuracy*, for "*little sums.*" A. B. H.

I should like to avail myself of this opportunity of paying my tribute to the memory of Greenwood.

I was first brought into a sustained intimacy with him when, together, for a number of years, we were joint Honorary Secretaries of the Section of Epidemiology of the Royal Society of Medicine. Previously I had been struck by his extraordinary power of talking extemporarily on abstruse problems, and by his rare gift, of speaking quite spontaneously in statistical terms, fluently, and with that precision of statement which statistical knowledge and training confers.

Greenwood was a clear-sighted and original thinker, with the courage to accept frankly conclusions which, however unacceptable otherwise, accorded with the dictates of clear thinking. If, at times, he was ironical in the dry vein, scornful of shams and voluble in denunciation of pretentious, egotistical posings, these were the quick reactions of a nature that was kindly, gentle, humorous and honest-minded. In the circles enriched by his contributions to current thought—

a privilege to hear, an invitation to controvert—he will be missed, for their loss is real. I think that among those mentioned as helpful to him in his career, the late Sir Shirley Murphy, first M.O.H. of the County of London, should not be omitted. Murphy was early in spotting the abilities of the young medical statistician, and interest in his advancement was a natural sequence. Greenwood ever held him in affectionate and respectful remembrance.

My personal sense of loss is deep. I have long regarded Major Greenwood as an old friend, learned, kind-hearted, of penetrating vision, with the aloofness of a sensitive nature instinctively seeking in reticence refuge from the slings and arrows he was prone quickly to repel with the rapier-like thrusts of a tongue which could be sharp.

WILLIAM BUTLER.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Advancement of Science—

October 1949—World population and world food supplies: *Sir J. Russell.* The planning of land use: *L. D. Stamp.* The nature of the mind's "factors": *Sir G. Thomson.*

Annals of Eugenics—

October 1949—A further note on discriminatory analysis: *M. H. Quenouille.* The association of characters as a result of inbreeding and linkage: *J. B. S. Haldane.* Data relating to seven genetical characters in six endogamous groups in Bombay: *L. D. Sanghvi* and *V. R. Khanolkar.* The combination of blood groups in a sample of 250 people: *R. Sanger* and *R. R. Race.*

The Banker—

October 1949—South Africa's relations with the sterling area: *C. G. Tether.*

British Journal of Psychology (Statistical Section)—

November 1949—Simple structure: a critical examination: *H. A. Reyburn* and *M. J. Raath.* A note on factor invariance and the identification of factors: *R. B. Cattell.* The progressive matrices as applied to school children: *G. Keir.* The two-factor theory: *C. Burt.* The Reciprocity Principle as an aid to factor analysis: *J. Sandler.*

Economica—

November 1949—A possible intra-European payments scheme: *R. F. Kahn.* A geometrical representation of balance of payments policy: *J. E. Meade.* Investment in a monetary economy: *J. S. G. Wilson.* Three notes on "expectation in economics": *R. Turvey, J. de V. Graaf, W. J. Baumol* and *G. L. S. Shackle.* Capital exports, and investment in building in Britain and the U.S.A., 1856–1914: *E. W. Cooney.* The rubber industry: *F. Benham.* A note on the interpretation of index numbers: *I. M. D. Little.*

Economic Journal—

December 1949—On sticking to one's last: *D. H. Robertson.* Economics: yesterday and tomorrow: *Sir A. Gray.* Demand for commodities is *not* demand for labour: *H. G. Johnson.* The multiplier as matrix: *R. M. Goodwin.* A pioneer of national income estimates: *H. W. Arndt.*

Eugenics Review—

October 1949—The Royal Commission on Population: the demographic position: *Sir C. Arden-Close.* The Royal Commission on Population and the Society's aims: *C. P. Blacker.* The Royal Commission on Population and P E P: *R. Rumbold.* The Royal Commission's Report and the press: *C. P. Blacker.* Some biological considerations in social evolution: *H. J. Fleure.*

Faculty of Actuaries, Transactions—

Vol. XIX, Part I—Inaugural Address by the President, Andrew R. Davidson, F.I.A. Insurance against inflation: *G. H. Recknell.*

Journal of Ecology—

July 1949—Statistical test for optimal conditions: note on a paper of Emmett and Ashby: *G. H. Jowett* and *G. Scurfield.* A statistical investigation into the distribution of *Holcus mollis* L. and *Deschampsia flexuosa* (L.) Trin.: *G. H. Jowett* and *G. Scurfield.*

Lloyds Bank Review—

October 1949—The sterling problem: *L. Robbins.* Benelux: *The Editor.*

Manchester School of Economic and Social Studies—

September 1949—The effects of an overseas slump on the British economy: *F. V. Meyer* and *W. A. Lewis*. The theory of profit: *P. Streeten*. Haslingden: a problem in balance: *I. W. Scarf*. A note on the size of establishments in factory trades: *K. S. Lomax*.

Manchester Statistical Society, Transactions—

Session 1948–49—The utility cloth and clothing scheme: *H. E. Wadsworth*. The cost of a family: *A. Henderson*. Colonial development: *W. A. Leslie*. Accountants and the inflation: *W. T. Baxter*. The development and organization of Irish industry: *R. C. Geary*.

Mathematical Tables and other Aids to Computation—

April 1949—Hilbert's double series theorem and principal latent roots of the resulting matrix. Piecewise polynomial approximation for large-scale digital calculators. The accuracy of linear interpolation in tables of the mathematics of finance.

July 1949—The solution of simultaneous linear equations with the aid of the 602 calculating punch.

Oxford University Institute of Statistics, Bulletin—

October 1949—Quarterly estimates of the national income: *D. Seers*. Human needs diets from 1936 to 1949: *T. Schulz*.

November 1949—The industrial distribution of juvenile labour: *R. Godson*. Capital movements, the terms of trade and the balance of payments: *K. Martin*.

December 1949—Quarterly estimates of the national income: *D. Seers*. A "human needs" diet in November 1949: *T. Schulz*. Trends in cocoa production: *P. Ady*.

Population Studies—

September 1949—The cost of children. Part I: *A. Henderson*. Optimum population, foreign trade and world economy: *M. Gottlieb*. Practical problems in sampling for social and demographic inquiries in undeveloped countries: *Y. P. Seng*. Some features of a peasant population in the Middle East: *D. A. Percival*. Supplementary maternal and child health services. Part I. Post-natal care. Part II. Nurseries: *J. W. B. Douglas* and *G. Rowntree*.

Review of Economic Studies—

Vol. XVI (2), No. 40—A new approach to the problem of business cycles: *M. Kalecki*. Welfare and tariffs: *I. M. D. Little*. A statistical measurement of the preference of private banks: *K. Philip*. A note on the theory of investment: *T. Haavelmo*. Utility cloth and clothing scheme: *H. E. Wadsworth*.

AUSTRALIA—

Economic Record—

August 1949—Marketing and home consumption of Australian primary products. (Whole number.)

INDIA—

Calcutta Statistical Association, Bulletin—

October 1949—A critique of U.P. Anthropometric Survey: *H. K. Nandi*. Interpenetrating (networks of) samples: *B. Ghosh*. Some features of graduation: *A. C. Nag*.

UNION OF SOUTH AFRICA—

South African Journal of Economics—

September 1949—Some notes on price inflation in South Africa, 1938–1948 (Part I): *T. van Waasdijk*. The 1849 settlers in Natal (Part I): *H. M. Robertson*. State intervention in the regulation of wages and working conditions in Great Britain and South Africa: *G. G. C. Routh*. A short historical note on the marine policy: *R. S. Shackell*. Population policy in Great Britain (review article): *R. Ross*.

UNITED STATES—

American Academy of Political and Social Science, Annals—

November 1949—Government finance in a stable and growing economy. (Whole number.)

Annals of Mathematical Statistics—

September 1949—On the theory of systematic sampling. II: *W. G. Madow*. Problems in plane sampling: *M. H. Quenouille*. Representation of probability distributions by Charlier Series: *R. P. Boas, Jr.* Heuristic approach to the Kolmogorov-Smirnov theorems: *J. L. Doob*. Pearsonian correlation coefficients associated with least squares theory: *P. S. Dwyer*. Inversion formulas in normal variable mapping: *J. Riordan*. On the determination of optimum probabilities in sampling: *M. H. Hansen* and *W. N. Hurwitz*. A solution to the problem of optimum classification: *P. G. Hoel* and *R. P. Peterson*. A generalization of Wald's Fundamental Identity: *G. Blom*. Spread of minima of large samples: *B. McMillan*. On the convergence of the classical iterative method of solving: *E. Reich*. Some recurrence formulae in the incomplete beta function ratio: *T. A. Bancroft*. On a theorem by Wald and Wolfowitz: *G. E. Noether*. On sums of symmetrically truncated normal random variables: *Z. W. Birnbaum* and *F. C. Andrews*. A certain cumulative probability function: *Sister M. A. Hatke*.

Biometrics—

September 1949—Optimum allocation and variance components in nested sampling with an application to chemical analysis: *S. Marcuse*. Fitting a straight line when both variables are subject to error: *M. S. Bartlett*. Relationship of catch to changes in population size of New England haddock: *H. A. Schuck*. One degree of freedom for non-additivity: *J. W. Tukey*. On a statistical approximation to the infection interval: *J. B. Chassan*.

Econometrica—

July-October 1949—Programming of interdependent activities: I. General discussion: *M. K. Wood* and *G. B. Dantzig*. II. Mathematical model: *G. B. Dantzig*. Bayes and Minimax solutions of sequential decision problems: *K. J. Arrow, D. Blackwell* and *M. A. Girshick*. Some conditions of macro-economic stability: *D. Hawkins* and *H. A. Simon*.

Estadística—

March 1949—Los métodos de registro y estadística vital nacional en el Perú: *C. A. Uriarte*. Concepts of the working population: *L. E. Truesdell*. Sobre errores en las investigaciones: *W. Edwards Deming*. Opinion polls and sample surveys: *T. N. E. Greville*. Polling methods: *S. A. Rice*. Plan para un Punto Focal Nacional de Intercambio de Estadística Internacional—cont. Life tables for Panama, 1941–1943: *A. Vives*. Disposiciones legales sobre el Censo de 1950: El Salvador, Costa Rica.

Harvard Business Review—

November 1949—Minnesota Labor Relations Act—an opinion survey: *J. Stieber*. The meaning of scientific management: *E. H. Anderson*. Merit rating as a management tool: *R. E. Shaeffer*. Mutual investment funds: *W. D. Carter*. Forecasting sales: *T. G. Macgowan*. Devaluation of the pound sterling: *S. E. Harris*.

Journal of Political Economy—

October 1949—English economists and the State-managed economy: *J. A. Schumpeter*. The International Monetary Fund: *R. F. Mikesell*. Cost behavior and the accounting pattern of public utility regulation: *E. Troxel*. The income tax and the supply of labor: *R. Goode*.

Milbank Memorial Fund Quarterly—

October 1949—The statistical association between the diet record of ascorbic acid intake and the blood content of the vitamin in surveyed populations: *P. Putnam, D. F. Milam, R. K. Anderson, W. J. Darby* and *P. A. Mead*. Demographic characteristics of women in "Who's Who": *C. V. Kiser* and *N. L. Schacter*.

Quarterly Journal of Economics—

November 1949—Isolationism in economic method: *G. J. Schuller*. The general theory of location and space-economy: *W. Isard*. Engineering production functions: *H. B. Chenery*. The physiocrats' concept of economics: *T. P. Neill*.

Review of Economics and Statistics—

November 1949—Wesley Mitchell, social scientist and social counselor: *A. H. Hansen*. The moral of the British crisis: *Sir H. Henderson*. Capital movements and international payments in post-war Europe: *M. L. Hoffman*. Future foreign financing: *H. Mendershausen*. Comments on the Steel Report (three articles); Introductory remarks: *S. E. Harris*. Raising the price of labor as a method of increasing employment: *S. H. Slichter*. Comments of S. H. Slichter: *R. R. Nathan*. Productivity and wages (four articles); Introduction: *S. E. Harris*. Wages and productivity: *J. C. Davis* and *T. K. Hitch*. The short-run behavior of physical productivity and average hourly earnings: *C. Kerr*. Of productivity statistics—an admonition: *S. Fabricant*.

CZECHOSLOVAKIA—

Statistický Obzor—

Vol. XXIX, Part 3 (1949)—Naléhavé úkoly československých statistiků. *Urgent tasks of the Czechoslovak statisticians: F. Herbst*. Stav obyvatelstva a jeho vývoj v SSSR. *Population and movement of population in the USSR: E. Čapek*.

FRANCE—

Journal de la Société de Statistique de Paris—

September-October 1949—Une étude sociologique des populations noires de l'Oubangui á l'aide de tests anthropobiologiques et psychologiques: *J. Genevay*. Chronique de production industrielle: *R. Remery*. Note sur la répartition géographique de la production minérale: *F. Blondel*. La part des salaires dans le revenu national: *M. Torti*.

November-December 1949—Les progrès de la statistique en France depuis 20 ans: *R. Rivet*. Chronique des statistiques bancaires et des questions monétaires: *P. Cauboue*. Taux d'intérêt, période de production et répartition des facteurs primaires de production: *M. Allais*. Quelques aspects statistiques du concours général de Français aux États-Unis: *M.-L. Dufrénoy*.

Revue d'Économie Politique—

November-December 1948—La France économique en 1947 (2e partie). (Whole number.)

GERMANY—

Mitteilungsblatt für Mathematische Statistik—

Vol. I, Part 1—Die typischen Schlussweisen der mathematischen Statistik: *Prof. Münzner*. Über Wahrscheinlichkeitsnetze und Wahrscheinlichkeitstafeln: *P. Lorenz*. Die Grundprobleme der Stichprobenmethode: *O. Anderson*. Näherungsverfahren zur Bestimmung einiger höherer statistischer Masszahlen der statistischen Kollektive: *M. Prodan*.

Vol. I, Part 2—Die typischen Schlussweisen der mathematischen Statistik. 2. Teil: *H. Münzner*. Die Grundprobleme der Stichprobenmethode. 2. Teil: *O. Anderson*. Die χ^2-Methode in der medizinischen Forschung: *M.-P. Geppert*. Elementare Ausführungen zur Theorie und Technik des Stichprobenverfahrens: *H. Kellerer*. Die Quotientenmethode, eine Variante der "Variate Difference" Methode: *H. Strecker*. "Um Aufklärung wird gebeten" (Zum Problem des "Wärmetodes"): *O. Anderson*.

Weltwirtschaftliches Archiv—

Vol. 63, Part I—Internationale Entwicklungstendenzen auf dem Gebiete der Einkommenbesteuerung: *F. Neumark*. Economic planning in the Welfare State: *H. von Beckerath*.

JAPAN—

Memoirs of the Faculty of Science, Series A—

Vol. IV, No. 1 (1949)—The harmonic analysis of stationary stochastic processes: *G. Maruyama.*

SWEDEN—

Ekonomisk Tidskrift—

September 1949—Nationalinkomst, växelkirs och handelsbalans: *K-O. Fåxen* and *K. Savosnik.*

Skandinavisk Aktuarietidskrift—

Parts 1–2, 1948—On some discrete frequency functions: *P. Ottestad.* The probability of decrements from a population. A study in discrete random processes: *H. L. Seal.* Some properties of the ruin function in the collective theory of risk: *C. O. Segerdahl.* On the theory of testing serial correlation: *T. W. Anderson.*

Parts 3–4, 1948—Über die Konvergenz des Iterationsverfahrens bei der Berechnung des effektiven Zinsfusses: *E. Kivikoski.* On divided differences and osculatory interpolation: *W. Simonsen.* A generalization of the collective theory of risk in regard to fluctuating basic-probabilities: *H. Ammeter.* On the probability of ruin in the collective risk theory for insurance enterprises with only negative risk sums: *T. Saxen.* On stationary point processes and Markov chains: *H. Wold.*

Parts 1–2, 1949—Sur les probabilités d'arrivée des évènements en nombre infini: *E. Franckx.* On the "birth-and-death" process: *N. Arley.* On the number of positive sums of random variables: *E. S. Andersen.* On the central limit theorem in the case of not equally distributed random variables: *H. Bergström.* Factorial moments and enumerating distributions: *A. Wintner.*

SWITZERLAND—

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker—

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